

SOLUTIONS-CHAPTER 2

2-1 A material has a hemispherical spectral emissivity that varies considerably with wavelength but is fairly independent of surface temperature (see, for example, the behavior of tungsten in Figs. 3-31 and 3-32). Radiation from a gray source at T_i is incident on the surface uniformly from all directions. Show that the total absorptivity for the incident radiation is equal to the total emissivity of the material evaluated at the source temperature T_i .

SOLUTION: For a gray source, the incident radiation is proportional to blackbody radiation at the source temperature; that is, $CE_{\lambda b}(T_i) d\lambda$ in the spectral range $d\lambda$.

From Eq. (2-18d) the total hemispherical absorptivity is

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_{\lambda}(T_A) dQ_{\lambda,i} d\lambda}{\int_{\lambda=0}^{\infty} dQ_{\lambda,i} d\lambda}$$

Substituting for the incident energy,

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_{\lambda}(T_A) CE_{\lambda b}(T_i) d\lambda}{\int_{\lambda=0}^{\infty} CE_{\lambda b}(T_i) d\lambda}$$

From Table 2-2, $\alpha_{\lambda}(T_A) = \epsilon_{\lambda}(T_A) = \epsilon_{\lambda}$, so that

$$\alpha = \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda} E_{\lambda b}(T_i) d\lambda}{\sigma T_i^4}$$

For properties independent of surface temperature,

$$\epsilon = \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda} E_{\lambda b}(T_i) d\lambda}{\sigma T_i^4} = \alpha$$

2-2 Using Fig. 3-31, estimate the hemispherical total emissivity of tungsten at 2800 K.

SOLUTION: Use a numerical or graphical integration to find

$$\epsilon = \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda} E_{\lambda b} d\lambda}{\sigma T^4}$$

A careful numerical integration with $\epsilon_{\lambda}(\lambda > 3 \mu\text{m}) = 0.1$ gives $\epsilon = 0.308$.

Answer: 0.308

2-3 Suppose that ϵ_λ is independent of λ (gray-body radiation). Show that $F_{0 \rightarrow \lambda T}$ represents the fraction of the total radiant emission of the gray body in the range from 0 to λT .

SOLUTION: The emission in a wavelength interval from $\lambda = 0$ to λ is

$$\int_{\lambda^*=0}^{\lambda} \epsilon_\lambda E_{\lambda b} d\lambda^* \text{ and, for all wavelengths the emission is } \int_{\lambda=0}^{\infty} \epsilon_\lambda E_{\lambda b} d\lambda.$$

The fraction of energy emitted for the range $0 \rightarrow \lambda$ is, if ϵ_λ is independent of λ ,

$$\text{Fraction } (0 \rightarrow \lambda) = \frac{\epsilon_\lambda \int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\epsilon_\lambda \int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda} = \frac{\int_{\lambda^*=0}^{\lambda} E_{\lambda b} d\lambda^*}{\int_{\lambda=0}^{\infty} E_{\lambda b} d\lambda}$$

From Eqs. (1-29) and (1-31), this is $F_{0 \rightarrow \lambda T}$.

2-4 For a surface with hemispherical spectral emissivity ϵ_λ , does the maximum of the E_λ distribution occur at the same λ as the maximum of the $E_{\lambda b}$ distribution at the same temperature? (*Hint*: examine the behavior of $dE_\lambda/d\lambda$.) Plot the distributions of E_λ as a function of λ for the data of Fig. 2-9 at 500 K and for the property data at 600K. At what λ is the maximum of E_λ ? How does this compare with the maximum of $E_{\lambda b}$?

SOLUTION:

$$\begin{aligned} E_\lambda &= \epsilon_\lambda E_{\lambda b} \\ \frac{dE_\lambda}{d\lambda} &= \frac{d \left\{ \frac{2\pi C_1 \epsilon_\lambda}{\lambda^5 \left[e^{(C_2/\lambda T)} - 1 \right]} \right\}}{d\lambda} \\ &= 2\pi C_1 \epsilon_t \frac{d}{d\lambda} \left[\frac{\lambda^{-5}}{\left[e^{(C_2/\lambda T)} - 1 \right]} \right] \end{aligned}$$

Where ϵ_t is total hemispherical emittance.

$$= 2\pi C_1 \epsilon_t \left\{ \frac{-5\lambda^{-6}}{\left[e^{(C_2/\lambda T)} - 1 \right]} - \lambda^{-5} \frac{\left(-C_2/\lambda^2 \right) e^{(C_2/\lambda T)}}{\left[e^{(C_2/\lambda T)} - 1 \right]^2} \right\}$$

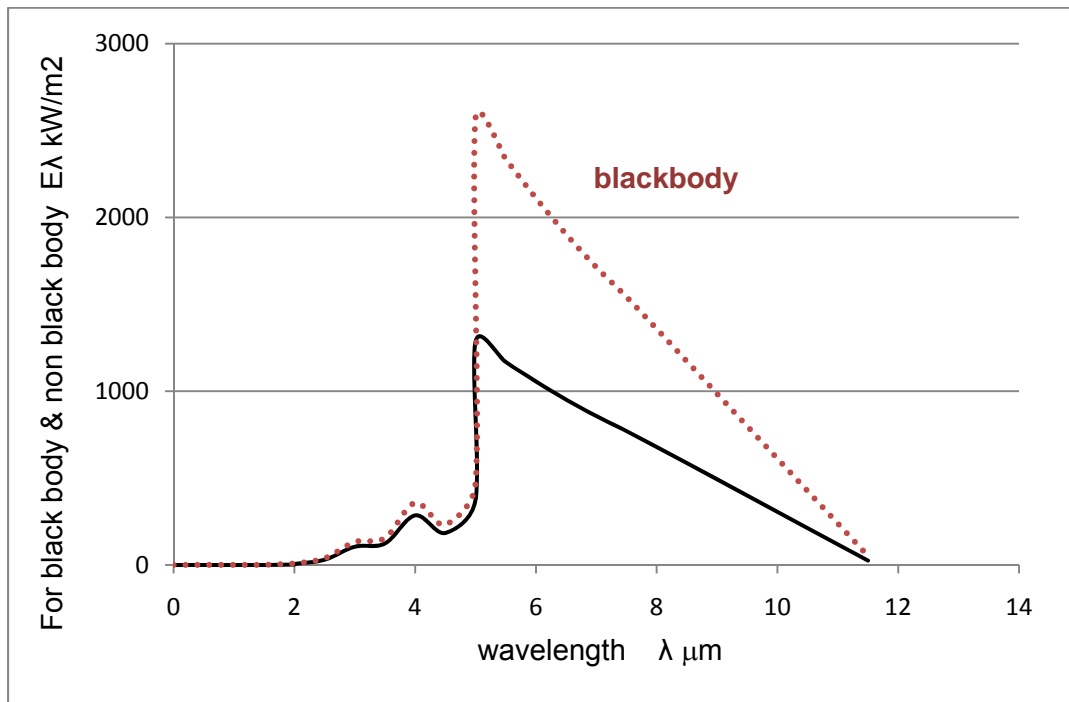
$$= \frac{2\pi C_1 \epsilon_t}{\lambda^6 [e^{(C_2/\lambda T)} - 1]} \left\{ -5 + \frac{C_2/\lambda}{[1 - e^{(-C_2/\lambda T)}]} \right\}$$

