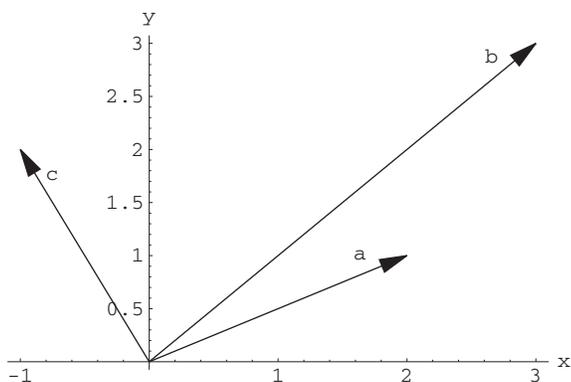


Chapter 1

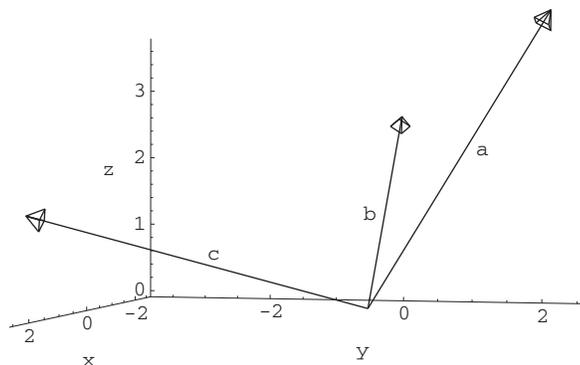
Vectors

1.1 Vectors in Two and Three Dimensions

1. Here we just connect the point $(0, 0)$ to the points indicated:



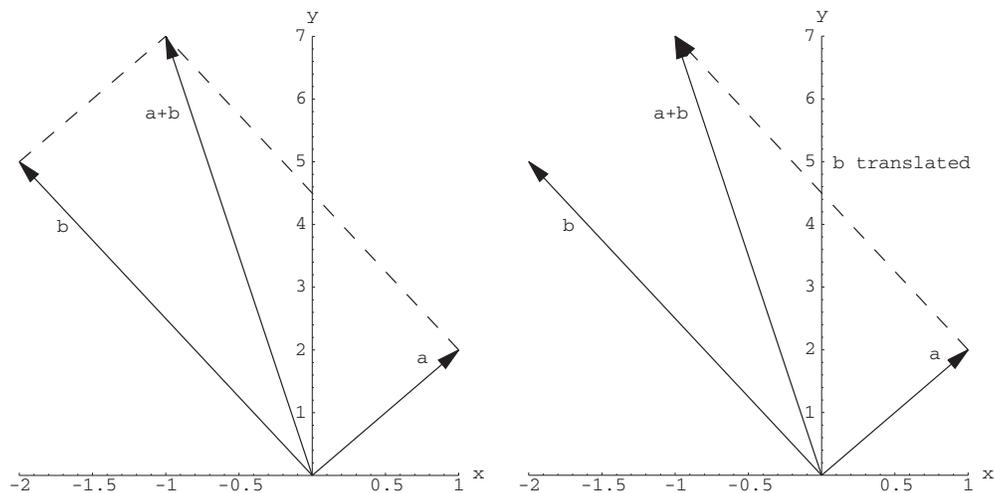
2. Although more difficult for students to represent this on paper, the figures should look something like the following. Note that the origin is not at a corner of the frame box but is at the tails of the three vectors.



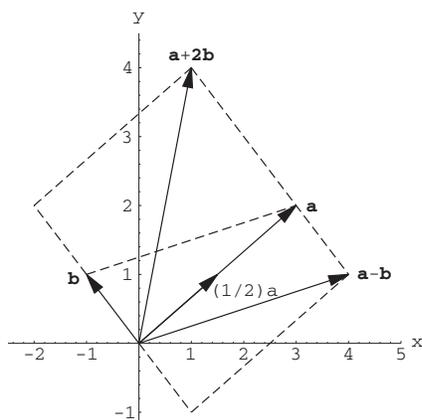
In problems 3 and 4, we supply more detail than is necessary to stress to students what properties are being used:

3. (a) $(3, 1) + (-1, 7) = (3 + [-1], 1 + 7) = (2, 8)$.
 (b) $-2(8, 12) = (-2 \cdot 8, -2 \cdot 12) = (-16, -24)$.
 (c) $(8, 9) + 3(-1, 2) = (8 + 3(-1), 9 + 3(2)) = (5, 15)$.
 (d) $(1, 1) + 5(2, 6) - 3(10, 2) = (1 + 5 \cdot 2 - 3 \cdot 10, 1 + 5 \cdot 6 - 3 \cdot 2) = (-19, 25)$.
 (e) $(8, 10) + 3((8, -2) - 2(4, 5)) = (8 + 3(8 - 2 \cdot 4), 10 + 3(-2 - 2 \cdot 5)) = (8, -26)$.
4. (a) $(2, 1, 2) + (-3, 9, 7) = (2 - 3, 1 + 9, 2 + 7) = (-1, 10, 9)$.
 (b) $\frac{1}{2}(8, 4, 1) + 2(5, -7, \frac{1}{4}) = (4, 2, \frac{1}{2}) + (10, -14, \frac{1}{2}) = (14, -12, 1)$.
 (c) $-2((2, 0, 1) - 6(\frac{1}{2}, -4, 1)) = -2((2, 0, 1) - (3, -24, 6)) = -2(-1, 24, -5) = (2, -48, 10)$.
5. We start with the two vectors **a** and **b**. We can complete the parallelogram as in the figure on the left. The vector from the origin to this new vertex is the vector $\mathbf{a} + \mathbf{b}$. In the figure on the right we have translated vector **b** so that its tail is the head of vector **a**. The sum $\mathbf{a} + \mathbf{b}$ is the directed third side of this triangle.

