## **CHAPTER 2**

### *How to Calculate Present Values*

*The values shown in the solutions may be rounded for display purposes. However, the answers were derived using a spreadsheet without any intermediate rounding.*

##

## **Answers to Problem Sets**

 1. a. False. The opportunity cost of capital varies with the risks associated with each individual project or investment. The cost of borrowing is unrelated to these risks.

 b. True. The opportunity cost of capital depends on the risks associated with each project and its cash flows.

c. True. The opportunity cost of capital is dependent on the rates of returns shareholders can

 earn on the own by investing in projects with similar risks

d. False. Bank accounts, within FDIC limits, are considered to be risk-free. Unless an investment is also risk-free, its opportunity cost of capital must be adjusted upward to account for the associated risks.

*Est time: 01-05*

2. The opportunity cost of capital refers to the rate of return a firm’s shareholders could earn on their own by investing at the same level of risk. Thus, when a firm considers a new project, it is the risk level of the project that determines opportunity cost of capital for that project.

*Est time: 01-05*

3. a. In the first year, you will earn $1,000 × 0.04 = $40.00

 b. In the second year, you will earn $1,040 × 0.04 = $41.60

c. By the end of the ninth year, you will accrued a principle of $1,040 × (1.049) = $1,423.31. Therefore, in the Tenth year, you will earn $1,423.31 × 0.04 = $56.93

*Est time: 01-05*

4. The “Rule of 72” is a rule of thumb that says with discrete compounding the time it takes for an investment to double in value is roughly 72/interest rate (in percent).

 Therefore, without a calculator, the Rule of 72 estimate is:

 Time to double = 72 / *r*

Time to double = 72 / 4

Time to double = 18 years , so less than 25 years.

 If you did have a calculator handy, this estimate is verified as followed:

 *Ct* = PV × (1 + *r*)*t*

*t* = ln2 / ln1.04

 *t* = 17.67 years

*Est time: 01-05*

5. a. Using the inflation adjusted 1958 price of $1,060, the real return per annum is:

*$450,300,000* = $1,060 × (1 + *r*)*(2017-1958)*

*r = [$450,300,000/$1,060](1/59 )* – 1 = 0.2456 or 24.56% per annum

b. Using the inflation adjusted 1519 price of $575,000, the real return per annum is:

