SOLUTIONS-CHAPTER 2

2.1 A material has a hemispherical spectral emissivity that varies considerably with wavelength but is fairly independent of surface temperature (see, for example, the behavior of tungsten in Figures 3.31 and 3.32. Radiation from a gray source at T_i is incident on the surface uniformly from all directions. Show that the total absorptivity for the incident radiation is equal to the total emissivity of the material evaluated at the source temperature T_i.

SOLUTION: For a gray source, the incident radiation is proportional to blackbody radiation at the source temperature; that is, $CE_{\lambda b}(T_i)\,d\lambda$ in the spectral range $d\lambda$.

From Equation 2.31 the total hemispherical absorptivity is

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (T_A) dQ_{\lambda,i} d\lambda}{\int_{\lambda=0}^{\infty} dQ_{\lambda,i} d\lambda}$$

Substituting for the incident energy, $\alpha = \dfrac{\int_{\lambda=0}^{\infty} \alpha_\lambda (T_A) CE_{\lambda b}\left(T_i\right) d\lambda}{\int_{\lambda=0}^{\infty} CE_{\lambda b}\left(T_i\right) d\lambda}$

From Table 2.2, $\alpha_\lambda (T_A) = \epsilon_\lambda (T_A) = \epsilon_\lambda$, so that $\alpha = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b}\left(T_i\right) d\lambda}{\sigma T_i^4}$

For properties independent of surface temperature, $\epsilon = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b}\left(T_i\right) d\lambda}{\sigma T_i^4} = \alpha$

2.2 Using Figure 3.31, estimate the hemispherical total emissivity of tungsten at 2600 K.

SOLUTION: Use a numerical or graphical integration to find $\epsilon = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda}{\sigma T^4}$

A careful numerical integration with ϵ_λ ($\lambda > 2.65\ \mu m$) = 0.1 gives $\epsilon = 0.284$.

Answer: 0.284.

2.3 Suppose that ϵ_λ is independent of λ (gray-body radiation). Show that $F_{0\to\lambda T}$ represents the fraction of the total radiant emission of the gray body in the range from 0 to λT.

SOLUTION: The emission in a wavelength interval from λ = 0 to λ is
$\int_{\lambda^*=0}^{\lambda} \epsilon_\lambda E_{\lambda b} d\lambda^*$ and, for all wavelengths the emission is $\int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda$.
The fraction of energy emitted for the range 0 → λ is, if ϵ ε_λ is independent of λ,

$$\text{Fraction } (0 \to \lambda) = \frac{\epsilon_\lambda \int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\epsilon_\lambda \int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda} = \frac{\int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda}$$

From Equation 1.33, this is $F_{0\to\lambda T}$.

2.4 For a surface with hemispherical spectral emissivity ϵ_λ, does the maximum of the E_λ distribution occur at the same λ as the maximum of the $E_{\lambda b}$ distribution at the same temperature? (*Hint*: examine the behavior of $dE_\lambda /d\lambda$.) Plot the distributions of E_λ as a function of λ for the data of Figure 2.9 at 600 K and for the property data at 700 K. At what λ is the maximum of E_λ? How does this compare with the maximum of $E_{\lambda b}$?

SOLUTION: $E_\lambda = \epsilon_\lambda E_{\lambda b}$

$$\frac{dE_\lambda}{d\lambda} = \frac{d\left\{\dfrac{2\pi C_1 \epsilon_\lambda}{\lambda^5\left[\exp\left(C_2/\lambda T\right)-1\right]}\right\}}{d\lambda} = 2\pi C_1 \epsilon_t \frac{d}{d\lambda}\left\{\frac{\lambda^{-5}}{\left[\exp\left(C_2/\lambda T\right)-1\right]}\right\}$$

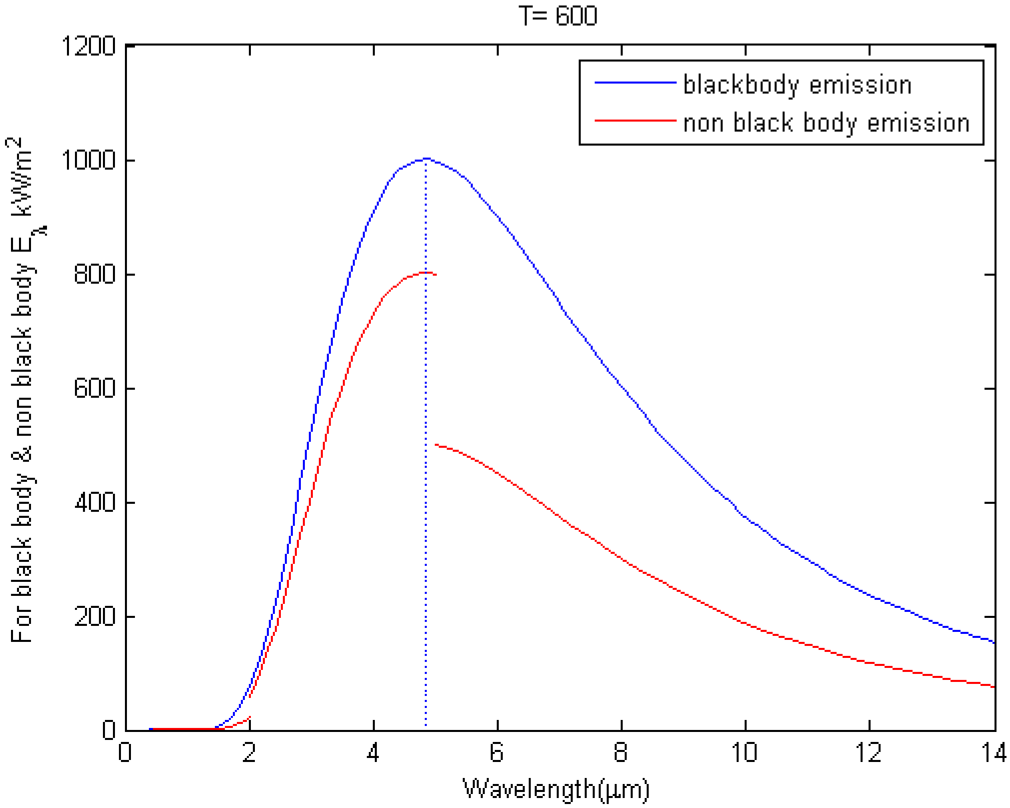
$$=2\pi C_1 \epsilon_t \left\{\frac{-5\lambda^{-6}}{\left[\exp\left(C_2/\lambda T\right)-1\right]} - \lambda^{-5}\frac{\left(-C_2/\lambda^2\right)\exp\left(C_2/\lambda\right)}{\left[\exp\left(C_2/\lambda T\right)-1\right]^2}\right\}$$