Setting the result = 0 to find maximum;

$$\frac{C_2/\lambda}{[1 - e^{(-C_2/\lambda T)}]} = 5$$

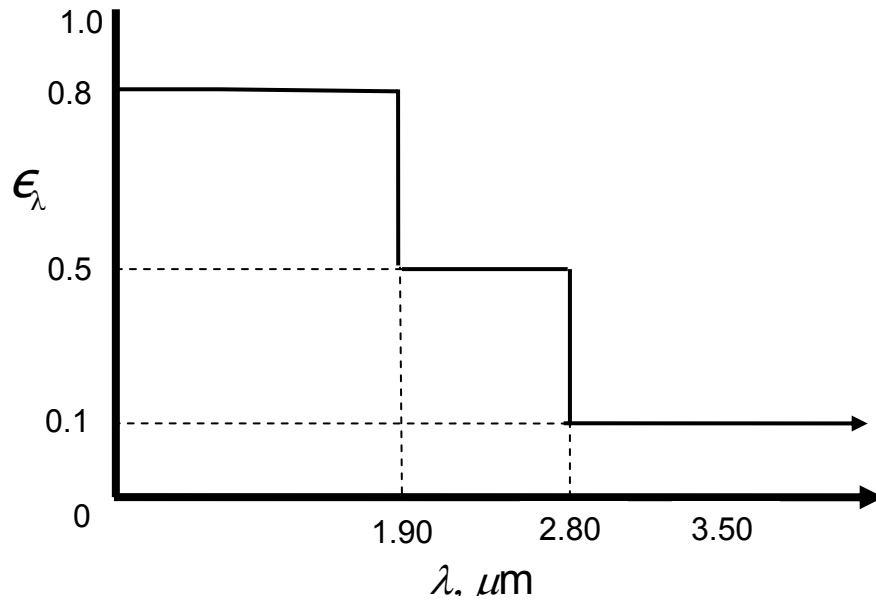
$(\lambda T)_{\max} = 2897.8 \mu\text{m.K}$ So for same temperature maximum occurs at the same point as $E_{\lambda b}$. Only the intensity of this power is reduced by the numerical value of emittance.

For data in Fig. 2-9 following figure is obtained



E_{λ} is maximum at $2897.8/600 = 4.83 \mu\text{m}$ which overlaps with the plot.

2-5 Find the emissivity at 300K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure.

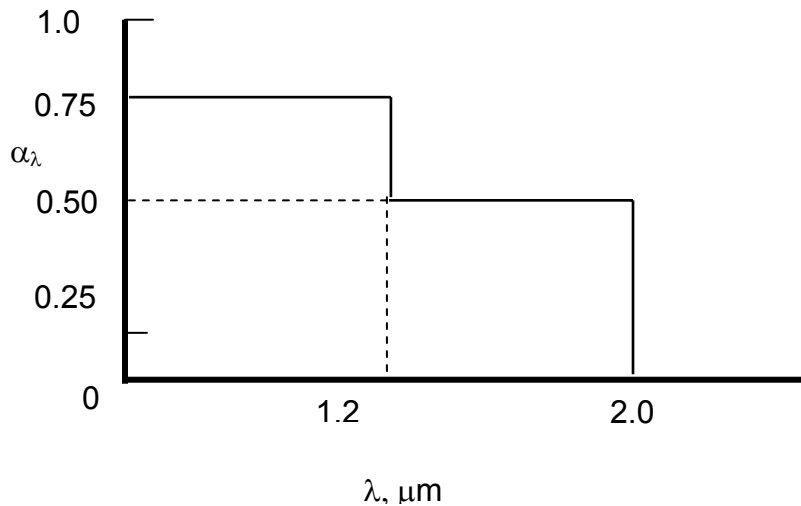


SOLUTION:

The emissivity using Eq. (2-3b) with $E_{\lambda b} = \pi I_{\lambda b}$

$$\begin{aligned}
 \epsilon(T) &= \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda}(T) E_{\lambda b}(T) d\lambda}{\sigma T^4} \\
 &= \frac{0.83 \int_{\lambda=0}^{1.90} E_{\lambda b}(T) d\lambda}{\sigma T^4} + \frac{0.50 \int_{\lambda=1.90}^{2.80} E_{\lambda b}(T) d\lambda}{\sigma T^4} + \frac{0.17 \int_{\lambda=2.80}^{\infty} E_{\lambda b}(T) d\lambda}{\sigma T^4} \\
 &= 0.83 F_{0-1.90T} + 0.50 F_{1.90T-2.80T} + 0.17 F_{2.80T-\infty} \\
 &= 0.83 F_{0-1.90T} + 0.50 (F_{0-2.80T} - F_{0-1.90T}) + 0.17 (1 - F_{0-2.80T}) \\
 &\text{so only two blackbody fractions need be found. Using Eq. 1-33 for the } F \text{ values,} \\
 \epsilon(T = 300K) &= 0.83 F_{0-1.90 \times 300} + 0.50 (F_{0-2.80 \times 300} - F_{0-1.90 \times 300}) + 0.17 (1 - F_{0-2.80 \times 300}) \\
 &\approx 0 + 0.50 \times (\approx 0 - \approx 0) + 0.17 \times (1 - \approx 0) = \underline{0.17} \\
 \epsilon(T = 5780K) &= 0.83 F_{0-1.90 \times 5780} + 0.50 (F_{0-2.80 \times 5780} - F_{0-1.90 \times 5780}) + 0.17 (1 - F_{0-2.80 \times 5780}) \\
 &= 0.83 \times 0.9316 + 0.50 \times (0.9745 - 0.9316) + 0.17 \times (1 - 0.9745) = \underline{0.7990} \\
 \text{Answer: } \epsilon(T=300 \text{ K}) &= 0.17; \alpha_s(T=5780 \text{ K}) = 0.7990
 \end{aligned}$$

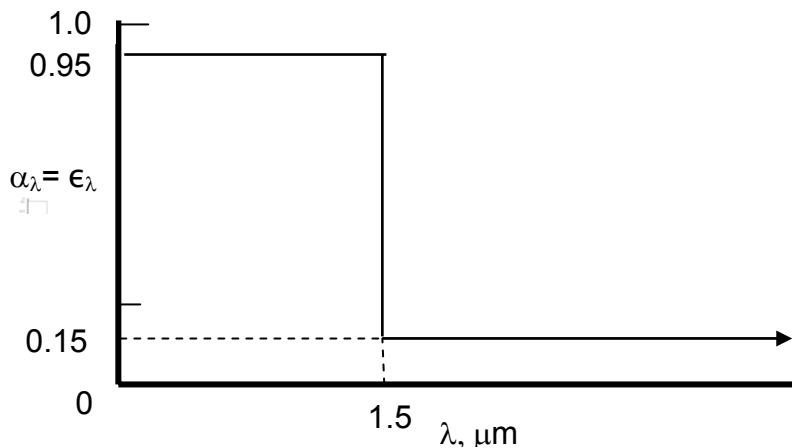
2-6 The surface temperature-independent hemispherical spectral absorptivity of a surface is measured when it is exposed to isotropic incident spectral intensity, and the results are approximated as shown below. What is the total hemispherical emissivity of this surface when it is at a temperature of 2000 K?