*$450,300,000* = $575,000 × (1 + *r*)*(2017-1519)*

*r = [$450,300,000/$575,000](1/498 )* – 1 = 0.0135 or 1.35% per annum

*Est time: 01-05*

6. *Ct*= PV × (1 + *r)t*

 *C8* = $100 × 1.158

 *C8* = $305.90

*Est time: 01-05*

7. a. *Ct*= PV × (1 + *r)t*

 *C10* = $100 × 1.0610

 *C10* = $179.08

 b. *Ct*= PV × (1 + *r)t*

 *C20* = $100 × 1.0620

 *C20* = $320.71

 c. *Ct*= PV × (1 + *r)t*

 *C10* = $100 × 1.0410

 *C10* = $148.02

 d. *Ct*= PV × (1 + *r)t*

 *C20* = $100 × 1.0420

 *C20* = $219.11

*Est time: 01-05*

8. *Ct*= PV × (1 + *r)t*

 *C2016* = $100 × 1.345

 *C2016* = $432.04

*Est time: 01-05*

9. a. PV = *Ct* × DF*t*

 *DFt* = $125 / $139

 *DFt*= .8993

b. *Ct*= PV × (1 + *r)t*

 *$139* = $125 × (1+r)5

 *r* = [$139/$125](1/5) – 1 = 0.0215 or 2.15%

*Est time: 01-05*

10. PV = *Ct* / (1 + *r*)t

 PV = $374 / 1.099

 PV = $172.20

*Est time: 01-05*

11. PV = *C1*/ (1 + *r*)1 *+ C2*/ (1 + *r*)2 + *C3*/ (1 + *r*)3

 PV = $432 / 1.15 + $137 / 1.152 + $797 / 1.153

 PV = $1,003.28

 NPV = PV – investment

 NPV = $1,003.28 – 1,200

 NPV = –$196.72

*Est time: 01-05*

12. The basic present value formula is: PV = *C* / (1 + *r*)*t*

a. PV = $100 / 1.0110

PV = $90.53

 b. PV = $100 / 1.1310

PV = $29.46

 c. PV = $100 / 1.2515

PV = $3.52

 d. PV = *C1* / (1 + *r*) + *C2* / (1 + *r*)2 + *C3* / (1 + *r*)3

 PV = $100 / 1.12 + $100 / 1.122 + $100 / 1.123

PV = $240.18

*Est time: 01-05*

13. 

NPV = –$380,000 + $50,000 / 1.12 + $57,000 / 1.122 + $75,000 / 1.123 + $80,000 / 1.124 +

 $85,000 / 1.125 + $92,000 / 1.126 + $92,000 / 1.127 + $80,000 / 1.128 + $68,000 / 1.129

 + $50,000 / 1.1210

 NPV = $23,696.15

*Est time: 01-05*

14. a. NPV = – Investment + *C* × ((1 / *r*) – {1 / [*r*(1 + *r*)*t*]})

 NPV = –$800,000 + $170,000 × ((1 / .14) – {1 / [.14(1.14)10]})

 NPV = $86,739.66

b. After five years, the factory’s value will be the present value of the five remaining year’s of cash flows.

PV = $170,000 × ((1 / .14) – {1 / [.14(1.14)(10 – 5)]})

 PV = $583,623.76

*Est time: 01-05*

15. Use the formula: NPV = –*C*0 + *C*1 / (1 + *r*) + *C2* / (1 + *r*)2

NPV5% = –$700,000 + $30,000 / 1.05 + $870,000 / 1.052

NPV5% = $117,687.07

NPV10%= –$700,000 + $30,000 / 1.10 + $870,000 / 1.102

NPV10%= $46,280.99

NPV15% = –$700,000 + $30,000 / 1.15 + $870,000 / 1.152

 NPV15% = –$16,068.05

The figure below shows that the project has a zero NPV at about 13.65%.

 NPV13.65% = –$700,000 + $30,000 / 1.1365 + $870,000 / 1.13652

 NPV13.65% = –$36.83



*Est time: 06-10*

16. a. NPV = –Investment + PVAoperating cash flows – PVrefits + PVscrap value

NPV = –$8,000,000 + ($5,000,000 – 4,000,000) × ((1 / .08) – {1 / [.08(1.08)15]}) –

 ($2,000,000 / 1.085 + $2,000,000 / 1.0810) + $1,500,000 / 1.0815

 NPV = –$8,000,000 + 8,559,479 – 2,287,553 + 472,863

 NPV = –$1,255,212

b. The cost of borrowing does not affect the NPV because the opportunity cost of capital depends on the use of the funds, not the source.

*Est time: 06-10*

17. NPV = *C* / *r* *–* investment

 NPV = $138 / .09 − $1,548

 NPV = −$14.67

*Est time: 01-05*

18. One way to approach this problem is to solve for the present value of:

(1) $100 per year for 10 years, and

(2) $100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, these present values must be equal, and thus we can solve for the interest rate (*r*).

The present value of $100 per year for 10 years is:

PV = *C* × ((1 / *r*) – {1 /[*r*(1 + *r*)*t*]})

PV = $100 × ((1 / *r*) – {1 /[*r*(1 + *r*)10]})

The present value, as of year 0, of $100 per year forever, with the first payment in year 11, is:

PV = (*C* / *r*) / (1 + *r*)t

PV = ($100 / *r*) / (1 + *r*)10

Equating these two present values, we have:

$100 × ((1 / *r*) – {1 /[*r*(1 + *r*)10]}) = ($100 / *r*) / (1 + *r*)10

Using trial and error or algebraic solution, we find that *r* = 7.18%.