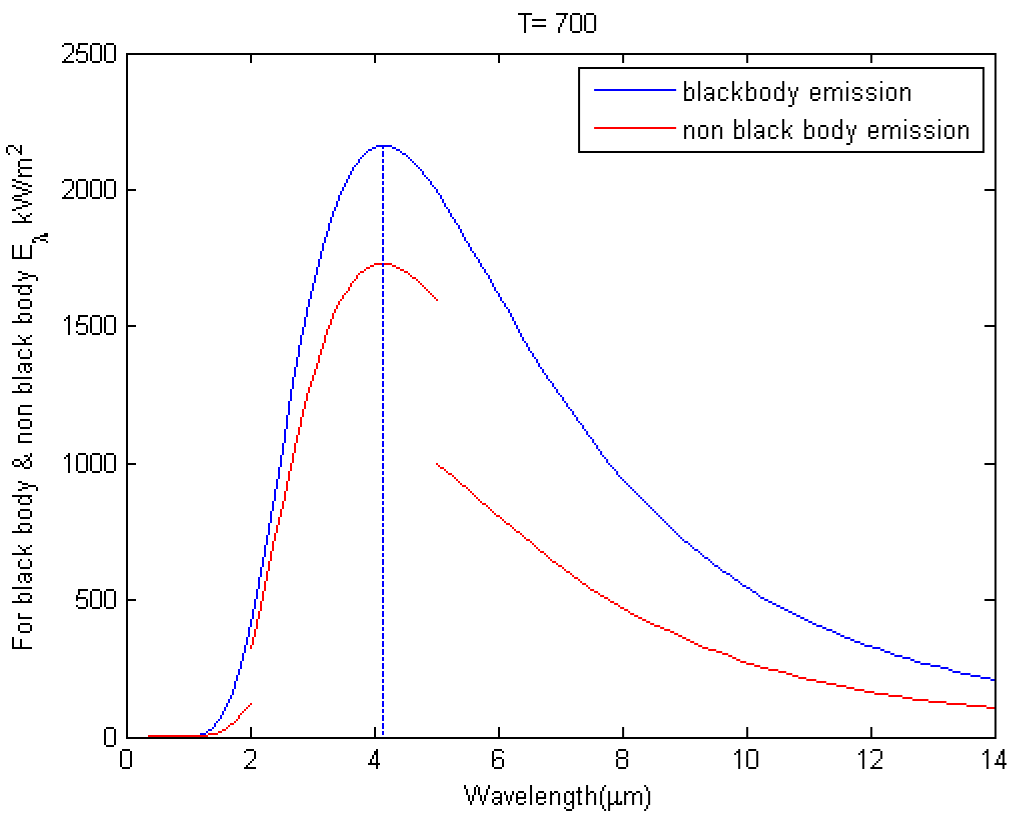
$$=\frac{2\pi C_1 \epsilon_t}{\lambda^6\left[\exp\left(C_2/\lambda T\right)-1\right]}\left\{-5+\frac{\left(C_2/\lambda\right)}{\left[1-\exp\left(C_2/\lambda T\right)\right]}\right\}$$

where ϵ_t is the total hemispherical emittance.

Setting the result = 0 to find the maximum; $\dfrac{(C_2/\lambda)}{1-\exp(-C_2/\lambda T)} = 5$

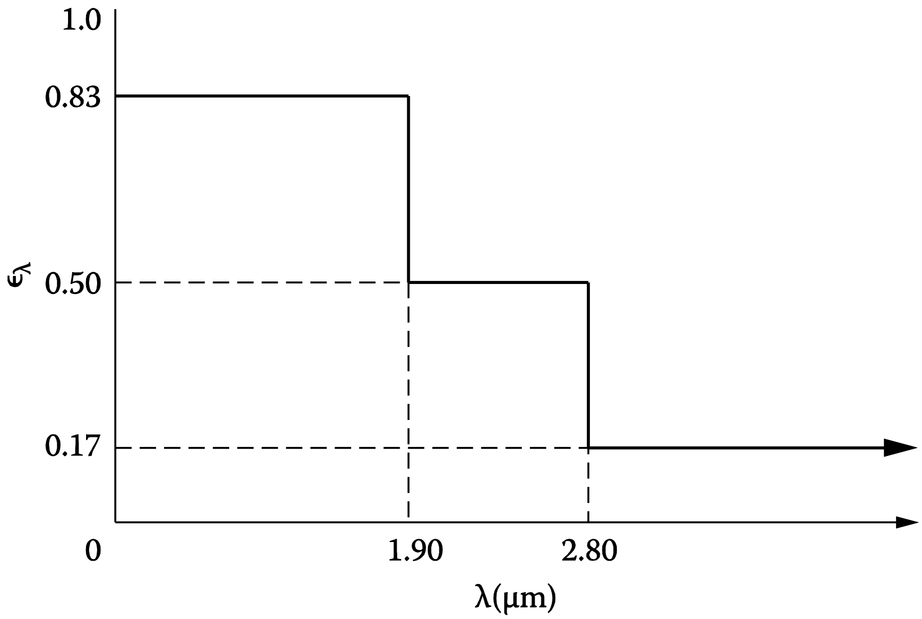
$(\lambda T)_{max}$=2897.8 μm.K. So for the same temperature, the maximum occurs at the same point as $E_{\lambda b}$. Only the intensity of this power is reduced by the numerical value of emittance. For data in Figure 2.9, the following figure is obtained


T= 600
blackbody emission
non black body emission
For black body & non black body Eλ kW/m2
Wavelength(μm)
1200
1000
800
600
400
200
0
0
2
4
6
8
10
12
14


T= 700
blackbody emission
non black body emission
For black body & non black body Eλ kW/m2
Wavelength(μm)
2500
2000
1500
1000
500
0
0
2
4
6
8
10
12
14

E_λ is maximum at 2897.8/600=4.83 μm which overlaps with the plot.
E_λ is maximum at 2897.8/700=4.14 μm which overlaps with the plot.

2-5 Find the emissivity at 400 K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure.



