Chapter 1: Logic and Proof

Exercise 1.1.2 Refer to Figure 1.1.1. The coordinates of the midpoints, M_1 and M_2, are $\left(\frac{c}{2}, \frac{d}{2}\right)$ and $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, respectively. Thus the slope of $\overline{M_1 M_2}$ is

$$\frac{\dfrac{b+d}{2} - \dfrac{d}{2}}{\dfrac{a+c}{2} - \dfrac{c}{2}} = \frac{a}{b},$$

the same as the slope of the segment joining $(0,0)$ with (a,b).

Exercise 1.1.6 The first occurrence is 90, 91, 92, 93, 94, 95, 96.

Exercise 1.2.2 (a) True (b) False (c) True (d) False

Exercise 1.3.3 (a) $3^2 + 4^2 = 5^2$ and $7^2 + 12^2 = 15^2$. (False)
(b) $3^2 + 4^2 = 5^2$ or $7^2 + 12^2 = 15^2$. (True)

Exercise 1.3.5(a) $7^2 + 24^2 \neq 25^2$.
(b) $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} < \dfrac{1+2+3}{2+3+4}$.
(c) In 1983, the ice went out of Lake Minnetonka on or after April 18.
(d) Doug cannot solve the equation $x^2 - 7x - 18 = 0$.
(e) Ambrose scored at least 90 on the last exam.

Exercise 1.3.7 The last column in each truth table is shown.

(a)	(b)	(c)	(d)	(e)
F	T	T	T	T
T	T	F	T	T
F	T	F	T	T
F	T	T	T	T

Exercise 1.3.10 (a)

P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

(b)

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(c)

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Exercise 1.3.11 (a) The function f is either not decreasing on $(-\infty, 0]$ or not increasing on $[0, \infty)$.
(b) The number 0 is in the domain of f and $\lim_{x \to 0} f(x) = f(0)$.

Exercise 1.3.12 (a) $\neg C\left(\tan, \frac{\pi}{2}\right) \wedge \neg C\left(\sec, \frac{\pi}{2}\right)$.
(b) $(a > 0) \vee \neg D(\ln, a)$.
(c) $C(|x|, 0) \wedge \neg D(|x|, 0)$.

Exercise 1.3.13

P	Q	$P \oplus Q$	$P \vee Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$(P \vee Q) \wedge \neg(P \wedge Q)$
T	T	F	T	T	F	F
T	F	T	T	F	T	T
F	T	T	T	F	T	T
F	F	F	F	F	T	F

Exercise 1.3.14

P	Q	R	$Q \oplus R$	$P \oplus (Q \oplus R)$	$P \oplus Q$	$(P \oplus Q) \oplus R$
T	T	T	F	T	F	T
T	T	F	T	F	F	F
T	F	T	T	F	T	F
T	F	F	F	T	T	T
F	T	T	F	F	T	F
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	F

Note that the fifth and last columns are identical.

Exercise 1.4.3

P	Q	$\neg Q$	$P \Rightarrow Q$	$\neg(P \Rightarrow Q)$	$P \wedge \neg Q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Exercise 1.4.7 (a) Converse: If you live in Minnesota, then you live in Minneapolis.
Contrapositive: If you don't live in Minnesota, then you don't live in Minneapolis.
(b) Converse: If n^2 is an even integer, then n is an even integer.
Contrapositive: If n^2 is not an even integer, then n is not an even integer.
(c) Converse: If f is continuous at $x = a$, then f is differentiable at $x = a$.
Contrapositive: If f is not continuous at $x = a$, then f is not differentiable at $x = a$.
(d) Converse: If $x^p + 1$ is not factorable, then p is not prime.
Contrapositive: If $x^p + 1$ is factorable, then p is prime.
(e) Converse: If you don't live in Syracuse, then you live in Minnesota.
Contrapositive: If you live in Syracuse, then you don't live in Minnesota.

Exercise 1.4.9

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$	$P \Leftrightarrow Q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Exercise 1.4.10

P	Q	R	$Q \Rightarrow R$	U	$P \Rightarrow Q$	V	$V \Rightarrow U$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

Note that U is not equivalent to V, so $\Rightarrow$ is not associative. Also note that $V \Rightarrow U$ is true, but the sixth line shows that $U \Rightarrow V$ is false.

Exercise 1.4.11

P	Q	R	$P \Leftrightarrow Q$	$Q \Leftrightarrow P$	$Q \Leftrightarrow R$	$(P \Leftrightarrow Q) \Leftrightarrow R$	$P \Leftrightarrow (Q \Leftrightarrow R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F
T	F	T	F	F	F	F	F
T	F	F	F	F	T	T	T
F	T	T	F	F	T	F	F
F	T	F	F	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	T	F	F

Exercise 1.4.12 (a) Let $P \Rightarrow Q$ be a conditional statement. Then $\neg Q \Rightarrow \neg P$ is its contrapositive. The contrapositive of $\neg Q \Rightarrow \neg P$ is $\neg(\neg P) \Rightarrow \neg(\neg Q)$ which is equivalent to $P \Rightarrow Q$ by Theorem 1.3.9(i).
(b) The converse of $Q \Rightarrow P$ is $P \Rightarrow Q$, the original conditional statement.
(c) Let $P \Rightarrow Q$ be a conditional statement. The contrapositive of $Q \Rightarrow P$ is $\neg P \Rightarrow \neg Q$. The converse of $\neg Q \Rightarrow \neg P$ is $\neg P \Rightarrow \neg Q$.
(d) The conditional $P \Rightarrow Q$ is false only when P is true and Q is false. In this case, $Q \Rightarrow P$ is true.

Exercise 1.4.14 (a)

P	Q	R	$P \Leftrightarrow Q$	$R \wedge P$	$R \wedge Q$	$(R \wedge P) \Leftrightarrow (R \wedge Q)$	$(P \Leftrightarrow Q) \Rightarrow ((R \wedge P) \Leftrightarrow (R \wedge Q))$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	F	T
T	F	F	F	F	F	T	T
F	T	T	F	F	T	F	T
F	T	F	F	F	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	F	F	T	T

(b)

P	Q	R	$P \Leftrightarrow Q$	$R \vee P$	$R \vee Q$	$(R \vee P) \Leftrightarrow (R \vee Q)$	$(P \Leftrightarrow Q) \Rightarrow ((R \vee P) \Leftrightarrow (R \vee Q))$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T

Exercise 1.4.15 (a) Proposition 1.3.9(i); (b) Proposition 1.3.9(ii); (c) Proposition 1.3.9(iii); (d) Proposition 1.3.9(iv); (e) Proposition 1.3.9(v); (f) Proposition 1.4.2; (g) Theorem 1.4.6.

Exercise 1.4.16 (a) Neither; (b) Tautology; (c) Contradiction (see Proposition 1.4.2); (d) Neither; (e) Tautology; (f) Tautology; (g) Tautology; (h) Neither.

Exercise 1.4.17

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \wedge P$	$((P \Rightarrow Q) \wedge P) \Rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T