*Est time: 06-10*

19. PV = *C* / (*r* – *g)*

 PV = $4 / (.14 − .04)

 PV = $40

*Est time: 01-05*

20. a. PV = *C* / *r*

 PV = $1 / .10

 PV = $10

 b. PV7 = (*C8* / *r*)

 PV0 approx = (*C8* / *r*) / 2

 PV0 approx = ($1 / .10) / 2

 PV0 approx = $5

c. A perpetuity paying $1 starting now would be worth $10 (part a), whereas a perpetuity starting in year 8 would be worth roughly $5 (part b). Thus, a payment of $1 for the next seven years would also be worth approximately $5 (= $10 – 5).

1. PV = *C* / ( *r* − *g*)

 PV = $10,000 / (.10 − .05)

 PV = $200,000

*Est time: 06-10*

21. a.DF1 = 1 / (1 + r)

 *r* = (1 – .905)/ .905

 *r* = .1050, or 10.50%

b. DF2 = 1 / (1 + *r*)2

 DF2 = 1 / 1.1052

 DF2 = .8190

c. PVAF2 = DF1 + DF2

PVAF2 = .905 + .819

PVAF2 = 1.7240

d. PVA = *C* × PVAF3

PVAF3 =$24.65 / $10

PVAF3 = 2.4650

e. PVAF3 = PVAF2 + DF3

DF3 = 2.465 – 1.7240

DF3 = .7410

*Est time: 06-10*

22. The fact that Kangaroo Autos is offering “free credit” tells us what the cash payments are; it does not change the fact that money has time value.

Present value of payments to Kangaroo Auto:

PV = Down payment + C × ((1 / *r)* – {1 / [*r*(1 + *r*)*t*]})

PV = $1,000 + $300 × ((1 + .0083) – {1 / [.0083(1 + .0083)30]})

PV= $8,938.02

Present value of car at Turtle Motors:

PV = price of car – discount

PV = $10,000 – 1,000

PV = $9,000

Kangaroo Autos offers the best deal because it has the lower present value of costs.

*Est time: 01-05*

23. PV = *Ct* / (1 + *r*)*t*

 PV = $20,000 / 1.105

 PV = $12,418.43

 *C* = PVA / ((1 / *r*) – {1 / [*r*(1 + *r*)*t*]})

 *C* = $12,418.43 / ((1 / .10) – {1 / [.10 (1 + .10)5]})

 *C* = $3,275.95

*Est time: 06-10*

24. *C* = PVA / ((1 / *r*) – {1 / [*r*(1 + *r*)*t*]})

 *C* = $20,000 / ((1 / .08) – {1 / [.08(1 + .08)12]})

 *C* = $2,653.90

*Est time: 01-05*

25. a. PV = *C* ×((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

 PV = ($9,420,713 / 19) × ((1 / .08) – {1 / [.08(1 + .08)19]})

 PV = $4,761,724

 b. PV = *C* ×((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

 $4,200,000 = ($9,420,713 / 19) × ((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

Using Excel or a financial calculator, we find that *r* = 9.81%.

*Est time: 06-10*

26. a. PV = *C* ×((1 / *r*) – {1 / [*r(1 + r*)*t*]})

 PV = $50,000 × ((1 / .055) – {1 / [.055(1 + .055)12]})

 PV = $430,925.89

b. Since the payments now arrive six months earlier than previously:

PV = $430,925.89 × {1 + [(1 + .055).5 – 1]}

PV = $442,617.74

*Est time: 06-10*

27. *Ct* = PV × (1 + *r*)t

*Ct* = $1,000,000 × (1.035)3

 *Ct* = $1,108,718

 Annual retirement shortfall = 12 × (monthly aftertax pension + monthly aftertax Social Security –

 monthly living expenses)

Annual retirement shortfall = 12 × ($7,500 + 1,500 – 15,000)

Annual retirement shortfall = –$72,000

The withdrawals are an annuity due, so:

PV = *C* × ((1 / r) – {1 / [*r*(1 + *r*)*t*]}) × (1 + *r*)

$1,108,718 = $72,000 × ((1 / .035) – {1 / [.035(1 + .035)*t*]})× (1 + .035)

14.878127 = (1 / .035) – {1 / [.035(1 + .035)t]}

13.693302 = 1 / [.035(1 + .035)*t*]

.073028 / .035 = 1.035*t*

*t* = ln2.086514 / ln1.035

*t* = 21.38 years

*Est time: 06-10*

28. a. PV = *C* / *r*

 PV = $1 billion / .08

 PV = $12.5 billion

1. PV = *C* / (*r* – *g)*

PV = $1 billion / (.08 – .04)

PV = $25.0 billion

c. PV = *C* × ((1 / *r*) – {1 / [*r* (1 + *r*)*t*]})

 PV = $1 billion × ((1 / .08) – {1 / [.08(1 + .08)20]})

 PV = $9.818 billion

d. The continuously compounded equivalent to an annually compounded rate of 8% is approximately 7.7%, which is computed as:

Ln(1.08) = .077, or 7.7%

PV = *C* × {(1 / *r*) – [1 / (*r* × e*rt*)]}

PV = $1 billion × {(1 / .077) – [1 / (.077 – e.077 × 20)]}

PV = $10.206 billion

This result is greater than the answer in Part (c) because the endowment is now earning interest during the entire year.

*Est time: 06-10*

29. a. PV = *C* × ((1 / *r*) – {1 / [*r*(1 + *r*)*t*]})

 PV = $2.0 million × ((1 / *.08*) – {1 / [*.08*(1.08)*20*]})

 PV = $19.64 million

b. If each cashflow arrives one year earlier, then you can simply compound the PV calculated in part a by (1+r)🡪 $19.64 million × (1.08) = $21.21 million

*Est time: 06-10*

30. a. Start by calculating the present value of an annuity due assuming a price of $1:

 PV = 0.25 + 0.25 × ((1 / *.05*) – {1 / [*.05*(1.05)*3*]})

 PV = 0.93, therefore it is better to pay instantly at a lower cost of 0.90 [1 × 0.9]

 b. Recalculate, except this time using an ordinary annuity:

 PV = 0.25 × ((1 / *.05*) – {1 / [*.05*(1.05)4]})

 PV = 0.89, therefore it is better to take the financing deal as it costs less than 0.90.

*Est time: 06-10*

31. a. Using the annuity formula:

 PV = $70,000 × ((1 / *.08*) – {1 / [*.08*(1 + .08)*8*]})

 PV = $402,264.73

 b. The amortization table follows:

 

*Est time: 06-10*

32. a. PV = *C* × ((1 / *r*) – {1 / [*r*(1 + *r*)*t*]})

 *C* = PV / ((1 / *r*) – {1 / [*r* (1 + *r*)*t*]})

 *C* = $200,000 / ((1 / .06) – {1 / [.06(1 + .06)20]})

 *C* = $17,436.91

 b.

c.Interest percent of first payment = Interest1 / Payment

Interest percent of first payment = (.06 × $200,000) / $17,436.91

Interest percent of first payment = .6882, or 68.82%

Interest percent of last payment = Interest20 / Payment

Interest percent of last payment = $986.99 / $17,436.91

Interest percent of last payment = .0566, or 5.66%

Without creating an amortization schedule, the interest percent of the last payment can be computed as:

Interest percent of last payment = 1 – {[Payment / (1 + *r*)] / Payment}

Interest percent of last payment = 1 – [($17,436.91 / 1.06) / $17,436.91]

Interest percent of last payment = .0566, or 5.66%

After 10 years, the balance is:

PV10 = *C* × ((1 + *r*) – {1 / [*r* × (1 + *r*)*t*]})