SOLUTION:
The emissivity using Equation 2.10 is

$$\epsilon(T) = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda(T) E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$= \frac{0.83\int_{\lambda=0}^{1.90} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{0.50\int_{\lambda=1.93}^{2.80} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{0.17\int_{\lambda=2.80}^{\infty} E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$= 0.83F_{0-1.90T} + 0.50F_{1.90T-2.80T} + 0.17F_{2.80T-\infty}$$

$$= 0.83F_{0-1.90T} + 0.50\left(F_{0-2.80T} - F_{0-1.90T}\right) + 0.17\left(1 - F_{0-2.80T}\right)$$

so only two blackbody fractions need be found. Using Equation 1.37 for the F values,

$$\epsilon(T=400) = 0.83F_{0-1.90\times400} + 0.5(F_{0-2.80\times400} - F_{0-1.90\times400}) \quad + 0.17(1 - F_{0-2.80\times400}) =$$

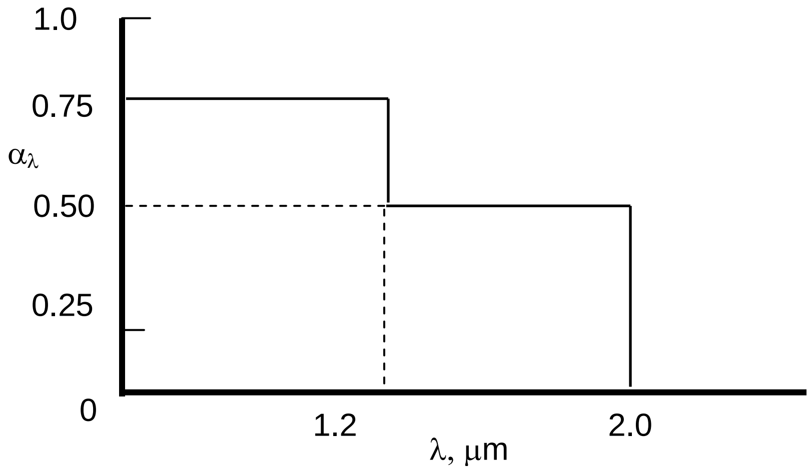
$$\approx 0 + 0.5 \times 0.50 \times (0.0011 - \approx 0) + 0.17 \times (1 - 0.0011) = 0.1704$$

$$\epsilon(T = 5780K) = 0.83F_{0-1.90\times5780} + 0.50\left(F_{0-2.80\times5780} - F_{0-1.90\times5780}\right) + 0.17\left(1 - F_{0-2.80\times5780}\right)$$

$$= 0.83\times0.9316 + 0.50\times\left(0.9745 - 0.9316\right) + 0.17\times(1 - 0.9745) = \underline{0.7990}.$$

Answer: ϵ (T=400 K) = 0.17; α_S(T=5780 K) = 0.7990

2.6 The surface temperature-independent hemispherical spectral absorptivity of a surface is measured when it is exposed to isotropic incident spectral intensity, and the results are approximated as shown below. What is the total hemispherical emissivity of this surface when it is at a temperature of 1000 K?

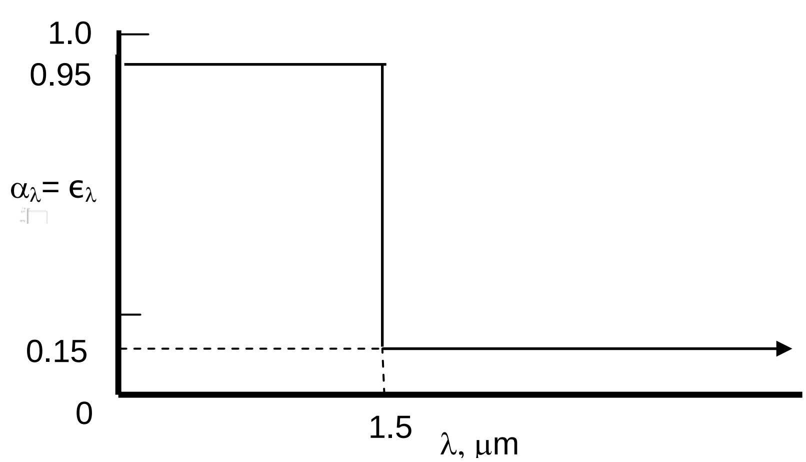


SOLUTION: $\epsilon_\lambda = \alpha_\lambda$ from Table 2.2 gives $\epsilon = \dfrac{\int_0^\infty \alpha_\lambda E_{\lambda b}(T_A)\,d\lambda}{\int_0^\infty E_{\lambda b}(T_A)\,d\lambda}$ so that

$\epsilon = 0.75\,F_{0\to1200} + 0.5\,F_{1200\to2000}$
$= 0.75 \times 0.002134 + 0.5 \times (0.06673 - 0.002134)$
$=$ <u>0.03390.</u>

Answer: <u>0.0339.</u>

2.7 (a) Obtain the total absorptivity of a diffuse surface with properties given in the figure for incident radiation from a blackbody with a temperature of 6200 K.
(b) What is the total emissivity of the diffuse surface with properties given in the figure if the surface temperature is 500 K?



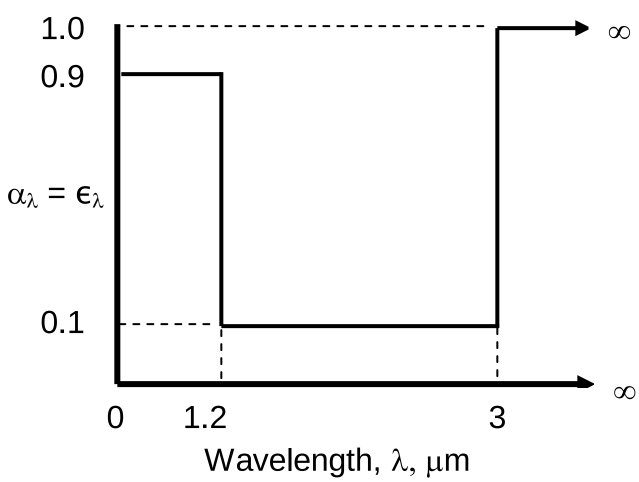
SOLUTION:

(a) Using $\alpha = \dfrac{\int_0^\infty \alpha_\lambda q_{\lambda,i} d\lambda}{\int_0^\infty q_{\lambda,i} d\lambda}$ with $q_{\lambda,i} = E_{\lambda b}(6200K)$ and Equation 1.37 for the blackbody fractions gives $\alpha = 0.95\, F_{0\to 1.5x6200} + 0.15\,(1 - F_{0\to 1.5x6200})$
$= 0.95 \times 0.8975 + 0.15(1-0.\ 0.8975) =$ <u>0.8680</u>

(b) Similarly, for emission, $\epsilon = 0.95\, F_{0\to 1.5x500} + 0.15\,(1 - F_{0\to 1.5x500})$
$= 0.95 \times (\approx 0) + 0.15[1-(\approx 0)] =$ <u>0.15.</u>

Answer: (a) 0.8680; (b) 0.15.

2.8 For the spectral properties given in the figure for a diffuse surface:
(a) what is the solar absorptivity of the surface (assume the solar temperature is 5800 K)?
(b) what is the total hemispherical emissivity of the surface if the surface temperature is 700 K?