SOLUTION: $\epsilon_\lambda = \alpha_\lambda$ from Table 2-2 gives $\epsilon = \frac{\int_0^\infty \alpha_\lambda E_{\lambda b}(T_A) d\lambda}{\int_0^\infty E_{\lambda b}(T_A) d\lambda}$ so that

$$\begin{aligned}\epsilon &= 0.75 F_{0 \rightarrow 2400} + 0.5 F_{2400 \rightarrow 4000} \\ &= 0.75 \times 0.14026 + 0.5 \times (0.48087 - 0.14026) \\ &= \underline{0.2755} \\ \text{Answer: } 0.2755\end{aligned}$$

2-7 (a) Obtain the total absorptivity of a diffuse surface with properties given in the figure for incident radiation from a blackbody with a temperature of 6000K. (b) What is the total emissivity of the diffuse surface with properties given in the figure if the surface temperature is 700K?



SOLUTION:

(a) Using $\alpha = \frac{\int_0^\infty \alpha_\lambda q_{\lambda,i} d\lambda}{\int_0^\infty q_{\lambda,i} d\lambda}$ with $q_{\lambda,i} = E_{\lambda b}(6000K)$ and Eq. (1-33) for the

blackbody fractions gives $\alpha = 0.95 F_{0 \rightarrow 1.5 \times 6000} + 0.15 (1 - F_{0 \rightarrow 1.5 \times 6000})$
 $= 0.95 \times 0.88948 + 0.15(1 - 0.88948) = \underline{0.8615}$

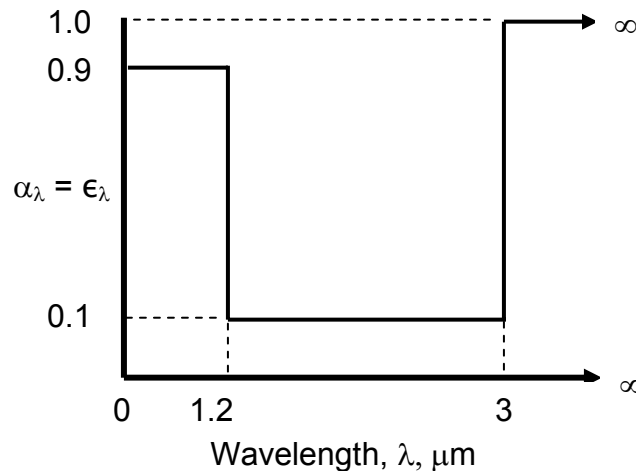
(b) Similarly, for emission, $\epsilon = 0.95 F_{0 \rightarrow 1.5 \times 700} + 0.15 (1 - F_{0 \rightarrow 1.5 \times 700})$
 $= 0.95 \times (\approx 0) + 0.15[1 - (\approx 0)] = \underline{0.15}$

Answer: (a) 0.8615; (b) 0.15

2-8 For the spectral properties given in the figure for a diffuse surface:

(a) what is the solar absorptivity of the surface (assume the solar temperature is 5800 K)?

(b) what is the total hemispherical emissivity of the surface if the surface temperature is 500 K?



SOLUTION: The definitions of ϵ and α in terms of ϵ_λ and α_λ are used as in Problems 2-5 and 2-6. Using Eq. (1-33),

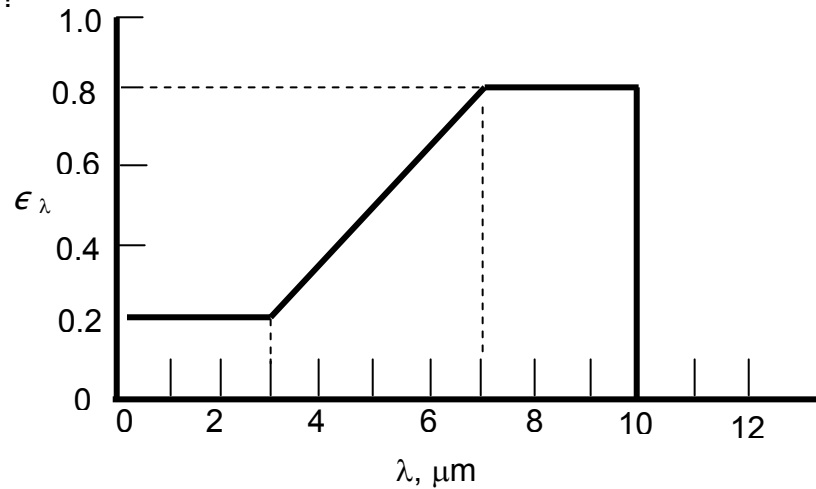
(a) $\alpha_\lambda = 0.9F_{0-1.2 \times 5800} + 0.1(F_{0-3 \times 5800} - F_{0-1.2 \times 5800}) + 1(1 - F_{0-3 \times 5800})$
 $= 0.72505 + 0.01704 + 0.02399 = 0.76608$

For a diffuse surface, Table 2-2 gives $\epsilon_\lambda = \alpha_\lambda$. Then

(b) $\epsilon = 0.9F_{0-1.2 \times 500} + 0.1(F_{0-3 \times 500} - F_{0-1.2 \times 500}) + 1(1 - F_{0-3 \times 500})$
 $= 0 + 0.00129 + 0.98715 = 0.98844$

Answer: (a) 0.766; (b) 0.988

2-9 A white ceramic surface has a hemispherical spectral emissivity distribution at 1800 K as shown. What is the hemispherical total emissivity of the surface at this surface temperature?