PV10 = $17,436.91 × {1.06 – [1 / (.06 × 1.0610)]}

PV10 = $128,337.19

Fraction of loan paid off = (Loan amount – PV10) / Loan amount

Fraction of loan paid off = ($200,000 – 128,337.19) / $200,000

Fraction of loan paid off = .3583, or 35.83%

*Est time: 16-20*

33. a. PV = *Ct* / (1 + *r*)*t*

 PV = $10,000 / 1.055

 PV = $7,835.26

 b. PV = *C*((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

 PV = $12,000((1 / .08) – {1 / [.08(1.08)6]})

 PV = $55,474.56

 c. *Ct* = PV × (1 + *r*)t

 *Ct* = ($60,476 − 55,474.56) × 1.086

*Ct =* $7,936.66

*Est time: 06-10*

34. a. PV = *C* ×((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

 *C* = $2,000,000 / ((1 / .08) – {1 / [.08(1 + .08)15]})

 *C* = $233,659.09

 b. *r* = (1 + R) / (1 + *h*) – 1

 *r* = 1.08 / 1.04 – 1

 *r* = .0385, or 3.85%

 PV = *C* ×((1 / *r*) – {1 / [*r*(*1 + r*)*t*]})

 *C* = $2,000,000 / ((1 / .0385) – {1 / [.0385(1 +.0385)15]})

 *C* = $177,952.49

 The retirement expenditure amount will increase by 4% annually.

*Est time: 06-10*

35. Calculate the present value of a growing annuity for option 1, then compare this amount with the option to pay instantly $12,750:

PV = *C* × ([1 / (*r* – *g*)] – {(1 + *g*)*t* / [(*r* – *g*) *× (1 + r*)*t*]})

 PV = $5,000 × ([1 / (.10 – .06)] – {(1 + .06)3 / [(.10 – .06) × (1 + .10)3]})

 PV = $13,146.51

Since the $13,147 present value of the three year growing annual membership dues exceeds the single $12,750 payment for three years, it is better to pay the lower upfront 3-year dues.

*Est time: 06-10*

36. a. PV = C0

 PV = $100,000

b. PV = *C*t / (1 + *r*)*t*

PV = $180,000 / 1.125

PV = $102,136.83

c. PV = *C* / *r*

PV = $11,400 / .12

PV = $95,000

d. PV = *C* × ((1 / r) – {1 / – [*r*(1 + *r*)t]})

 PV = $19,000 × ((1 / .12) – {1 / – [.12(1.12)10]})

 PV = $107,354.24

e. PV = *C* / (*r* – *g)*

PV = $6,500 / (.12 − .05)

PV = $92,857.14

Prize (d) is the most valuable because it has the highest present value.

*Est time: 01-05*

37. a. PV =*C* / *r*

PV = $2,000,000 / .12

 PV = $16,666,667

 b. PV = *C ×* ((1 / *r*) – {1 / [*r*(1 + r)*t*]})

 PV = $2,000,000 × ((1 / .12) – {1 / [.12(1 + .12)20]})

 PV = $14,938,887

 c. PV = *C* / (*r – g*)

 PV = $2,000,000 / (.12 – .03)

 PV = $22,222,222

 d. PV = *C* × ([1 / (*r* – *g*)] – {(1 + *g*)*t* / [(*r* – *g*) *× (1 + r*)*t*]})

 PV = $2,000,000 × ([1 / (.12 – .03)] – {(1 + .03)20 / [(.12 – .03) × (1 + .12)20]})

 PV = $18,061,473

*Est time: 06-10*

38. First, find the semiannual rate that is equivalent to the annual rate:

1 + *r* = (1 + *r* / 2)2

1.08 = (1 + *r* / 2)2

*r* / 2 = 1.08.5 – 1

*r* = .039230, or 3.9230%

PV = *C* 0 + *C* × ((1 / *r*) – {1 / [*r*(*1 +* r)t]})

PV = $100,000 + $100,000 × ((1 / .039230) – {1 / [.039230(1 + .039230)9]})