SOLUTION: The definitions of ϵ and α in terms of ϵ_λ and α_λ are used as in Problems 2.5 and 2.6. Using Equation1.37,

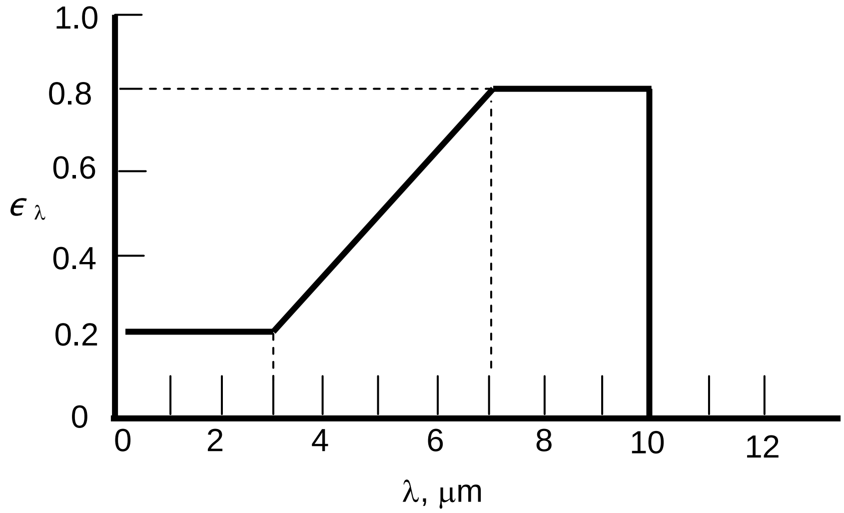
(a) $\alpha_\lambda = 0.9F_{0\text{-}1.2x5800} + 0.1(F_{0\text{-}3x5800} - F_{0\text{-}1.2x5800}) + 1(1 - F_{0\text{-}3x5800})$
$= 0.72505 + 0.01704 + 0.02399 = 0.76608$

For a diffuse surface, Table 2.2 gives $\epsilon_\lambda = \alpha_\lambda$. Then

(b) $\epsilon = 0.9F_{0\text{-}1.2x700} + 0.1(F_{0\text{-}3x700} - F_{0\text{-}1.2x700}) + 1(1 - F_{0\text{-}3x700})$
$= 0 + 0.00830 + 0.91695 = 0.92528.$

Answer: (a) 0.766; (b) 0.925.

2.9 A white ceramic surface has a hemispherical spectral emissivity distribution at 1600 K as shown. What is the hemispherical total emissivity of the surface at this surface temperature?



SOLUTION: Numerical or graphical integratıon ıs necessary, as no analytical integration appears possible even for this simple variation in spectral emissivity. From Equation 2,10 with $E_{\lambda b}=\pi I_{\lambda b}$

$$\epsilon(1600K) = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda (1600K) E_{\lambda b}(1600K)\, d\lambda}{\sigma (1600)^4}$$

Now,

$$\underset{\varepsilon\varepsilon}{\epsilon}(1600K) = \frac{1}{\sigma T_A^4}\int_{\lambda=0}^{\lambda_1} \epsilon_1 E_{\lambda b}\, d\lambda + \frac{1}{\sigma T_A^4}\int_{\lambda=\lambda}^{\lambda_2} \epsilon\left[\epsilon_1 + (\epsilon_2 - \epsilon_1)\left(\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1}\right)\right] E_{\lambda b}\, d\lambda$$

$$(+ \frac{1}{\sigma T_A^4}\int_{\lambda=\lambda_2}^{\lambda_3} \epsilon_2 E_{\lambda b}\, d\lambda$$

where ϵ_1=0.2, ϵ_2=0.8, λ_1=3 μm, λ_2=7 μm, λ_3= 10 μm

Numerical Romberg integration of the ϵ (1600 K) equation gives 0.28128.

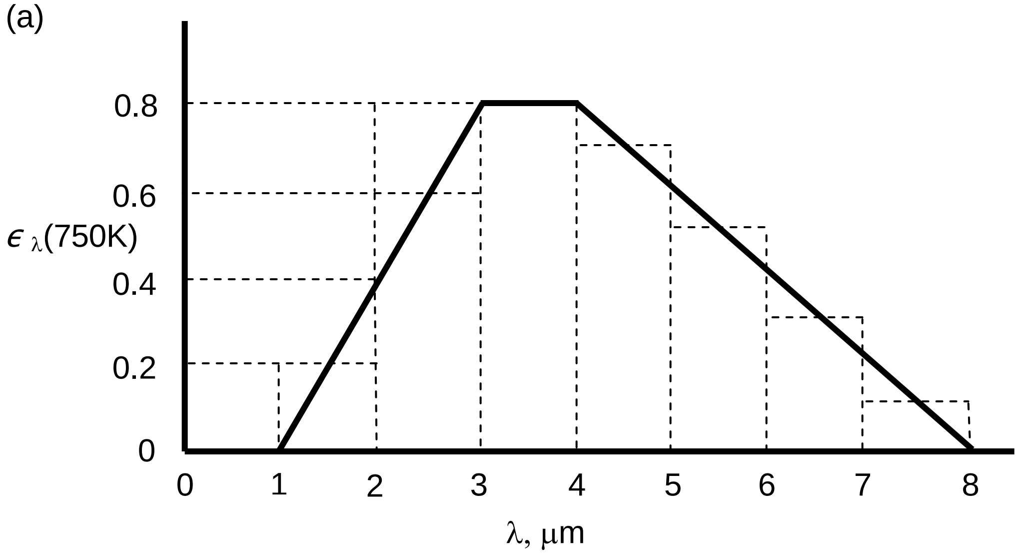
Answer: 0.281

2.10 A surface has the following values of hemispherical spectral emissivity at a temperature of 800K.

λ, μm	ϵ_λ(800 K)
<1	0
1	0
1.5	0.2
2	0.4
2.5	0.6
3	0.8
3.5	0.8
4	0.8
4.5	0.7
5	0.6
6	0.4
7	0.2
8	0
>8	0

(a) What is the hemispherical total emissivity of the surface at 800 K?
(b) What is the hemispherical total absorptivity of the surface at 800 K if the incident radiation is from a gray source at 1800 K that has an emissivity of 0.815? The incident radiation is uniform over all incident angles.

SOLUTION:
(a)



From Equation 2.10,

$$\epsilon(800K) = \frac{\int_{\lambda=0}^{\infty}\epsilon_\lambda\left(800K\right)E_{\lambda b}\left(800K\right)d\lambda}{\sigma T_A^4}$$
$$= \frac{1}{\sigma T_A^4}\int_{\lambda=\lambda_1}^{\lambda_2}\left[\epsilon_1 + \left(\epsilon_2 - \epsilon_1\right)\frac{\lambda-\lambda_1}{\lambda_2-\lambda_1}\right]E_{\lambda b}\left(800K\right)d\lambda$$
$$= \frac{1}{\sigma T_A^4}\int_{\lambda=\lambda_2}^{\lambda_3}\left[\epsilon_2 + \left(\epsilon_3 - \epsilon_2\right)\frac{\lambda-\lambda_2}{\lambda_3-\lambda_2}\right]E_{\lambda b}\left(800K\right)d\lambda$$
$$= \frac{1}{\sigma T_A^4}\int_{\lambda=\lambda_3}^{\lambda_4}\left[\epsilon_3 + \left(\epsilon_4 - \epsilon_3\right)\frac{\lambda-\lambda_3}{\lambda_4-\lambda_3}\right]E_{\lambda b}\left(800K\right)d\lambda$$

where T_A = 800K, ϵ_1=0, ϵ_2=0.8, ϵ_3=0.8, ϵ_4=0, λ_1=1 μm, λ_2 = 3 μm, λ_3 = 4 μm, λ_4 = 8 μm. Accurate numerical integration of the ϵ (800 K) equation gives <u>0.43826</u>.