SOLUTION: Numerical or graphical integration is necessary, as no analytical integration appears possible even for this simple variation in spectral emissivity. From Eq. 2-3b with $E_{\lambda b} = \pi I_{\lambda b}$

$$\epsilon(1800K) = \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda}(1800K) E_{\lambda b}(1800K) d\lambda}{\sigma(1800)^4}$$

Now,

$$\epsilon(1800K) = \frac{1}{\sigma T_A^4} \int_{\lambda=0}^{\lambda_1} \epsilon_1 E_{\lambda b} d\lambda + \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_1}^{\lambda_2} \epsilon \left[\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right] E_{\lambda b} d\lambda + \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_2}^{\lambda_3} \epsilon_2 E_{\lambda b} d\lambda$$

where $\epsilon_1 = 0.2$, $\epsilon_2 = 0.8$, $\lambda_1 = 3 \mu\text{m}$, $\lambda_2 = 7 \mu\text{m}$, $\lambda_3 = 10 \mu\text{m}$

Numerical Romberg integration of the $\epsilon(1800 \text{ K})$ equation gives 0.26440.

Answer: 0.264

2-10 A surface has the following values of hemispherical spectral emissivity at a temperature of 750K.

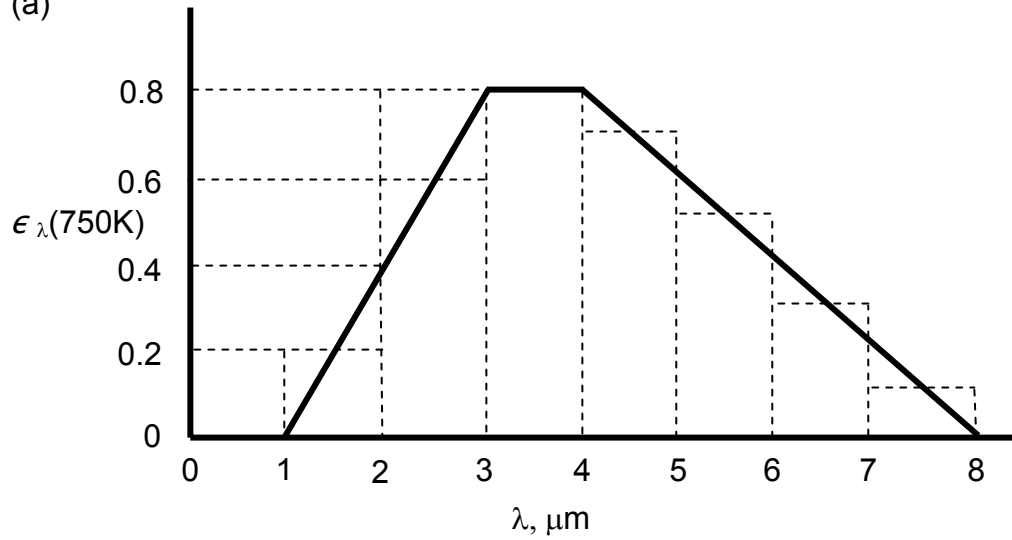
$\lambda, \mu\text{m}$	$\epsilon_{\lambda}(750 \text{ K})$
<1	0
1	0
1.5	0.2
2	0.4
2.5	0.6
3	0.8
3.5	0.8
4	0.8

4.5	0.7
5	0.6
6	0.4
7	0.2
8	0
>8	0

- (a) What is the hemispherical total emissivity of the surface at 750K?
(b) What is the hemispherical total absorptivity of the surface at 750K if the incident radiation is from a gray source at 1600 K that has an emissivity of 0.815? The incident radiation is uniform over all incident angles.

SOLUTION:

(a)



From Eq. 2-3b with $E_{\lambda b} = \pi I_{\lambda b}$,

$$\begin{aligned}
 \epsilon(750K) &= \frac{\int_0^{\infty} \epsilon_{\lambda}(750K) E_{\lambda b}(750K) d\lambda}{\sigma T_A^4} \\
 &= \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_1}^{\lambda_2} \left[\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right] E_{\lambda b}(750K) d\lambda \\
 &\quad + \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_2}^{\lambda_3} \left[\epsilon_2 + (\epsilon_3 - \epsilon_2) \frac{\lambda - \lambda_2}{\lambda_3 - \lambda_2} \right] E_{\lambda b}(750K) d\lambda \\
 &\quad + \frac{1}{\sigma T_A^4} \int_{\lambda=\lambda_3}^{\lambda_4} \left[\epsilon_3 + (\epsilon_4 - \epsilon_3) \frac{\lambda - \lambda_3}{\lambda_4 - \lambda_3} \right] E_{\lambda b}(750K) d\lambda
 \end{aligned}$$

where $T_A = 750K$, $\epsilon_1=0$, $\epsilon_2=0.8$, $\epsilon_3=0.8$, $\epsilon_4=0$, $\lambda_1=1 \mu m$, $\lambda_2 = 3 \mu m$, $\lambda_3 = 4 \mu m$, $\lambda_4 = 8 \mu m$. Accurate numerical integration of the ϵ (750 K) equation gives 0.41284.

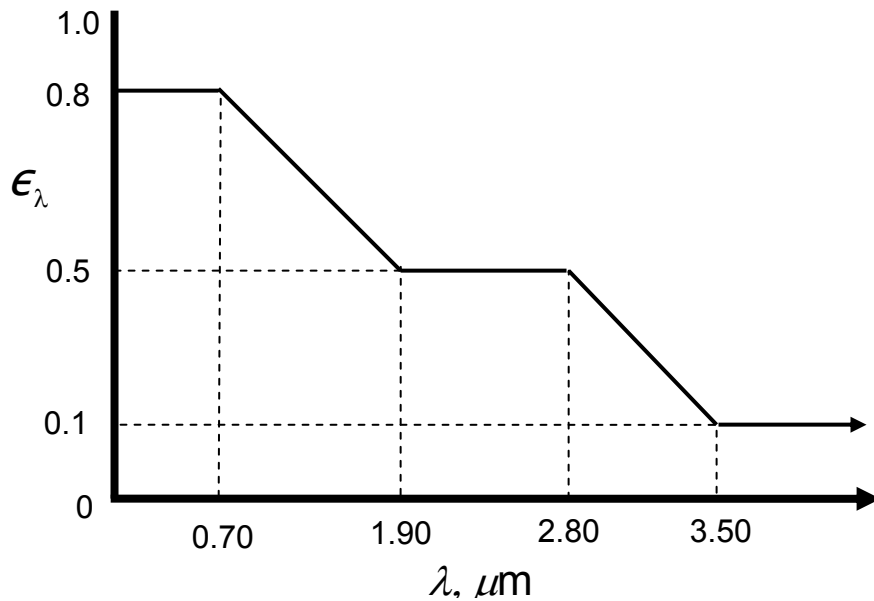
(b) From Eq. 2-18b

$$\begin{aligned}\alpha(750K) &= \frac{\int_0^\infty \alpha_\lambda(750K) 0.815 E_{\lambda b}(1600K) d\lambda}{\int_0^\infty 0.815 E_{\lambda b}(1600K) d\lambda} = \frac{\int_0^\infty \epsilon_\lambda(750K) E_{\lambda b}(1600K) d\lambda}{\sigma 1600^4} \\ &= \frac{1}{\sigma 1600^4} \int_{\lambda=\lambda_1}^{\lambda_2} \left[\epsilon_1 + (\epsilon_2 - \epsilon_1) \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right] E_{\lambda b}(1600K) d\lambda \\ &\quad + \frac{1}{\sigma 1600^4} \int_{\lambda=\lambda_2}^{\lambda_3} \left[\epsilon_2 + (\epsilon_3 - \epsilon_2) \frac{\lambda - \lambda_2}{\lambda_3 - \lambda_2} \right] E_{\lambda b}(1600K) d\lambda \\ &\quad + \frac{1}{\sigma 1600^4} \int_{\lambda=\lambda_3}^{\lambda_4} \left[\epsilon_3 + (\epsilon_4 - \epsilon_3) \frac{\lambda - \lambda_3}{\lambda_4 - \lambda_3} \right] E_{\lambda b}(1600K) d\lambda\end{aligned}$$