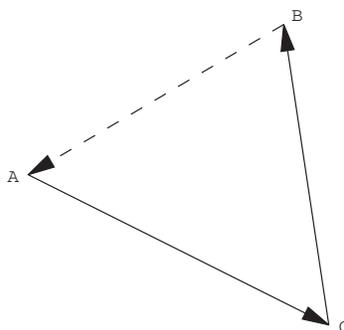
2 Chapter 1 Vectors



6. $a = (3, 2)$ $b = (-1, 1)$
 $a - b = (3 - (-1), 2 - 1) = (4, 1)$ $\frac{1}{2}a = (\frac{3}{2}, 1)$ $a + 2b = (1, 4)$



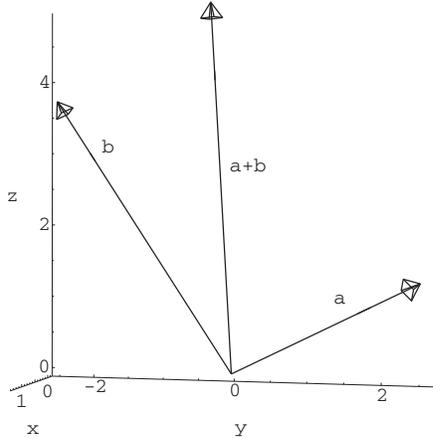
7. (a) $\vec{AB} = (-3 - 1, 3 - 0, 1 - 2) = (-4, 3, -1)$ $\vec{BA} = -\vec{AB} = (4, -3, 1)$
 (b) $\vec{AC} = (2 - 1, 1 - 0, 5 - 2) = (1, 1, 3)$
 $\vec{BC} = (2 - (-3), 1 - 3, 5 - 1) = (5, -2, 4)$
 $\vec{AC} + \vec{CB} = (1, 1, 3) - (5, -2, 4) = (-4, 3, -1)$
 (c) This result is true in general:



Head-to-tail addition demonstrates this.

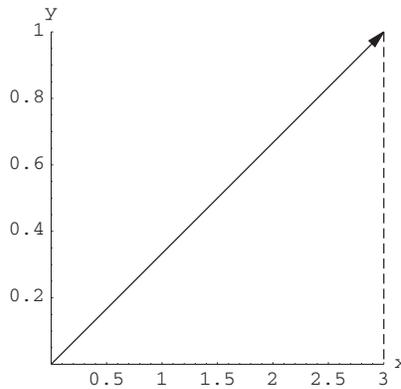
8. The vectors $\mathbf{a} = (1, 2, 1)$, $\mathbf{b} = (0, -2, 3)$ and $\mathbf{a} + \mathbf{b} = (1, 2, 1) + (0, -2, 3) = (1, 0, 4)$ are graphed below. *Again note that the origin is at the tails of the vectors in the figure.*

Also, $-1(1, 2, 1) = (-1, -2, -1)$. This would be pictured by drawing the vector $(1, 2, 1)$ in the opposite direction. Finally, $4(1, 2, 1) = (4, 8, 4)$ which is four times vector \mathbf{a} and so is vector \mathbf{a} stretched four times as long in the same direction.

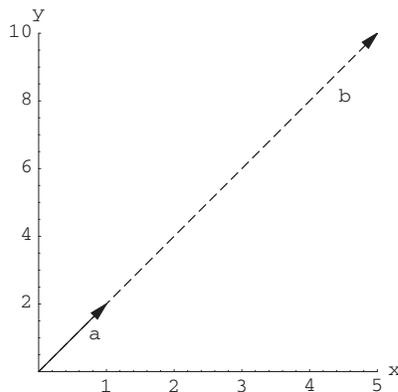


9. *Since the sum on the left must equal the vector on the right componentwise:*
 $-12 + x = 2$, $9 + 7 = y$, and $z - 3 = 5$. Therefore, $x = 14$, $y = 16$, and $z = 8$.

10. If we drop a perpendicular from $(3, 1)$ to the x -axis we see that by the Pythagorean Theorem the length of the vector $(3, 1) = \sqrt{3^2 + 1^2} = \sqrt{10}$.