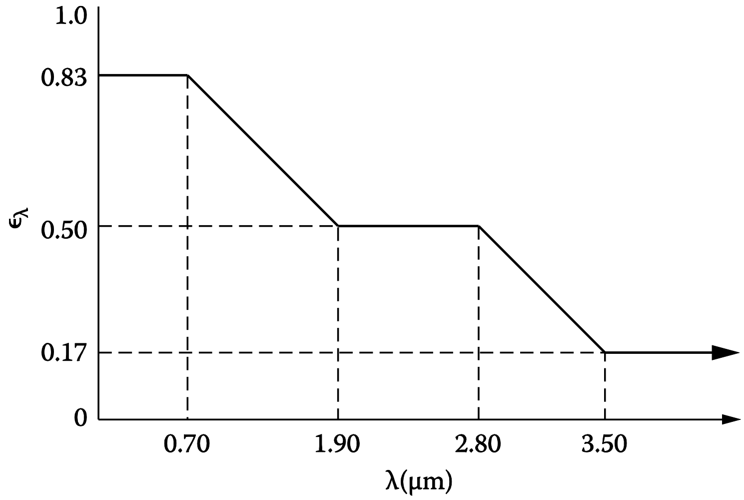
(b) From Equation 2.25

$$\alpha(800K) = \frac{\int_{\lambda=0}^{\infty}\alpha_\lambda\left(800K\right)0.815E_{\lambda b}\left(1800K\right)d\lambda}{\int_0^{\infty}0.815E_{\lambda b}\left(1800K\right)d\lambda} = \frac{\int_{\lambda=0}^{\infty}\epsilon_\lambda\left(800K\right)E_{\lambda b}\left(1800K\right)d\lambda}{\sigma 1800^4}$$
$$= \frac{1}{\sigma 1800^4}\int_{\lambda=\lambda_1}^{\lambda_2}\left[\epsilon_1 + \left(\epsilon_2 - \epsilon_1\right)\frac{\lambda-\lambda_1}{\lambda_2-\lambda_1}\right]E_{\lambda b}\left(1800K\right)d\lambda$$
$$= \frac{1}{\sigma 1800^4}\int_{\lambda=\lambda_2}^{\lambda_3}\left[\epsilon_2 + \left(\epsilon_3 - \epsilon_2\right)\frac{\lambda-\lambda_2}{\lambda_3-\lambda_2}\right]E_{\lambda b}\left(1800K\right)d\lambda$$
$$= \frac{1}{\sigma 1800^4}\int_{\lambda=\lambda_3}^{\lambda_4}\left[\epsilon_3 + \left(\epsilon_4 - \epsilon_3\right)\frac{\lambda-\lambda_3}{\lambda_4-\lambda_3}\right]E_{\lambda b}\left(1800K\right)d\lambda$$

Numerical integration of the $\int_{\lambda=0}^{\infty}\epsilon_\lambda E_{\lambda b}d\lambda$ gives <u>0.42709</u>.

Answer: (a) 0.438; (b) 0.427.

2.11 Find the emissivity at 950 K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure. This will require numerical integration.



SOLUTION:

The emissivity in the various ranges can be expressed as

$$0 \le \lambda < 0.70 : \epsilon_\lambda = 0.83$$

$$0.70 \le \lambda < 1.90 : \epsilon_\lambda = 0.83 - 0.33\left(\frac{\lambda - 0.70}{1.20}\right)$$

$$1.90 \le \lambda < 2.80 : \epsilon_\lambda = 0.50$$

$$2.80 \le \lambda < 3.50 : \epsilon_\lambda = 0.50 - 0.33\left(\frac{\lambda - 2.80}{0.70}\right)$$

$$\lambda \ge 3.50; \epsilon_\lambda = 0.17$$

The total emissivity is then

$$\epsilon(T) = \frac{\int_{\lambda=0}^{\infty} \epsilon_\lambda(T) E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$= \frac{0.83\int_{\lambda=0}^{0.70} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{\int_{\lambda=0.70}^{1.90}\left[0.83 - 0.33\left(\frac{\lambda - 0.70}{1.20}\right)\right] E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$+ \frac{0.50\int_{\lambda=1.90}^{2.80} E_{\lambda b}(T)\, d\lambda}{\sigma T^4} + \frac{\int_{\lambda=2.80}^{3.50}\left[0.50 - 0.33\left(\frac{\lambda - 2.80}{0.70}\right)\right] E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

$$+ \frac{0.17\int_{\lambda=3.50}^{\infty} E_{\lambda b}(T)\, d\lambda}{\sigma T^4}$$

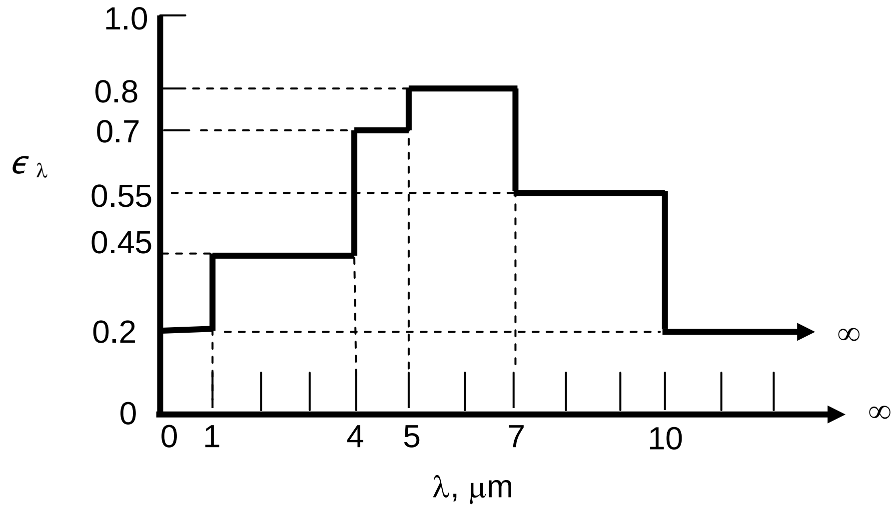
The blackbody fractions can be used to evaluate the first, third and fifth integrals, but the second and third probably require numerical integration. The results using numerical integration are ϵ (950 K) = 0.0000+0.0684+0.1185+0.0556+0.0823 = <u>0.3249</u>; ϵ (5780 K) = 0.4060+0.3225+0.0215+0.0041+0.0024 = <u>0.7564.</u>

Answer: ϵ *(*950 K) = 0.3248; ϵ (5780 K) = α_s(5780 K) = <u>0.7564.</u>

2.12 A diffuse surface at 1000 K has a hemispherical spectral emissivity that can be approximated by the solid line shown.