Numerical integration of the $\int_{\lambda=0}^\infty \epsilon_\lambda E_{\lambda b} d\lambda$ gives 0.46342.

Answer: (a) 0.413; (b) 0.463

2-11 Find the emissivity at 1000K and the solar absorptivity of the diffuse material with the measured spectral emissivity shown in the figure. This will require numerical integration.



SOLUTION:

The emissivity in the various ranges can be expressed as

$$0 \leq \lambda < 0.70: \epsilon_{\lambda} = 0.83$$

$$0.70 \leq \lambda < 1.90: \epsilon_{\lambda} = 0.83 - 0.33 \left(\frac{\lambda - 0.70}{1.20} \right)$$

$$1.90 \leq \lambda < 2.80: \epsilon_{\lambda} = 0.50$$

$$2.80 \leq \lambda < 3.50: \epsilon_{\lambda} = 0.50 - 0.33 \left(\frac{\lambda - 2.80}{0.70} \right)$$

$$\lambda \geq 3.50: \epsilon_{\lambda} = 0.17$$

The total emissivity is then

$$\begin{aligned} \epsilon(T) &= \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda}(T) E_{\lambda b}(T) d\lambda}{\sigma T^4} \\ &= \frac{0.83 \int_{\lambda=0}^{0.70} E_{\lambda b}(T) d\lambda}{\sigma T^4} + \frac{\int_{\lambda=0.70}^{1.90} \left[0.83 - 0.33 \left(\frac{\lambda - 0.70}{1.20} \right) \right] E_{\lambda b}(T) d\lambda}{\sigma T^4} \\ &\quad + \frac{0.50 \int_{\lambda=1.90}^{2.80} E_{\lambda b}(T) d\lambda}{\sigma T^4} + \frac{\int_{\lambda=2.80}^{3.50} \left[0.50 - 0.33 \left(\frac{\lambda - 2.80}{0.70} \right) \right] E_{\lambda b}(T) d\lambda}{\sigma T^4} \\ &\quad + \frac{0.17 \int_{\lambda=3.50}^{\infty} E_{\lambda b}(T) d\lambda}{\sigma T^4} \end{aligned}$$

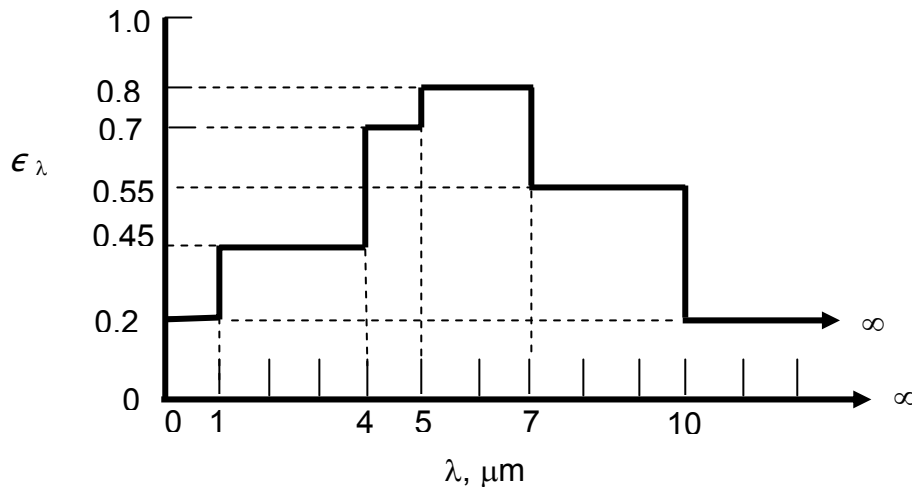
The blackbody fractions can be used to evaluate the first, third and fifth integrals, but the second and third probably require numerical integration. The results using numerical integration are $\epsilon(1200 \text{ K}) = 0.0000 + 0.0684 + 0.1185 + 0.0556 + 0.0823 = \underline{0.3248}$; $\epsilon(5780 \text{ K}) = 0.4060 + 0.3225 + 0.0215 + 0.0041 + 0.0024 = \underline{0.7564}$

Answer: $\epsilon(1200 \text{ K}) = 0.3248$; $\epsilon(5780 \text{ K}) = \underline{0.7564}$

2-12 A diffuse surface at 900 K has a hemispherical spectral emissivity that can be approximated by the solid line shown.

(a) What is the hemispherical-total emissive power of the surface? What is the total intensity emitted in a direction 60° from the normal to the surface?

(b) What percentage of the total emitted energy is in the wavelength range $5 < \lambda < 10 \mu\text{m}$? How does this compare with the percentage emitted in this wavelength range by a gray body at 900 K with an emissivity $\epsilon = 0.611$?



SOLUTION:

$$(a): \epsilon(T_A) = \frac{\int_{\lambda=0}^{\infty} \epsilon_{\lambda}(T_A) E_{\lambda b}(T_A) d\lambda}{\sigma T_A^4}; \text{ the blackbody fractions for each}$$

spectral band are obtained from Eq. (1-33) as given in the table below:

λ range	ϵ_{λ}	λT range	$F_{0 \rightarrow \lambda T} - F_{0 \rightarrow \lambda T}$	$\Delta F_{0 \rightarrow \lambda T}$	$\epsilon_{\lambda} \Delta F_{0 \rightarrow \lambda T}$
0 - 1	0.2	0 - 900	0.00009	0.00009	≈ 0
1 - 4	0.45	900-3600	0.40360-0.00009	0.40351	0.18158
4 - 5	0.7	3600-4500	0.56430-0.40360	0.16070	0.11249
5 - 7	0.8	4500-6300	0.76173-0.56430	0.19743	0.15794
7 - 10	0.55	6300-9000	0.88948-0.76173	0.12775	0.07026
10 - ∞	0.2	9000 - ∞	1 - 0.88948	0.11052	0.02210

$$\epsilon = \sum \epsilon_{\lambda} \Delta F_{0 \rightarrow \lambda T} = \underline{0.54437}$$

$$E_{\lambda b} = \epsilon \sigma T^4 = 0.54437 \times 5.6704 \times 10^{-8} \times 900^4 = \underline{20,252 \text{ W/m}^2}$$

