

Test Bank Questions

Chapter 1

- Fill in the blanks to rewrite the following statement with variables: Is there an integer with a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7?
 - Is there an integer n such that n has _____?
 - Does there exist _____ such that if n is divided by 4 the remainder is 1 and if _____?
- Fill in the blanks to rewrite the following statement with variables:
Given any positive real number, there is a positive real number that is smaller.
 - Given any positive real number r , there is _____ s such that s is _____.
 - For any _____, _____ such that $s < r$.
- Rewrite the following statement less formally, without using variables:
There is an integer n such that $1/n$ is also an integer.
- Fill in the blanks to rewrite the following statement:
For all objects T , if T is a triangle then T has three sides.
 - All triangles _____.
 - Every triangle _____.
 - If an object is a triangle, then it _____.
 - If T _____, then T _____.
 - For all triangles T , _____.
- Fill in the blanks to rewrite the following statement:
Every real number has an additive inverse.
 - All real numbers _____.
 - For any real number x , there is _____ for x .
 - For all real numbers x , there is real number y such that _____.
- Fill in the blanks to rewrite the following statement:
There is a positive integer that is less than or equal to every positive integer.
 - There is a positive integer m such that m is _____.
 - There is a _____ such that _____ every positive integer.
 - There is a positive integer m which satisfies the property that given any positive integer n , m is _____.
- Write in words how to read the following out loud $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 9\}$.
 - Use the set-roster notation to indicate the elements in the set.

8. (a) Is $\{5\} \in \{1, 3, 5\}$?
 (b) Is $\{5\} \subseteq \{1, 3, 5\}$?
 (c) Is $\{5\} \in \{\{1\}, \{3\}, \{5\}\}$?
 (d) Is $\{5\} \subseteq \{\{1\}, \{3\}, \{5\}\}$?
9. Let $A = \{a, b, c\}$ and $B = \{u, v\}$. Write *a.* $A \times B$ and *b.* $B \times A$.
10. Let $A = \{3, 5, 7\}$ and $B = \{15, 16, 17, 18\}$, and define a relation R from A to B as follows: For all $(x, y) \in A \times B$,
- $$(x, y) \in R \iff \frac{y}{x} \text{ is an integer.}$$
- (a) Is $3 R 15$? Is $3 R 16$? Is $(7, 17) \in R$? Is $(3, 18) \in R$?
 (b) Write R as a set of ordered pairs.
 (c) Write the domain and co-domain of R .
 (d) Draw an arrow diagram for R .
 (e) Is R a function from A to B ? Explain.
11. Define a relation R from \mathbf{R} to \mathbf{R} as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$, $(x, y) \in R$ if, and only if, $x = y^2 + 1$.
- (a) Is $(2, 5) \in R$? Is $(5, 2) \in R$? Is $(-3) R 10$? Is $10 R (-3)$?
 (b) Draw the graph of R in the Cartesian plane.
 (c) Is R a function from \mathbf{R} to \mathbf{R} ? Explain.
12. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Define a function $G: A \rightarrow B$ as follows:
- $$G = \{(1, b), (2, c), (3, b), (4, c)\}.$$
- (a) Find $G(2)$.
 (b) Draw an arrow diagram for G .
13. Define functions F and G from \mathbf{R} to \mathbf{R} by the following formulas:
- $$F(x) = (x + 1)(x - 3) \quad \text{and} \quad G(x) = (x - 2)^2 - 7.$$
- Does $F = G$? Explain.

Chapter 2

1. Which of the following is a negation for “Jim is inside and Jan is at the pool.”
- (a) Jim is inside or Jan is not at the pool.
 (b) Jim is inside or Jan is at the pool.
 (c) Jim is not inside or Jan is at the pool.
 (d) Jim is not inside and Jan is not at the pool.
 (e) Jim is not inside or Jan is not at the pool.

Answers for Test Bank Questions: Chapters 1-4

Please use caution when using these answers. Small differences in wording, notation, or choice of examples or counterexamples may be acceptable.