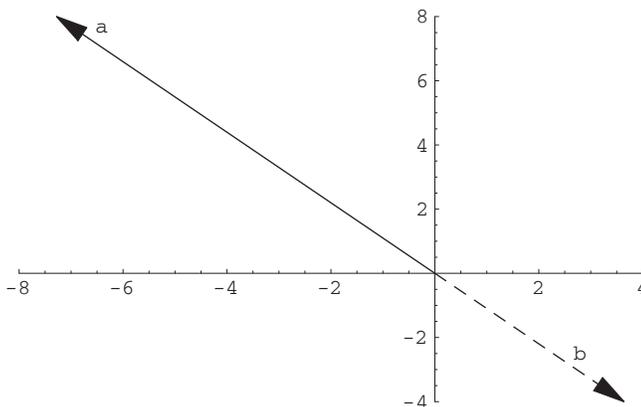


11. Notice that \mathbf{b} (represented by the dotted line) $= 5\mathbf{a}$ (represented by the solid line).



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12. Here the picture has been projected into two dimensions so that you can more clearly see that \mathbf{a} (represented by the solid line) $= -2\mathbf{b}$ (represented by the dotted line).



13. The natural extension to higher dimensions is that we still add componentwise and that multiplying a scalar by a vector means that we multiply each component of the vector by the scalar. In symbols this means that:

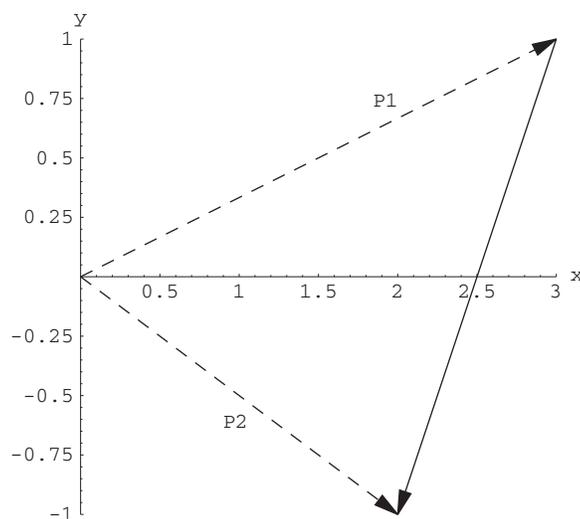
$$\mathbf{a} + \mathbf{b} = (a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n) = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n) \text{ and } k\mathbf{a} = (ka_1, ka_2, \dots, ka_n).$$

In our particular examples, $(1, 2, 3, 4) + (5, -1, 2, 0) = (6, 1, 5, 4)$, and $2(7, 6, -3, 1) = (14, 12, -6, 2)$.

14. The diagrams for parts (a), (b) and (c) are similar to Figure 1.12 from the text. The displacement vectors are:

- (a) $(1, 1, 5)$
- (b) $(-1, -2, 3)$
- (c) $(1, 2, -3)$
- (d) $(-1, -2)$

Note: The displacement vectors for (b) and (c) are the same but in opposite directions (i.e., one is the negative of the other). The displacement vector in the diagram for (d) is represented by the solid line in the figure below:

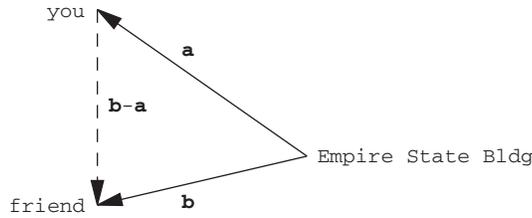


15. In general, we would define the displacement vector from (a_1, a_2, \dots, a_n) to (b_1, b_2, \dots, b_n) to be $(b_1 - a_1, b_2 - a_2, \dots, b_n - a_n)$.

In this specific problem the displacement vector from P_1 to P_2 is $(1, -4, -1, 1)$.

16. Let B have coordinates (x, y, z) . Then $\overrightarrow{AB} = (x - 2, y - 5, z + 6) = (12, -3, 7)$ so $x = 14, y = 2, z = 1$ so B has coordinates $(14, 2, 1)$.

17. If \mathbf{a} is your displacement vector from the Empire State Building and \mathbf{b} your friend's, then the displacement vector from you to your friend is $\mathbf{b} - \mathbf{a}$.



18. Property 2 follows immediately from the associative property of the reals:

$$\begin{aligned}
 (\mathbf{a} + \mathbf{b}) + \mathbf{c} &= ((a_1, a_2, a_3) + (b_1, b_2, b_3)) + (c_1, c_2, c_3) \\
 &= ((a_1 + b_1, a_2 + b_2, a_3 + b_3) + (c_1, c_2, c_3)) \\
 &= ((a_1 + b_1) + c_1, (a_2 + b_2) + c_2, (a_3 + b_3) + c_3) \\
 &= (a_1 + (b_1 + c_1), a_2 + (b_2 + c_2), a_3 + (b_3 + c_3)) \\
 &= (a_1, a_2, a_3) + ((b_1 + c_1), (b_2 + c_2), (b_3 + c_3)) \\
 &= \mathbf{a} + (\mathbf{b} + \mathbf{c}).
 \end{aligned}$$