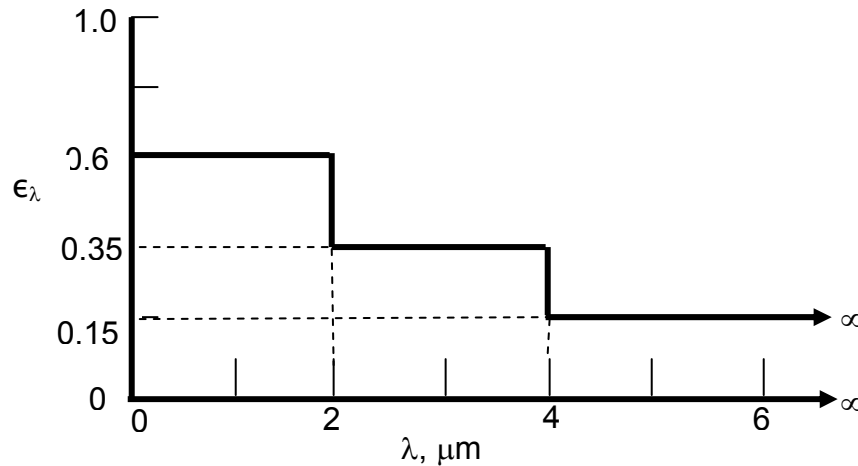
$$I_{\lambda b} = E_{\lambda b} / \pi = \underline{6446.6 \text{ W/m}^2 \cdot \text{sr}}$$

(b) For the range $5 < \lambda < 10$, the percentage is $[(0.15794 + 0.07026) / 0.54437] \times 100 = 41.92\%$. For a gray surface, the percentage is the same as for a black surface, or

$$100 (F_{0 \rightarrow 9000} - F_{0 \rightarrow 4500}) = 100 (0.88948 - 0.56430) = \underline{32.52 \%}.$$

Answer: (a) 20,252 W/m²; 6447 W/m²·sr (b) 41.92 %; 32.52 %

2-13 The ϵ_λ for a metal at 1100 K is approximated as shown, and it does not vary significantly with the metal temperature. The surface is diffuse.



- (a) What is α for incident radiation from a gray source at 1300K with $\epsilon_{\text{source}} = 0.822$?
- (b) What is α for incident radiation from a source at 1300 K made from the same metal as the receiving plate?

SOLUTION:

$$(a) \alpha = \frac{\int_{\lambda=0}^{\infty} \alpha_\lambda (= \epsilon_\lambda) 0.822 E_{\lambda b}(1300K) d\lambda}{0.822 \sigma 1600^4} = \sum \epsilon_\lambda \Delta F$$

λ range	ϵ_λ	λT_i range	$F_{0 \rightarrow \lambda T_i} - F_{0 \rightarrow \lambda T_i}$	ΔF	$\epsilon_\lambda \Delta F$
0 - 2	0.6	0-2600	0.18312 - 0	0.18312	0.10987
2 - 4	0.35	2600-5200	0.65793-0.18312	0.47481	0.16618
4 - ∞	0.15	5200- ∞	1 - 0.65793	0.34207	0.05131

$$\alpha = \sum \epsilon_\lambda \Delta F = \underline{0.32736}$$

$$(b) \alpha = \sum \alpha_\lambda (\alpha_\lambda \Delta F) / (\sum \alpha_\lambda \Delta F)$$

$$= [0.6^2 \times 0.18312 + 0.35^2 \times 0.47481 + 0.15^2 \times 0.34207] / 0.32736 = \underline{0.40257}$$

Answer: (a) 0.3274; (b) 0.4026

2-14 The directional total absorptivity of a gray surface is given by the expression $\alpha(\theta) = 0.667 \cos^2 \theta$ where θ is the angle away from the normal to the surface.

- (a) What is the hemispherical total emissivity of the surface?
- (b) What is the hemispherical-hemispherical total reflectivity of this surface for diffuse incident radiation (uniform incident intensity)?
- (c) What is the hemispherical-directional total reflectivity for diffuse incident radiation reflected into a direction 75° from the normal?

SOLUTION:

(a)

$$\begin{aligned}\epsilon = \alpha &= \frac{1}{\pi} \int_{\omega=0}^{4\pi} \alpha(\theta) \cos\theta d\omega \\ &= \frac{2\pi}{\pi} \int_{\omega=0}^{4\pi} 0.667 \cos^3 \theta d\omega = \left. \frac{-1.5 \cos^4 \theta}{4} \right|_0^{\pi/2} = 0.3335\end{aligned}$$

(b) $\rho = 1 - \alpha = 1 - \epsilon = 1 - 0.3335 = \underline{0.6665}$.

(c) Using Eq. (2-28), $\rho(\theta_r, \phi_r) = \rho(\theta, \phi) = 1 - \alpha(\theta) = 1 - 0.667 \cos^2 (75^\circ) = \underline{0.9553}$.

Answer: (a) 0.334; (b) 0.667; (c) 0.955

2-15 Using Fig. 3-24, estimate the total absorptivity of typewriter paper for normally incident radiation from a blackbody source at 1050 K.

SOLUTION: From Eqs. (2-55) and (2-24) for a gray surface and assuming that there is no dependence on circumferential angle ϕ ,

$$\alpha(\theta = 0, T_A) = 1 - \rho(\theta = 0, T_A) = 1 - \pi \int_{\theta_r = -\pi/2}^{\pi/2} \rho(\theta = 0, \theta_r, T_A) \cos \theta_r \sin \theta_r d\theta_r$$

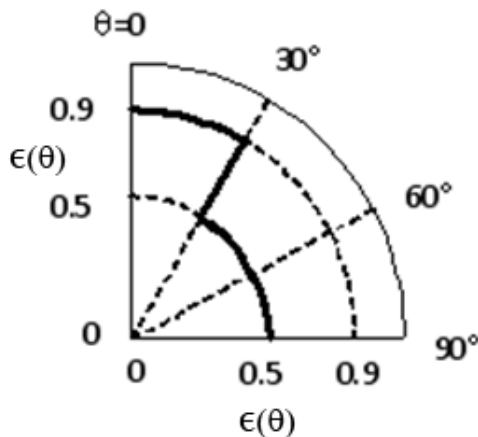
Evaluation requires numerical integration.

2-16 A gray surface has a directional emissivity as shown. The properties are isotropic with respect to circumferential angle ϕ .

(a) What is the hemispherical emissivity of this surface?

(b) If the energy from a blackbody source at 500K is incident uniformly from all directions, what fraction of the incident energy is absorbed by this surface?

(c) If the surface is placed in a very cold environment, at what rate must energy be added per unit area to maintain the surface temperature at 800K?