(a) What is the hemispherical-total emissive power of the surface? What is the total intensity emitted in a direction 60° from the normal to the surface?

(b) What percentage of the total emitted energy is in the wavelength range 5 < λ < 10 μm? How does this compare with the percentage emitted in this wavelength range by a gray body at 1000 K with an emissivity ϵ = 0.611?



SOLUTION:

(a): $\epsilon(T_A) = \dfrac{\int_{\lambda=0}^{\infty} \epsilon_\lambda(T_A) E_{\lambda b}(T_A)\, d\lambda}{\sigma T_A^4}$; the blackbody fractions for each spectral band are obtained from Equation 1.37 as given in the table below:

λ range	ϵ_λ	λT range	$F_{0\to\lambda T}$ - $F_{0\to\lambda T}$	$\Delta F_{0\to\lambda T}$	$\epsilon_\lambda \Delta F_{0-\lambda T}$
0 - 1	0.2	0 - 1000	0.000032	0.00032	≈ 0
1 - 4	0.45	1000-4000	0.48086-0.00032	0.48054	0.21624
4 - 5	0.7	4000-5000	0.63371-0.48086	0.15285	0.10699
5 - 7	0.8	5000-7000	0.80793-0.63371	0.17422	0.13937
7 - 10	0.55	7000-10000	0.9134-0.80793	0.10547	0.05801
10 - ∞	0.2	10000 - ∞	1 - 0.9134	0.08660	0.017321

$\epsilon = \Sigma\, \epsilon_\lambda \Delta F_{0\to\lambda T}$ = <u>0.53800</u>

$E_{\lambda b} = \epsilon \sigma T^4$ = 0.53800x 5.6704x10^{-8} x 900^4 = <u>30506.9 W/m^2</u>

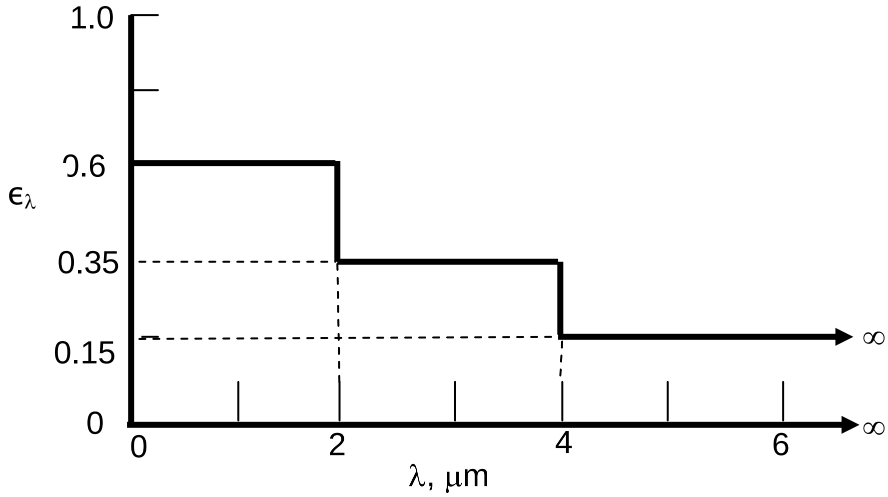
$I_{\lambda b} = E_{\lambda b}/\pi$ = <u>9710.7 W/m^2•sr</u>

(b) For the range 5<λ<10, the percentage is [(0.13937+0.05801)/ 0.53800]x100=36.69%. For a gray surface, the percentage is the same as for a black surface, or

100 ($F_{0\to 9000}$ - $F_{0\to 4500}$) = 100 (0.91339 - 0.63371) = <u>27.97 %</u>.

Answer: (a) <u>30507</u> W/m^2; <u>9711</u> W/m^2•sr (b) 36.69 %; <u>27.97</u> %

2.13 The ϵ_λ for a metal at 1000 K is approximated as shown, and it does not vary significantly with the metal temperature. The surface is diffuse.



(a) What is α for incident radiation from a gray source at 1200 K with ϵ_{source} = 0.822?

(b) What is α for incident radiation from a source at 1200 K made from the same metal as the receiving plate?

SOLUTION:

$$\text{(a)}\quad \alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (=\epsilon_\lambda) 0.822 E_{\lambda b}(1200K)\, d\lambda}{0.822\sigma 1200^4} = \sum \epsilon_\lambda \Delta F$$

λ range	ϵ_λ	λT_i range	$F_{0\to\lambda T_i} - F_{0\to\lambda T_i}$	ΔF	$\epsilon_\lambda \Delta F$
0 - 2	0.6	0-2400	0.14026- 0	0.14026	0.08415
2 - 4	0.35	2400-4800	0.60753-0.14026	0.46727	0.16355
4 - ∞	0.15	4800-∞	1 - 0.60753	0.39247	0.05887

$\alpha = \Sigma\, \epsilon_\lambda \Delta F = \underline{0.30657}$

(b) $\alpha = \Sigma\, \alpha_\lambda (\alpha_\lambda \Delta F) / (\Sigma\, \alpha_\lambda \Delta F)$

$= [\, 0.6^2 \times 0.14026 + 0.35^2 \times 0.46727 + 0.15^2 \times 0.39247\,] / \underline{0.30657} = \underline{0.38022}$

Answer: (a) 0.30657; (b) 0.38022.

2.14 The directional total absorptivity of a gray surface is given by the expression $\alpha(\theta) = 0.450\cos^2\theta$ where θ is the angle away from the normal to the surface.

(a) What is the hemispherical total emissivity of the surface?

(b) What is the hemispherical- hemispherical total reflectivity of this surface for diffuse incident radiation (uniform incident intensity)?

(c) What is the hemispherical-directional total reflectivity for diffuse incident radiation reflected into a direction 75° from the normal?

SOLUTION:

(a)
$$\epsilon = \alpha = \frac{1}{\pi}\int_{\omega=0}^{4\pi} \alpha(\theta)\cos\theta d\omega$$
$$= \frac{2\pi}{\pi}\int_{\omega=0}^{4\pi} 0.450\cos^3\theta d\omega = \left.\frac{-0.9\cos^4\theta}{4}\right|_0^{\pi/2} = 0.2250$$

(b) $\rho = 1 - \alpha = 1 - \epsilon = 1 - 0.2250 =$ <u>0.7750</u>.