Property 3 also follows from the corresponding componentwise observation:

$$\mathbf{a} + \mathbf{0} = (a_1 + 0, a_2 + 0, a_3 + 0) = (a_1, a_2, a_3) = \mathbf{a}.$$

19. We provide the proofs for \mathbf{R}^3 :

$$\begin{aligned}
 (1) \quad (k + l)\mathbf{a} &= (k + l)(a_1, a_2, a_3) = ((k + l)a_1, (k + l)a_2, (k + l)a_3) \\
 &= (ka_1 + la_1, ka_2 + la_2, ka_3 + la_3) = k\mathbf{a} + l\mathbf{a}. \\
 (2) \quad k(\mathbf{a} + \mathbf{b}) &= k((a_1, a_2, a_3) + (b_1, b_2, b_3)) = k(a_1 + b_1, a_2 + b_2, a_3 + b_3) \\
 &= (k(a_1 + b_1), k(a_2 + b_2), k(a_3 + b_3)) = (ka_1 + kb_1, ka_2 + kb_2, ka_3 + kb_3) \\
 &= (ka_1, ka_2, ka_3) + (kb_1, kb_2, kb_3) = k\mathbf{a} + k\mathbf{b}. \\
 (3) \quad k(l\mathbf{a}) &= k(l(a_1, a_2, a_3)) = k(la_1, la_2, la_3) \\
 &= (kla_1, kla_2, kla_3) = (lka_1, lka_2, lka_3) \\
 &= l(ka_1, ka_2, ka_3) = l(k\mathbf{a}).
 \end{aligned}$$

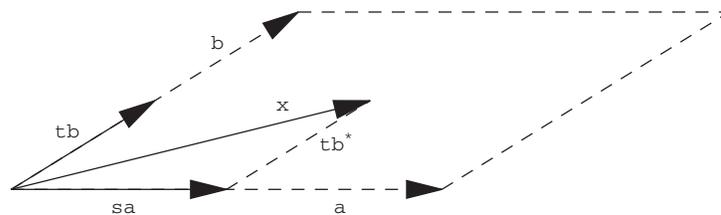
20. (a) $0\mathbf{a}$ is the zero vector. For example, in \mathbf{R}^3 :

$$0\mathbf{a} = 0(a_1, a_2, a_3) = (0 \cdot a_1, 0 \cdot a_2, 0 \cdot a_3) = (0, 0, 0).$$

(b) $1\mathbf{a} = \mathbf{a}$. Again in \mathbf{R}^3 :

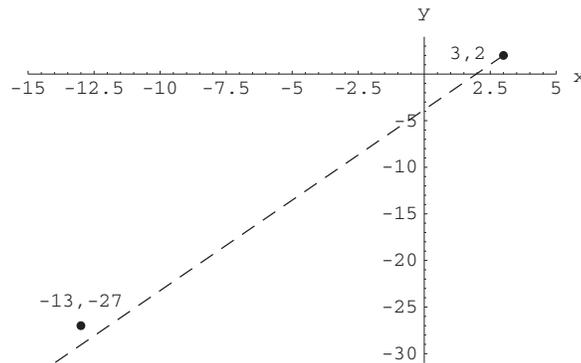
$$1\mathbf{a} = 1(a_1, a_2, a_3) = (1 \cdot a_1, 1 \cdot a_2, 1 \cdot a_3) = (a_1, a_2, a_3) = \mathbf{a}.$$

21. (a) The head of the vector $s\mathbf{a}$ is on the x -axis between 0 and 2. Similarly the head of the vector $t\mathbf{b}$ lies somewhere on the vector \mathbf{b} . Using the head-to-tail method, $s\mathbf{a} + t\mathbf{b}$ is the result of translating the vector $t\mathbf{b}$, in this case, to the right by $2s$ (represented in the figure by $t\mathbf{b}^*$). The result is clearly inside the parallelogram determined by \mathbf{a} and \mathbf{b} (and is only on the boundary of the parallelogram if either t or s is 0 or 1).



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- (b) Again the vectors \mathbf{a} and \mathbf{b} will determine a parallelogram (with vertices at the origin, and at the heads of \mathbf{a} , \mathbf{b} , and $\mathbf{a} + \mathbf{b}$). The vectors $s\mathbf{a} + t\mathbf{b}$ will be the position vectors for all points in that parallelogram determined by $(2, 2, 1)$ and $(0, 3, 2)$.
22. Here we are translating the situation in Exercise 21 by the vector $\overrightarrow{OP_0}$. The vectors will all be of the form $\overrightarrow{OP_0} + s\mathbf{a} + t\mathbf{b}$ for $0 \leq s, t \leq 1$.
23. (a) The speed of the flea is the length of the velocity vector $= \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$ units per minute.
 (b) After 3 minutes the flea is at $(3, 2) + 3(-1, -2) = (0, -4)$.
 (c) We solve $(3, 2) + t(-1, -2) = (-4, -12)$ for t and get that $t = 7$ minutes. Note that *both* $3 - 7 = -4$ and $2 - 14 = -12$.
 (d) We can see this algebraically or geometrically: Solving the x part of $(3, 2) + t(-1, -2) = (-13, -27)$ we get that $t = 16$. But when $t = 16$, $y = -30$ not -27 . Also in the figure below we see the path taken by the flea will miss the point $(-13, -27)$.