SOLUTION:

(a) From Eq. (2-6b),

$$\begin{aligned}\epsilon &= 2 \int_{\theta=0}^{\pi/2} \epsilon(\theta) \cos \theta \sin \theta \, d\theta = 2 \left[\int_{\sin \theta=0}^{1/2} 0.9 \sin \theta \, d(\sin \theta) + \int_{\sin \theta=1/2}^1 0.5 \sin \theta \, d(\sin \theta) \right] \\ &= 2 \left[0.9 \frac{\sin^2 \theta}{2} \Big|_{\sin \theta=0}^{1/2} + 0.5 \frac{\sin^2 \theta}{2} \Big|_{\sin \theta=1/2}^1 \right] = 2 \left[0.9 \frac{\left(\frac{1}{4} - 0\right)}{2} + 0.5 \frac{\left(1 - \frac{1}{4}\right)}{2} \right] = \underline{0.600}\end{aligned}$$

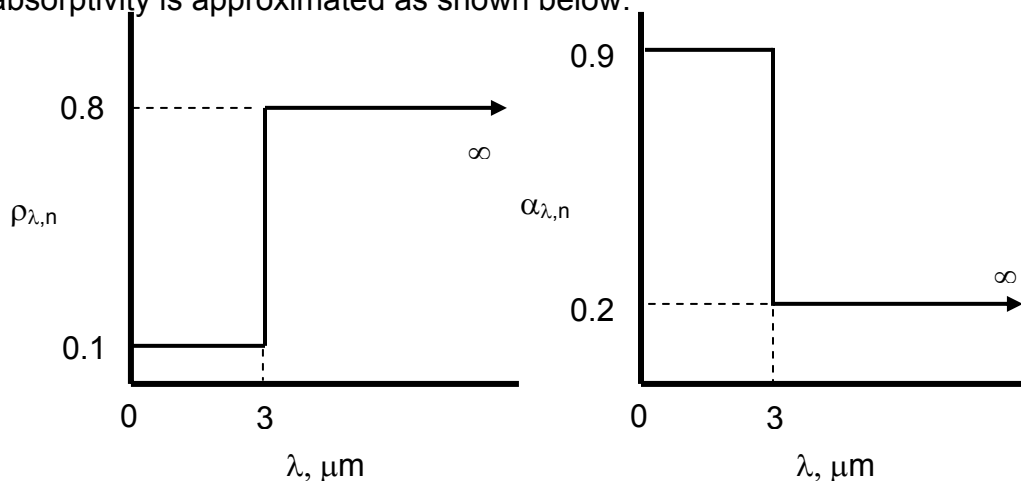
(b) Because the surface is gray, from Table 2-2, $\alpha = \epsilon$. Hence, $\alpha = 0.600$ = fraction absorbed.

(c) Emissive power = $\epsilon \sigma T^4 = 0.600 \times 5.67040 \times 10^{-8} \times 800^4 = \underline{13,936 \text{ W/m}^2}$.

Answer: (a) 0.600; (b) 0.600; (c) 13,936 W/m²

2-17 Using Fig. 3-44, estimate the ratio of normal total solar absorptivity to hemispherical total emissivity for aluminum at a surface temperature of 550 K with a coating of 0.1- μm dendritic lead sulfide crystals. Assume the surface is diffuse. (The solar temperature can be taken as 5780 K.)

SOLUTION: From the figure, approximate the normal hemispherical spectral reflectivity as shown below. Using $\alpha_{\lambda,n} = 1 - \rho_{\lambda,n}$, the spectral normal absorptivity is approximated as shown below:



For $\alpha_{n,\text{solar}}$, use $T_{\text{solar}} = 5780 \text{ K}$: then $(\lambda T)_{\text{cutoff}} = 3 \times 5780$

$= 17340 \mu\text{m} \cdot \text{K}$, and $F_{0 \rightarrow (\lambda T)_{\text{cutoff}}} = 0.97880$ [Eq. (1-33)].

Then $\alpha_{n,\text{solar}} = 0.9 F_{0 \rightarrow (\lambda T)_{\text{cutoff}}} + 0.2 (1 - F_{0 \rightarrow (\lambda T)_{\text{cutoff}}})$

$$= 0.9 \times 0.97880 + 0.2 (1 - 0.97880) = \underline{0.8852}$$

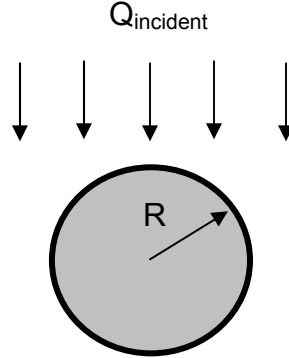
$\epsilon(T = 550 \text{ K}) \approx \epsilon_n(T = 550 \text{ K}) = 0.9 F_{0 \rightarrow 1650} + 0.2 (1 - F_{0 \rightarrow 1650})$

$$= 0.9 \times 0.02388 + 0.2 (1 - 0.02388) = \underline{0.2167} \text{ (NOTE: This assumes that the hemispherical } \epsilon \approx \epsilon_n.)$$

Thus, $\alpha_{n,\text{solar}} / \epsilon(T = 550 \text{ K}) = 0.8852 / 0.2167 = \underline{4.085}$

Answer: 4.085

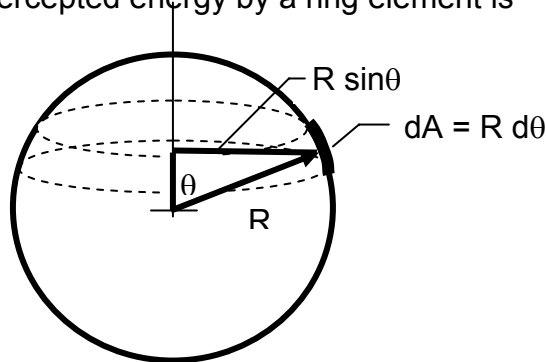
2-18 A gray surface has a directional total emissivity that depends on angle of incidence as $\epsilon(\theta) = 0.850 \cos \theta$. Uniform radiant energy from a single direction normal to the cylinder axis is incident on a long cylinder of radius R . What fraction of energy striking the cylinder is reflected? What is the result if the body is a sphere rather than a cylinder?