(c) Using Equation 2.43, $\rho(\theta_r,\phi_r) = \rho(\theta,\phi) = 1 - \alpha(\theta) = 1 - 0.450\cos^2(75°) =$ <u>0.9699</u>.

Answer: (a) 0.225; (b) 0.775; (c) 0.97.

2.15 Using Figure 3.24, estimate the total absorptivity of typewriter paper for normally incident radiation from a blackbody source at 1200 K.

SOLUTION: Use Equations 2.81 and 2.39 for a gray surface and assume that there is no dependence on circumferential angle ϕ. From the plot we have symmetry for the values of $\rho(\theta = 0, \theta_r, T_A)$.

$$\alpha(\theta=0,T_A) = 1 - \rho(\theta=0,T_A) = 1 - 2\pi\int_{\theta_r=0}^{\pi/2} \rho(\theta=0,\theta_r,T_A)\cos\theta_r \sin\theta_r d\theta_r = 1 - 0.1079 =$$
$$= 0.8921$$

Evaluation requires numerical integration.

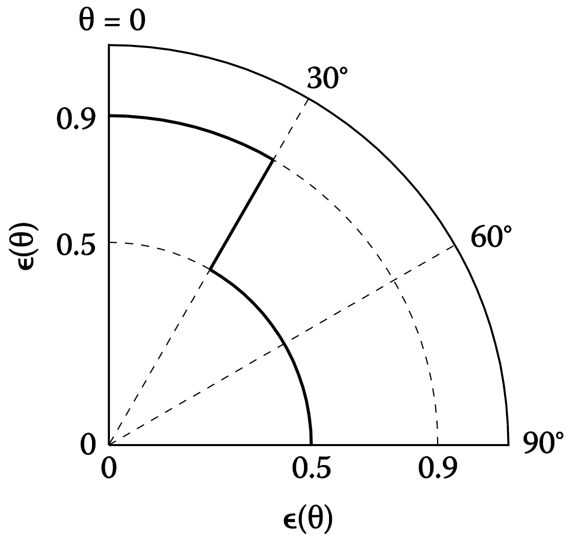
Answer: 0.8921.

2.16 A gray surface has a directional emissivity as shown in the figure. The properties are isotropic with respect to circumferential angle ϕ.

(a) What is the hemispherical emissivity of this surface?

(b) If the energy from a blackbody source at 650 K is incident uniformly from all directions, what fraction of the incident energy is absorbed by this surface?

(c) If the surface is placed in a very cold environment, at what rate must energy be added per unit area to maintain the surface temperature at 1000 K?



SOLUTION:
(a) From Equation 2.9,

$$\epsilon = 2\int_{\theta=0}^{\pi/2}\epsilon(\theta)\cos\theta\sin\theta\ d\theta = 2\left[\int_{\sin\theta=0}^{1/2}0.9\sin\theta\ d(\sin\theta) + \int_{\sin\theta=1/2}^{1}0.5\sin\theta\ d(\sin\theta)\right]$$

$$= 2\left[0.9\frac{\sin^2\theta}{2}\bigg|_{\sin\theta=0}^{1/2} + 0.5\frac{\sin^2\theta}{2}\bigg|_{\sin\theta=1/2}^{1}\right] = 2\left[0.9\frac{\left(\frac{1}{4}-0\right)}{2} + 0.5\left(\frac{1-\frac{1}{4}}{2}\right)\right] = \underline{0.600}$$

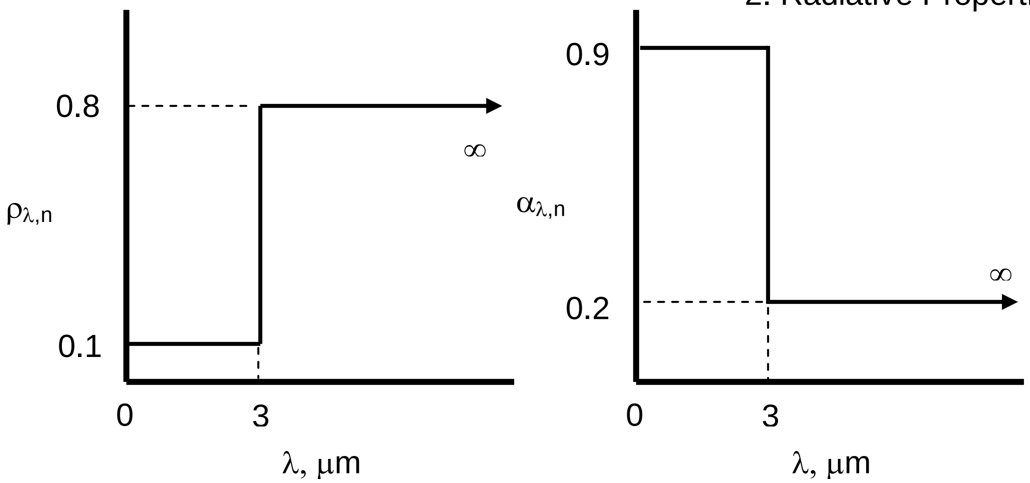
(b) Because the surface is gray, from Table 2.2, $\alpha = \epsilon$. Hence, α = 0.600 = fraction absorbed.

(c) Emissive power = $\epsilon\sigma T^4$ = 0.600 x 5.67040x10^{-8} x 1000^4 = 34,022W/m^2.

Answer: (a) 0.600; (b) 0.600; (c) 34,022 W/m^2 .

2.17 Using Figure 3.44, estimate the ratio of normal total solar absorptivity to hemispherical total emissivity for aluminum at a surface temperature of 650 K with a coating of 0.1-μm dendritic lead sulfide crystals. Assume the surface is diffuse. (The solar temperature can be taken as 5780 K.)

SOLUTION: From the figure, approximate the normal hemispherical spectral reflectivity as shown below. Using $\alpha_{\lambda,n} = 1 - \rho_{\lambda,n}$, the spectral normal absorptivity is approximated as shown below:



For $\alpha_{n,solar}$, use T_{solar} = 5780 K: then $(\lambda T)_{cutoff}$ = 3x5780 = 17340 μm•K, and $F_{0\rightarrow(\lambda T)cutoff}$ = 0.97880 [Equation 1.37].

Then $\alpha_{n,solar} = 0.9\ F_{0\rightarrow(\lambda T)cutoff} + 0.2\ (1 - F_{0\rightarrow(\lambda T)cutoff})$

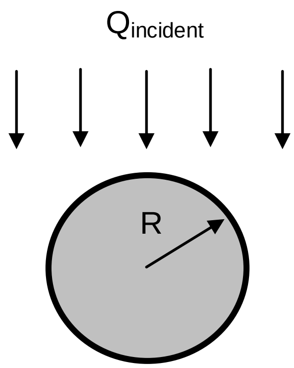
= 0.9 x 0.97880 + 0.2 (1 - 0.97880) = <u>0.8852</u>

ϵ (T = 650 K) ≈ ϵ_n(T = 650 K) = 0.9 $F_{0\rightarrow1950}$ + 0.2 (1 - $F_{0\rightarrow1950}$)

= 0.9 x 0.05920 + 0.2 (1 - 0.05920)= <u>0.2414</u> (NOTE: This assumes that the hemispherical $\epsilon \approx \epsilon_n$.) Thus, $\alpha_{n,solar}/\varepsilon$(T = 650 K) = 0.8852/0.2414 = <u>3.6662.</u>

Answer: 3.666.