PV = $846,147.28

*Est time: 06-10*

39. **a.** *Ct* = PV × (1 + *r*)*t*

 *C1* = $1 × 1.121 = $1.1200

*C5*  = $1 × 1.125 = $1.7623

*C10* = $1 × 1.1210 = $9.6463

 **b.** *Ct* = PV × (1 + *r / m*)m*t*

 *C1* = $1 × [1 + (.117 / 2)2 × 1 = $1.1204

*C5*  = $1 × [1 + (.117 / 2)2 × 5 = $1.7657

*C10* = $1 × [1 + (.117 / 2)2 × 20 = $9.7193

 **c.** *Ct* = PV × em*t*

*C1* = $1 × e(.115 × 1)  = $1.1219

 *C5*  = $1 × e(.115 × 5)  = $1.7771

*C10* = $1 × e(.115 × 20) = $9.9742

The preferred investment is (c) because it compounds interest faster and produces the highest future value at any point in time.

*Est time: 06-10*

40.

* 1. *Ct* = PV × (1 + *r*)*t*

 *Ct* = $10,000,000 x (1.06)4

 *Ct* = $12,624,770

b. *Ct* = PV × [1+ (*r / m*)*mt*

 *Ct* = $10,000,000 × [1 + (.06 / 12)]12 × 4

 *Ct* = $12,704,892

* 1. *Ct* = PV × *ert*

 *Ct* = $10,000,000 × *e*.06 × 4

 *Ct* = $12,712,492

*Est time: 01-05*

41. a. PVOA = *C* / *r*

 PVOA = $100/ .07

 PVOA = $1,428.57

 b. PVAD = *C* / *r* × (1 + *r*)

 PVAD = $100/ .07× (1 + .07)

 PVAD = $1,528.57

c. To find the present value with payments spread evenly over the year, use the continuously compounded rate that equates to 7% compounded annually. This rate is found using natural logarithms.

 PVCC = *C* / *r*CC

 PVCC = $100/ Ln(1 + .07)

 PVCC = $1,478.01

For informational purposes, the continuously compounded rate is:

Ln(1 + .07) = .0677, or 6.77%

The sooner payments are received, the more valuable they are.

*Est time: 06-10*

42. Annual compounding:

*Ct* = PV × (1 + *r*)*t*

*C20* = $100 × 1.1520

*C20* = $1,636.65

Continuous compounding:

*Ct* = PV × e*rt*

*C20* = $100 × e.15 × 20

*C20* = $2,008.55

*Est time: 01-05*

43. a. FV = *C × ert*

 FV = $1,000 × *e*.12 x 5

 FV = $1,822.12

 b. PV = *C / ert*

 PV = $5,000,000 / e.12 × 8

 PV = $1,914,464

 c. PV = *C* (1 / *r*  – 1 / *rert*)

 PV = $2,000 (1 / .12 – 1 / .12*e* .12 x 15)

 PV = $13,911.69

*Est time: 01-05*

44. a. Rule of 72 estimate:

 Time to double = 72 / *r*

Time to double = 72 / 12

Time to double = 6 years

Exact time to double:

 *Ct* = PV × (1 + *r*)*t*

*t* = ln2 / ln1.12

 *t* = 6.12 years

1. With continuous compounding for interest rate *r* and time period *t*:

e*rt* = 2

*rt* = ln2

Solving for *t* when *r* is expressed as a decimal:

*rt* = .693

*t* = .693 / *r*

*Est time: 06-10*

45. Spreadsheet exercise

*Est time: 11-15*

46. a. PV = *C* / (*r* – *g*)

 PV = $2,000,000 / [.10 – (–.04)]

 PV = $14,285,714

b. PV20 = *C*21 / (*r* – *g*)

 PV20 = {$2,000,000 × [1 + (–.04)]20} / [.10 – (–.04)]

 PV20 = $6,314,320

 PV cash flows 1-20 = PV – PV20 / (1 + *r*)20

PV cash flows 1-20 = $14,285,714 – ($6,314,320 / 1.1020)

PV cash flows 1-20 = $13,347,131

*Est time: 06-10*