SOLUTION: $\alpha(\theta) = \epsilon(\theta) = 0.850 \cos \theta$. $\rho(\theta) = 1 - \alpha(\theta) = 1 - 0.850 \cos \theta$. For the cylinder, the intercepted energy on dA per unit length of cylinder is $q dA \cos \theta = q (R d\theta) \cos \theta$, and the reflected energy per unit length is $q (R d\theta) \cos \theta (1 - 0.850 \cos \theta)$. The ratio is

$$\frac{\text{reflected}}{\text{incident}} = \frac{q \int_{\theta=0}^{\pi/2} R \cos \theta (1 - 0.850 \cos \theta) d\theta}{q \int_{\theta=0}^{\pi/2} R \cos \theta d\theta} = 1 - \frac{0.850\pi}{4} = 0.3324$$

For the sphere, the intercepted energy by a ring element is



$q 2\pi R \sin \theta R d\theta \cos \theta$, and the ratio of reflected to incident energy is

$$\begin{aligned} \frac{\text{reflected}}{\text{incident}} &= \frac{q \int_{\theta=0}^{\pi/2} 2\pi R^2 \sin \theta \cos \theta (1 - 0.850 \cos \theta) d\theta}{q \int_{\theta=0}^{\pi/2} 2\pi R^2 \sin \theta \cos \theta d\theta} \\ &= \frac{\left(\frac{\sin^2 \theta}{2} + \frac{0.850 \cos^3 \theta}{3} \right) \Big|_0^{\pi/2}}{\frac{\sin^2 \theta}{2} \Big|_0^{\pi/2}} = \frac{\frac{1}{2} - \frac{0.850}{3}}{\frac{1}{2}} = 0.433 \end{aligned}$$

Answer: 0.332; 0.433

2-19 A flat metal plate 0.1 m wide by 1.0 m long has a temperature that varies only along the long direction. The temperature is 1100 K at one end, and decreases linearly over the one meter length to 350 K. The hemispherical spectral emissivity of the plate does not change significantly with temperature but is a function of wavelength. The wavelength dependence is approximated by a linear function decreasing from $\epsilon_\lambda = 0.80$ at $\lambda = 0$ to $\epsilon_\lambda = 0.02$ at $\lambda = 10 \mu\text{m}$.

What is the rate of radiative energy loss from one side of the plate? The surroundings are at a very low temperature.

SOLUTION: The rate of energy loss is given by the total emissive power integrated over the plate length, or

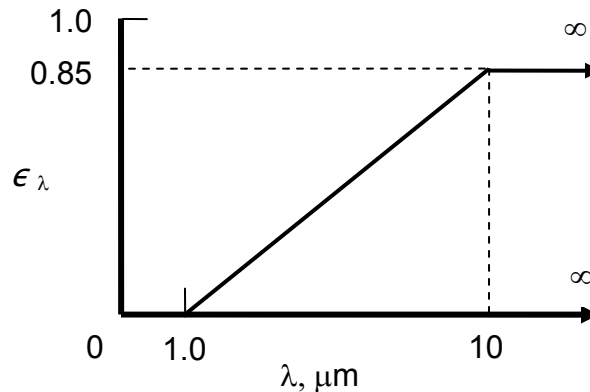
$$Q = 0.1(\text{m}^2) \int_{x=0}^1 \int_{\lambda=0}^{10} \epsilon_\lambda E_{\lambda b}(T) d\lambda dx$$

$$= 0.1 \int_{x=0}^1 \int_{\lambda=0}^{10} \left(0.80 - \frac{0.78\lambda}{10}\right) \frac{2\pi C_1}{\lambda^5 \left\{ \exp \left[\frac{C_2}{\lambda(350 + 750x)} \right] - 1 \right\}} d\lambda dx$$

= 947.1W where the final result is obtained by numerical integration of the double integral.

Answer: 947.1 W

2-20 A thin ceramic plate, insulated on one side, is radiating energy from its exposed side into a vacuum at very low temperature. The plate is initially at 1400 K, and is to cool to 350 K. At any instant, the plate is assumed to be at uniform temperature across its thickness and over its exposed area. The plate is 0.25 cm thick, and the surface hemispherical-spectral emissivity is as shown and is independent of temperature. What is the cooling time? The density of the ceramic is 3200 kg/m^3 , and its specific heat is $710 \text{ J/(kg}\cdot\text{K)}$.



SOLUTION: For an area A, the energy equation is

$$\begin{aligned}
 -V\rho c \frac{dT}{dt} &= qA = A \int_{\lambda=0}^{\infty} \epsilon_{\lambda} \frac{2\pi C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} d\lambda \\
 &= A \int_{\lambda=0}^{10} \frac{0.85(\lambda - 1)}{9} \frac{2\pi C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} d\lambda + A \int_{\lambda=10}^{\infty} 0.85 \frac{2\pi C_1}{\lambda^5 \left[\exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} d\lambda
 \end{aligned}$$

Integrating gives

$$t = \frac{\rho c V}{A} \int_{T=350}^{1400} \frac{dT}{\int_{\lambda=0}^{10} \frac{0.85(\lambda - 1)}{9} \frac{2\pi C_1}{\lambda^5 [\exp(\frac{C_2}{\lambda T}) - 1]} d\lambda + \int_{\lambda=10}^{\infty} 0.85 \frac{2\pi C_1}{\lambda^5 [\exp(\frac{C_2}{\lambda T}) - 1]} d\lambda}$$

Numerical integration is required, and results in t = 1231 s = 20.5 min. (Note: V/A = 0.25x10⁻² m.)

Answer: t = 20.5 min

SOLUTIONS-CHAPTER 3

3-1 An electrical insulator has a refractive index of $n = 1.375$ and has a smooth surface radiating into air. What is the directional emissivity for the direction normal to the surface? What is it for the direction 70° away from the normal?

SOLUTION: From Eq. 3-9 with $n_1 = 1$, $n_2 = 1.375$,

$$\epsilon_n = 1 - [(n_2 - 1)/(n_2 + 1)]^2 = 1 - [(1.375 - 1)/(1.375 + 1)]^2 = \underline{0.9751}.$$

Find χ from Eq. 3-3 using $n_1 = 1$, $n_2 = 1.375$, $\theta = 70^\circ$.

$$\sin \chi = (n_1/n_2) \sin \theta = (1/1.375) \sin 70^\circ = 0.6834$$

Then $\chi = \sin^{-1}(0.6834) = 43.11^\circ$.

$\epsilon(70^\circ) = 1 - \rho(70^\circ)$; using Eq. 3-7a

$$\begin{aligned} \epsilon(70^\circ) &= 1 - (1/2)[\sin^2(\theta - \chi)/\sin^2(\theta + \chi)] \{1 + [\cos^2(\theta + \chi)/\cos^2(\theta - \chi)]\} \\ &= 1 - (1/2)[\sin^2(26.9^\circ)/\sin^2(113.1^\circ)] \{1 + [\cos^2(113.1^\circ)/\cos^2(26.9^\circ)]\} \\ &= 1 - (1/2)(0.4524/0.9198)^2 [1 + (-0.3923/0.8918)^2] = \underline{0.8556} \end{aligned}$$

Answer: 0.9751; 0.8556

3-2 A smooth hot ceramic dielectric sphere with an index of refraction $n = 1.41$ is photographed with an infrared camera. Calculate how bright the image is at locations B and C relative to that at A. (Camera is distant from sphere.)