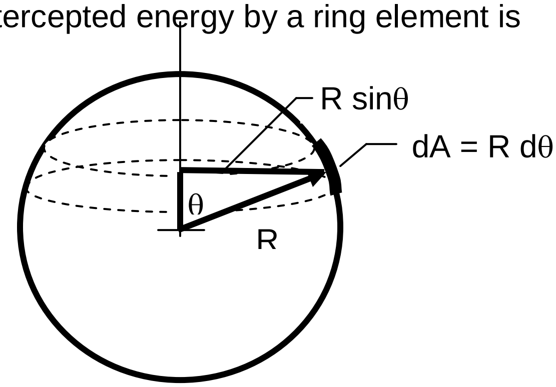
2.18 A gray surface has a directional total emissivity that depends on angle of incidence as $\epsilon(\theta) = 0.788 \cos\theta$. Uniform radiant energy from a single direction normal to the cylinder axis is incident on a long cylinder of radius R. What fraction of energy striking the cylinder is reflected? What is the result if the body is a sphere rather than a cylinder?



SOLUTION: $\alpha(\theta) = \epsilon(\theta) = 0.788 \cos\theta$. $\rho(\theta) = 1 - \alpha(\theta) = 1 - 0.788 \cos\theta$.
For the cylinder, the intercepted energy on dA per unit length of cylinder is q dA cos θ = q (Rdθ) cos θ, and the reflected energy per unit length is q (Rdθ) cos θ (1 - 0.788 cos θ). The ratio is

$$\frac{\text{reflected}}{\text{incident}} = \frac{q\int_{\theta=0}^{\pi/2} R\cos\theta\left(1-0.788\cos\theta\right)d\theta}{q\int_{\theta=0}^{\pi/2} R\cos\theta d\theta} = 1-\frac{0.788\pi}{4} = 0.3811$$

For the sphere, the intercepted energy by a ring element is



q 2πR sinθ Rdθ cosθ, and the ratio of reflected to incident energy is

$$\frac{\text{reflected}}{\text{incident}} = \frac{q\int_{\theta=0}^{\pi/2} 2\pi R^2 \sin\theta\cos\theta\left(1-0.788\cos\theta\right)d\theta}{q\int_{\theta=0}^{\pi/2} 2\pi R^2 \sin\theta\cos\theta d\theta}$$

$$= \frac{\left(\frac{\sin^2\theta}{2}+\frac{0.788\cos^3\theta}{3}\right)\Bigg|_0^{\pi/2}}{\frac{\sin^2\theta}{2}\Bigg|_0^{\pi/2}} = \frac{\frac{1}{2}-\frac{0.788\pi}{3}}{\frac{1}{2}} = 0.4747$$

Answer: 0.381; 0.475.

2.19 A flat metal plate 0.1 m wide by 1.0 m long has a temperature that varies only along the long direction. The temperature is 900 K at one end, and decreases linearly over the one meter length to 350 K. The hemispherical spectral emissivity of the plate does not change significantly with temperature but is a function of wavelength. The wavelength dependence is approximated by a linear function decreasing from ϵ_λ = 0.85 at λ = 0 to ϵ_λ = 0.02 at λ = 10 μm. What is the rate of radiative energy loss from one side of the plate? The surroundings are at a very low temperature.

SOLUTION: The rate of energy loss is given by the total emissive power integrated over the plate length, or

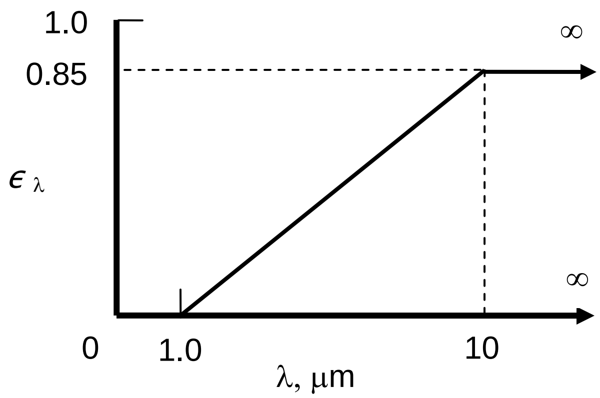
$$Q = 0.1(\text{m}^2)\int_{x=0}^{1}\int_{\lambda=0}^{10}\epsilon_\lambda E_{\lambda b}\left(T\right)d\lambda dx$$

$$= 0.1\int_{x=0}^{1}\int_{\lambda=0}^{10}\left(0.85-\frac{0.83\lambda}{10}\right)\frac{2\pi C_1}{\lambda^3\left\{\exp\left[\frac{C_2}{\lambda\left(350+550x\right)}\right]-1\right\}}d\lambda dx = 416.6\text{W} \cdot$$

where the final result is obtained by numerical integration of the double integral.

Answer: 416.6 W.

2.20 A thin ceramic plate, insulated on one side, is radiating energy from its exposed side into a vacuum at very low temperature. The plate is initially at 1200 K, and is to cool to 300 K. At any instant, the plate is assumed to be at uniform temperature across its thickness and over its exposed area. The plate is 0.25 cm thick, and the surface hemispherical-spectral emissivity is shown in the figure and is independent of temperature. What is the cooling time? The density of the ceramic is 3200 kg/m^3, and its specific heat is 710 J/(kg•K).



SOLUTION: For an area A, the energy equation is

$$-V\rho c\frac{dT}{dt} = qA = A\int_{\lambda=0}^{\infty}\epsilon_\lambda \frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda$$

$$= A\int_{\lambda=0}^{10}\frac{0.85(\lambda-1)}{9}\frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda + A\int_{\lambda=10}^{\infty}0.85\frac{2\pi C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]}d\lambda$$

Integrating gives

$$t = \frac{\rho c V}{A}\int_{T=300}^{1200}\frac{dT}{\int_{\lambda=1}^{10}\frac{0.85(\lambda-1)}{9}\frac{2\pi C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}d\lambda + \int_{\lambda=10}^{\infty}0.85\frac{2\pi C_1}{\lambda^5\left[\exp(C_2/\lambda T)-1\right]}d\lambda}$$

Numerical integration is required, and results in <u>t = 1796 s = </u>29.9 <u>min</u>. (Note: V/A = 0.25x10^{-2} m.)

Answer: t = 29.9 min.