24. (a) The plane is climbing at a rate of 4 miles per hour.
 (b) To make sure that the axes are oriented so that the plane passes over the building, the positive x direction is east and the positive y direction is north. Then we are heading east at a rate of 50 miles per hour at the same time we're heading north at a rate of 100 miles per hour. We are directly over the skyscraper in $1/10$ of an hour or 6 minutes.
 (c) Using our answer in (b), we have traveled for $1/10$ of an hour and so we've climbed $4/10$ of a mile or 2112 feet. The plane is $2112 - 1250$ or 862 feet about the skyscraper.
25. (a) Adding we get: $\mathbf{F}_1 + \mathbf{F}_2 = (2, 7, -1) + (3, -2, 5) = (5, 5, 4)$.
 (b) You need a force of the same magnitude in the opposite direction, so $\mathbf{F}_3 = -(5, 5, 4) = (-5, -5, -4)$.
26. (a) Measuring the force in pounds we get $(0, 0, -50)$.
 (b) The z components of the two vectors along the ropes must be equal and their sum must be opposite of the z component in part (a). Their y components must also be opposite each other. Since the vector points in the direction $(0, \pm 2, 1)$, the y component will be twice the z component. Together this means that the vector in the direction of $(0, -2, 1)$ is $(0, -50, 25)$ and the vector in the direction $(0, 2, 1)$ is $(0, 50, 25)$.
27. The force \mathbf{F} due to gravity on the weight is given by $\mathbf{F} = (0, 0, -10)$. The forces along the ropes are each parallel to the displacement vectors from the weight to the respective anchor points. That is, the tension vectors along the ropes are

$$\mathbf{F}_1 = k((3, 0, 4) - (1, 2, 3)) = k(2, -2, 1)$$

$$\mathbf{F}_2 = l((0, 3, 5) - (1, 2, 3)) = l(-1, 1, 2),$$

where k and l are appropriate scalars. For the weight to remain in equilibrium, we must have $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F} = \mathbf{0}$, or, equivalently, that

$$k(2, -2, 1) + l(-1, 1, 2) + (0, 0, -10) = (0, 0, 0).$$

Taking components, we obtain a system of three equations:

$$\begin{cases} 2k - l = 0 \\ -2k + l = 0 \\ k + 2l = 10. \end{cases}$$

Solving, we find that $k = 2$ and $l = 4$, so that

$$\mathbf{F}_1 = (4, -4, 2) \text{ and } \mathbf{F}_2 = (-4, 4, 8).$$

1.2 More about Vectors

It may be useful to point out that the answers to Exercises 1 and 5 are the “same”, but that in Exercise 1, $\mathbf{i} = (1, 0)$ and in Exercise 5, $\mathbf{i} = (1, 0, 0)$. This comes up when going the other direction in Exercises 9 and 10. In other words, it’s not always clear whether the exercise “lives” in \mathbf{R}^2 or \mathbf{R}^3 .

1. $(2, 4) = 2(1, 0) + 4(0, 1) = 2\mathbf{i} + 4\mathbf{j}$.
2. $(9, -6) = 9(1, 0) - 6(0, 1) = 9\mathbf{i} - 6\mathbf{j}$.
3. $(3, \pi, -7) = 3(1, 0, 0) + \pi(0, 1, 0) - 7(0, 0, 1) = 3\mathbf{i} + \pi\mathbf{j} - 7\mathbf{k}$.
4. $(-1, 2, 5) = -1(1, 0, 0) + 2(0, 1, 0) + 5(0, 0, 1) = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$.
5. $(2, 4, 0) = 2(1, 0, 0) + 4(0, 1, 0) = 2\mathbf{i} + 4\mathbf{j}$.
6. $\mathbf{i} + \mathbf{j} - 3\mathbf{k} = (1, 0, 0) + (0, 1, 0) - 3(0, 0, 1) = (1, 1, -3)$.
7. $9\mathbf{i} - 2\mathbf{j} + \sqrt{2}\mathbf{k} = 9(1, 0, 0) - 2(0, 1, 0) + \sqrt{2}(0, 0, 1) = (9, -2, \sqrt{2})$.
8. $-3(2\mathbf{i} - 7\mathbf{k}) = -6\mathbf{i} + 21\mathbf{k} = -6(1, 0, 0) + 21(0, 0, 1) = (-6, 0, 21)$.
9. $\pi\mathbf{i} - \mathbf{j} = \pi(1, 0) - (0, 1) = (\pi, -1)$.
10. $\pi\mathbf{i} - \mathbf{j} = \pi(1, 0, 0) - (0, 1, 0) = (\pi, -1, 0)$.

Note: You may want to assign both Exercises 11 and 12 together so that the students may see the difference. You should stress that the reason the results are different has nothing to do with the fact that Exercise 11 is a question about \mathbf{R}^2 while Exercise 12 is a question about \mathbf{R}^3 .