Chapter 1

1. a. a remainder of 1 when it is divided by 4 and a remainder of 3 when it is divided by 7
b. an integer n ; n is divided by 7 the remainder is 3
2. a. a positive real number; smaller than r
b. positive real number r ; there is a positive real number s
Fill in the blanks to rewrite the following statement with variables:
3. There is an integer whose reciprocal is also an integer.
4. a. have three sides
b. has three sides
c. has three sides
d. is a triangle; has three sides
e. T has three sides
5. a. have additive inverses
b. an additive inverse
c. y is an additive inverse for x
6. a. less than or equal to every positive integer
b. positive integer m ; less than or equal to every positive integer
c. less than or equal to n
7. (a) The set of all integers n such that n is a factor of 9.
Or: The set of all elements n in \mathbf{Z} such that n is a factor of 9.
Or: The set of all elements n in the set of all integers such that n is a factor of 9.
(b) $\{1, 3, 9\}$
8. (a) No
(b) Yes
(c) Yes
(d) No
9. a. $\{(a, u), (a, v), (b, u), (b, v), (c, u), (c, v)\}$
b. $\{(u, a), (v, a), (u, b), (v, b), (u, c), (v, c)\}$
10. a. Yes; No; No; Yes
b. $\{(3, 15), (3, 18), (5, 15)\}$
c. domain is $\{3, 5, 7\}$; co-domain is $\{15, 16, 17, 18\}$.
d. Draw an arrow diagram for R .
e. No: R fails both conditions for being a function from A to B . (1) Elements 5 and 7 in A are not related to any elements in B , and (2) there is an element in A , namely 3, that is related to two different elements in B , namely 15 and 18.

11. a. No; Yes; No; Yes
 b. Draw the graph of R in the Cartesian plane.
 c. No: R fails both conditions for being a function from \mathbf{R} to \mathbf{R} . (1) There are many elements in \mathbf{R} that are not related to any element in \mathbf{R} . For instance, none of 0, $1/2$, and -1 is related to any element of \mathbf{R} . (2) there are elements in \mathbf{R} that are related to two different elements in \mathbf{R} . For instance 2 is related to both 1 and -1 .
12. a. $G(2) = c$
 b. Draw an arrow diagram for G .
13. $F \neq G$. Note that for every real number x ,

$$G(x) = (x - 2)^2 - 7 = x^2 - 4x + 4 - 7 = x^2 - 4x - 3,$$

whereas

$$F(x) = (x + 1)(x - 3) = x^2 - 2x - 3.$$

Thus, for instance,

$$F(1) = (1 + 1)(1 - 3) = -4 \quad \text{whereas} \quad G(1) = (1 - 2)^2 - 7 = -6.$$

Chapter 2

1. e
2. e
3. a. The variable S is not undeclared or the data are not out of order.
 b. The variable S is not undeclared and the data are not out of order.
 c. Al was with Bob on the first, and Al is not innocent.
 d. $-5 > x$ or $x \geq 2$
4. The statement forms are not logically equivalent.

Truth table:

p	q	$\sim p$	$p \vee q$	$\sim p \wedge q$	$p \vee q \rightarrow p$	$p \vee (\sim p \wedge q)$
T	T	F	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	F	T
F	F	T	F	F	T	F

Explanation: The truth table shows that $p \vee q \rightarrow p$ and $p \vee (\sim p \wedge q)$ have different truth values in rows 3 and 4, i.e., when p is false. Therefore $p \vee q \rightarrow p$ and $p \vee (\sim p \wedge q)$ are not logically equivalent.

5. *Sample answers:*

Two statement forms are logically equivalent if, and only if, they always have the same truth values.

Or: Two statement forms are logically equivalent if, and only if, no matter what statements are substituted in a consistent way for their statement variables the resulting statements have the same truth value.

6. *Solution 1:* The given statements are not logically equivalent. Let p be "Sam bought it at Crown Books," and q be "Sam didn't pay full price." Then the two statements have the following form:

$$p \rightarrow q \quad \text{and} \quad p \vee \sim q.$$

The truth tables for these statement forms are

p	q	$\sim q$	$p \rightarrow q$	$p \vee \sim q$
T	T	F	T	T
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T