11. (a) $(3, 1) = c_1(1, 1) + c_2(1, -1) = (c_1 + c_2, c_1 - c_2)$, so $\begin{cases} c_1 + c_2 = 3, \text{ and} \\ c_1 - c_2 = 1. \end{cases}$

Solving simultaneously (for instance by adding the two equations), we find that $2c_1 = 4$, so $c_1 = 2$ and $c_2 = 1$. So $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$.

- (b) Here $c_1 + c_2 = 3$ and $c_1 - c_2 = -5$, so $c_1 = -1$ and $c_2 = 4$. So $\mathbf{b} = -\mathbf{a}_1 + 4\mathbf{a}_2$.

- (c) More generally, $(b_1, b_2) = (c_1 + c_2, c_1 - c_2)$, so $\begin{cases} c_1 + c_2 = b_1, \text{ and} \\ c_1 - c_2 = b_2. \end{cases}$

Again solving simultaneously, $c_1 = \frac{b_1 + b_2}{2}$ and $c_2 = \frac{b_1 - b_2}{2}$. So

$$\mathbf{b} = \left(\frac{b_1 + b_2}{2}\right)\mathbf{a}_1 + \left(\frac{b_1 - b_2}{2}\right)\mathbf{a}_2.$$

12. Note that $\mathbf{a}_3 = \mathbf{a}_1 + \mathbf{a}_2$, so really we are only working with two (linearly independent) vectors.

- (a) $(5, 6, -5) = c_1(1, 0, -1) + c_2(0, 1, 0) + c_3(1, 1, -1)$; this gives us the equations:

$$\begin{cases} 5 = c_1 + c_3 \\ 6 = c_2 + c_3 \\ -5 = -c_1 - c_3. \end{cases}$$

The first and last equations contain the same information and so we have infinitely many solutions. You will quickly see one by letting $c_3 = 0$. Then $c_1 = 5$ and $c_2 = 6$. So we could write $\mathbf{b} = 5\mathbf{a}_1 + 6\mathbf{a}_2$. More generally, you can choose any value for c_1 and then let $c_2 = c_1 + 1$ and $c_3 = 5 - c_1$.

- (b) We cannot write $(2, 3, 4)$ as a linear combination of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 . Here we get the equations:

$$\begin{cases} c_1 + c_3 = 2 \\ c_2 + c_3 = 3 \\ -c_1 - c_3 = 4. \end{cases}$$

The first and last equations are inconsistent and so the system cannot be solved.

- (c) As we saw in part (b), not all vectors in \mathbf{R}^3 can be written in terms of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 . In fact, only vectors of the form $(a, b, -a)$ can be written in terms of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 . For your students who have had linear algebra, this is because the vectors \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 are not linearly independent.

Note: As pointed out in the text, the answers for 13–21 are not unique.

13. $\mathbf{r}(t) = (2, -1, 5) + t(1, 3, -6)$ so $\begin{cases} x = 2 + t \\ y = -1 + 3t \\ z = 5 - 6t. \end{cases}$

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14. $\mathbf{r}(t) = (12, -2, 0) + t(5, -12, 1)$ so $\begin{cases} x = 12 + 5t \\ y = -2 - 12t \\ z = t. \end{cases}$

15. $\mathbf{r}(t) = (2, -1) + t(1, -7)$ so $\begin{cases} x = 2 + t \\ y = -1 - 7t. \end{cases}$

16. $\mathbf{r}(t) = (2, 1, 2) + t(3 - 2, -1 - 1, 5 - 2)$ so $\begin{cases} x = 2 + t \\ y = 1 - 2t \\ z = 2 + 3t. \end{cases}$

17. $\mathbf{r}(t) = (1, 4, 5) + t(2 - 1, 4 - 4, -1 - 5)$ so $\begin{cases} x = 1 + t \\ y = 4 \\ z = 5 - 6t. \end{cases}$

18. $\mathbf{r}(t) = (8, 5) + t(1 - 8, 7 - 5)$ so $\begin{cases} x = 8 - 7t \\ y = 5 + 2t. \end{cases}$

Note: In higher dimensions, we switch our notation to x_i .

19. $\mathbf{r}(t) = (1, 2, 0, 4) + t(-2, 5, 3, 7)$ so $\begin{cases} x_1 = 1 - 2t \\ x_2 = 2 + 5t \\ x_3 = 3t \\ x_4 = 4 + 7t. \end{cases}$

20. $\mathbf{r}(t) = (9, \pi, -1, 5, 2) + t(-1 - 9, 1 - \pi, \sqrt{2} + 1, 7 - 5, 1 - 2)$ so $\begin{cases} x_1 = 9 - 10t \\ x_2 = \pi + (1 - \pi)t \\ x_3 = -1 + (\sqrt{2} + 1)t \\ x_4 = 5 + 2t \\ x_5 = 2 - t. \end{cases}$

21. (a) $\mathbf{r}(t) = (-1, 7, 3) + t(2, -1, 5)$ so $\begin{cases} x = -1 + 2t \\ y = 7 - t \\ z = 3 + 5t. \end{cases}$

(b) $\mathbf{r}(t) = (5, -3, 4) + t(0 - 5, 1 + 3, 9 - 4)$ so $\begin{cases} x = 5 - 5t \\ y = -3 + 4t \\ z = 4 + 5t. \end{cases}$

(c) Of course, there are infinitely many solutions. For our variation on the answer to (a) we note that a line parallel to the vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ is also parallel to the vector $-(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$ so another set of equations for part (a) is:

$$\begin{cases} x = -1 - 2t \\ y = 7 + t \\ z = 3 - 5t. \end{cases}$$

For our variation on the answer to (b) we note that the line passes through both points so we can set up the equation with respect to the other point:

$$\begin{cases} x = -5t \\ y = 1 + 4t \\ z = 9 + 5t. \end{cases}$$

(d) The symmetric forms are:

$$\frac{x + 1}{2} = 7 - y = \frac{z - 3}{5} \quad (\text{for (a)})$$

$$\frac{5 - x}{5} = \frac{y + 3}{4} = \frac{z - 4}{5} \quad (\text{for (b)})$$

$$\frac{x + 1}{-2} = y - 7 = \frac{z - 3}{-5} \quad (\text{for the variation of (a)})$$

$$\frac{x}{-5} = \frac{y - 1}{4} = \frac{z - 9}{5} \quad (\text{for the variation of (b)})$$

22. Solve for t in each of the parametric equations. Thus

$$t = \frac{x-5}{-2}, t = \frac{y-1}{3}, t = \frac{z+4}{6}$$

and the symmetric form is

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z+4}{6}.$$

23. Solving for t in each of the parametric equations gives $t = x - 7$, $t = (y + 9)/3$, and $t = (z - 6)/(-8)$, so that the symmetric form is

$$\frac{x-7}{1} = \frac{y+9}{3} = \frac{z-6}{-8}.$$

24. Set each piece of the equation equal to t and solve:

$$\frac{x-2}{5} = t \Rightarrow x-2 = 5t \Rightarrow x = 2 + 5t$$

$$\frac{y-3}{-2} = t \Rightarrow y-3 = -2t \Rightarrow y = 3 - 2t$$

$$\frac{z+1}{4} = t \Rightarrow z+1 = 4t \Rightarrow z = -1 + 4t.$$

25. Let $t = (x+5)/3$. Then $x = 3t - 5$. In view of the symmetric form, we also have that $t = (y-1)/7$ and $t = (z+10)/(-2)$. Hence a set of parametric equations is $x = 3t - 5$, $y = 7t + 1$, and $z = -2t - 10$.

Note: In Exercises 26–29, we could say for certain that two lines are not the same if the vectors were not multiples of each other. In other words, it takes two pieces of information to specify a line. You either need two points, or a point and a direction (or in the case of \mathbf{R}^2 , equivalently, a slope).

26. The first line is parallel to the vector $\mathbf{a}_1 = (5, -3, 4)$, while the second is parallel to $\mathbf{a}_2 = (10, -5, 8)$. Since \mathbf{a}_1 and \mathbf{a}_2 are not parallel, the lines cannot be the same.
27. If we multiply each of the pieces in the second symmetric form by -2 , we are effectively just traversing the same path at a different speed and with the opposite orientation. So the second set of equations becomes:

$$\frac{x+1}{3} = \frac{y+6}{7} = \frac{z+5}{5}.$$

This looks a lot more like the first set of equations. If we now subtract one from each piece of the second set of equations (as suggested in the text), we are effectively just changing our initial point but we are still on the same line:

$$\frac{x+1}{3} - \frac{3}{3} = \frac{y+6}{7} - \frac{7}{7} = \frac{z+5}{5} - \frac{5}{5}.$$

We have transformed the second set of equations into the first and therefore see that they both represent the same line in \mathbf{R}^3 .

28. If you first write the equation of the two lines in vector form, we can see immediately that their direction vectors are the same so either they are parallel or they are the same line:

$$\mathbf{r}_1(t) = (-5, 2, 1) + t(2, 3, -6)$$

$$\mathbf{r}_2(t) = (1, 11, -17) - t(2, 3, -6).$$

The first line contains the point $(-5, 2, 1)$. If the second line contains $(-5, 2, 1)$, then the equations represent the same line. Solve just the x component to get that $-5 = 1 - 2t \Rightarrow t = 3$. Checking we see that $\mathbf{r}_2(3) = (1, 11, -17) - 3(2, 3, -6) = (-5, 2, 1)$ so the lines are the same.

29. Here again the vector forms of the two lines can be written so that we see their headings are the same:

$$\mathbf{r}_1(t) = (2, -7, 1) + t(3, 1, 5)$$

$$\mathbf{r}_2(t) = (-1, -8, -3) + 2t(3, 1, 5).$$

The point $(2, -7, 1)$ is on line one, so we will check to see if it is also on line two. As in Exercise 28 we check the equation for the x component and see that $-1 + 6t = 2 \Rightarrow t = 1/2$. Checking we see that $\mathbf{r}_2(1/2) = (-1, -8, -3) + (1/2)(2)(3, 1, 5) = (2, -7, 2) \neq (2, -7, 1)$ so the equations do not represent the same lines.

10 Chapter 1 Vectors

Note: It is a good idea to assign both Exercises 30 and 31 together. Although they look similar, there is a difference that students might miss.

30. If you make the substitution $u = t^3$, the equations become:
$$\begin{cases} x = 3u + 7, \\ y = -u + 2, \text{ and} \\ z = 5u + 1. \end{cases}$$

The map $u = t^3$ is a bijection. The important fact is that u takes on exactly the same values that t does, just at different times. Since u takes on all reals, the parametric equations do determine a line (it's just that the speed along the line is not constant).

31. This time if you make the substitution $u = t^2$, the equations become:
$$\begin{cases} x = 5u - 1, \\ y = 2u + 3, \text{ and} \\ z = -u + 1. \end{cases}$$

The problem is that u cannot take on negative values so these parametric equations are for a ray with endpoint $(-1, 3, 1)$ and heading $(5, 2, -1)$.

32. (a) The vector form of the equations is: $\mathbf{r}(t) = (7, -2, 1) + t(2, 1, -3)$. The initial point is then $\mathbf{r}(0) = (7, -2, 1)$, and after 3 minutes the bird is at $\mathbf{r}(3) = (7, -2, 1) + 3(2, 1, -3) = (13, 1, -8)$.
 (b) $(2, 1, -3)$
 (c) We only need to check one component (say the x): $7 + 2t = 34/3 \Rightarrow t = 13/6$. Checking we see that $\mathbf{r}(\frac{13}{6}) = (7, -2, 1) + (\frac{13}{6})(2, 1, -3) = (\frac{34}{3}, \frac{1}{6}, -\frac{11}{2})$.
 (d) As in part (c), we'll check the x component and see that $7 + 2t = 17$ when $t = 5$. We then check to see that $\mathbf{r}(5) = (7, -2, 1) + 5(2, 1, -3) = (17, 3, -14) \neq (17, 4, -14)$ so, no, the bird doesn't reach $(17, 4, -14)$.
33. We can substitute the parametric forms of x , y , and z into the equation for the plane and solve for t . So $(3t - 5) + 3(2 - t) - (6t) = 19$ which gives us $t = -3$. Substituting back in the parametric equations, we find that the point of intersection is $(-14, 5, -18)$.
34. Using the same technique as in Exercise 33, $5(1 - 4t) - 2(t - 3/2) + (2t + 1) = 1$ which simplifies to $t = 2/5$. This means the point of intersection is $(-3/5, -11/10, 9/5)$.
35. We will set each of the coordinate equations equal to zero in turn and substitute that value of t into the other two equations.

$$x = 2t - 3 = 0 \Rightarrow t = 3/2. \text{ When } t = 3/2, y = 13/2 \text{ and } z = 7/2.$$

$$y = 3t + 2 = 0 \Rightarrow t = -2/3, \text{ so } x = -13/3 \text{ and } z = 17/3.$$

$$z = 5 - t = 0 \Rightarrow t = 5, \text{ so } x = 7 \text{ and } y = 17.$$

The points are $(0, 13/2, 7/2)$, $(-13/3, 0, 17/3)$, and $(7, 17, 0)$.

36. We could show that two points on the line are also in the plane or that for points on the line: $2x - y + 4z = 2(5 - t) - (2t - 7) + 4(t - 3) = 5$, so they are in the plane.
37. For points on the line we see that $x - 3y + z = (5 - t) - 3(2t - 3) + (7t + 1) = 15$, so the line does not intersect the plane.
38. First we parametrize the line by setting $t = (x - 3)/6$, which gives us $x = 6t + 3$, $y = 3t - 2$, $z = 5t$. Plugging these parametric values into the equation for the plane gives

$$2(6t + 3) - 5(3t - 2) + 3(5t) + 8 = 0 \iff 12t + 24 = 0 \iff t = -2.$$

The parameter value $t = -2$ yields the point $(6(-2) + 3, 3(-2) - 2, 5(-2)) = (-9, -8, -10)$.

39. We find parametric equations for the line by setting $t = (x - 3)/(-2)$, so that $x = 3 - 2t$, $y = t + 5$, $z = 3t - 2$. Plugging these parametric values into the equation for the plane, we find that

$$3(3 - 2t) + 3(t + 5) + (3t - 2) = 9 - 6t + 3t + 15 + 3t - 2 = 22$$

for all values of t . Hence the line is contained in the plane.

40. Again we find parametric equations for the line. Set $t = (x + 4)/3$, so that $x = 3t - 4$, $y = 2 - t$, $z = 1 - 9t$. Plugging these parametric values into the equation for the plane, we find that

$$2(3t - 4) - 3(2 - t) + (1 - 9t) = 7 \iff 6t - 8 - 6 + 3t + 1 - 9t = 7 \iff -13 = 7.$$

Hence we have a contradiction; that is, no value of t will yield a point on the line that is also on the plane. Thus the line and the plane do not intersect.