|  |  |
| --- | --- |
| **P1.1** A steel bar of rectangular cross section, 15 mm by 60 mm, is loaded by a compressive force of 110 kN that acts in the longitudinal direction of the bar. Compute the average normal stress in the bar. |  |

### Solution

The cross-sectional area of the steel bar is



The normal stress in the bar is

 **Ans.**

|  |  |
| --- | --- |
| **P1.2** A circular pipe with outside diameter of 4.5 in. and wall thickness of 0.375 in. is subjected to an axial tensile force of 42,000 lb. Compute the average normal stress in the pipe. |  |

### Solution

The outside diameter *D*, the inside diameter *d*, and the wall thickness *t* are related by



Therefore, the inside diameter of the pipe is



The cross-sectional area of the pipe is



The average normal stress in the pipe is

 **Ans.**

|  |
| --- |
| **P1.3** A circular pipe with an outside diameter of 80 mm is subjected to an axial compressive force of 420 kN. The average normal stress may not exceed 130 MPa. Compute the minimum wall thickness required for the pipe. |

### Solution

From the definition of normal stress, solve for the minimum area required to support a 420 kN load without exceeding a normal stress of 130 MPa



The cross-sectional area of the pipe is given by



Set this expression equal to the minimum area and solve for the maximum inside diameter *d*



The outside diameter *D*, the inside diameter *d*, and the wall thickness *t* are related by



Therefore, the minimum wall thickness required for the aluminum tube is

 **Ans.**

|  |
| --- |
| **P1.4** Three solid bars, each with square cross sections, make up the axial assembly shown in Figure P1.4/5. Two loads of *P* = 30 kN are applied to the assembly at flange *B*, two loads of *Q* = 18 kN are applied at *C*, and one load of *R* = 42 kN is applied at end *D*. The bar dimensions are *b*1 = 60 mm, *b*2 = 20 mm, and *b*3 = 40 mm. Determine the normal stress in each bar. |
| FIGURE P1.4/5 |

### Solution

|  |
| --- |
| Cut an FBD through bar (1). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (1) is tension. From equilibrium, the force in bar (1) is |
|  |
| From the given width of bar (1), the cross-sectional area of bar (1) is    and thus, the normal stress in bar (1) is  **Ans.** |
|  |

|  |
| --- |
| Cut an FBD through bar (2). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (2) is tension. From equilibrium, the force in bar (2) is |
|  |
| From the given width of bar (2), the cross-sectional area of bar (2) is    The normal stress in bar (2) is  **Ans.** |

|  |
| --- |
| Cut an FBD through bar (3). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (3) is tension. From equilibrium, the force in bar (3) is |
|  |
| The cross-sectional area of bar (3) is    The normal stress in bar (3) is  **Ans.** |
|  |

|  |
| --- |
| **P1.5** Three solid bars, each with square cross sections, make up the axial assembly shown in Figure P1.4/5. Two loads of *P* = 25 kN are applied to the assembly at flange *B*, two loads of *Q* = 15 kN are applied at *C*, and one load of *R* = 35 kN is applied at end *D*. Bar (1) has a width of *b*1 = 90 mm. Calculate the width *b*2 required for bar (2) if the normal stress magnitude in bar (2) must equal the normal stress magnitude in bar (1). |
| FIGURE P1.4/5 |

### Solution

|  |
| --- |
| Cut an FBD through bar (1). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (1) is tension. From equilibrium, the force in bar (1) is |
|  |
| From the given width of bar (1), the cross-sectional area of bar (1) is    and thus, the normal stress in bar (1) is |
|  |

|  |
| --- |
| Cut an FBD through bar (2). The FBD should include the free end of the assembly at *D*. We will assume that the internal force in bar (2) is tension. From equilibrium, the force in bar (2) is |
|  |
| The normal stress in bar (2) must equal the normal stress in bar (1). Thus,    Solve for the required area of bar (2):    The width of bar (2) is therefore:  **Ans.** |

|  |
| --- |
| **P1.6** Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.6/7. One load of *P* = 1,500 lb is applied to the assembly at *A*, two loads of *Q* = 900 lb are applied at *B*, and two loads of *R* = 1,300 lb are applied at *C*. The diameters of rods (1), (2), and (3) are *d*1 = 0.625 in., *d*2 = 0.500 in., and *d*3 = 0.875 in. Determine the axial normal stress in each of the three rods. |
| FIGURE P1.6/7 |

### Solution

|  |
| --- |
| Cut an FBD through rod (1). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (1) is tension. From equilibrium, the force in rod (1) is |
|  |
| Use the given diameter to calculate the cross-sectional area of rod (1):    The normal stress in rod (1) is  **Ans.** |

|  |
| --- |
| Cut an FBD through rod (2). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (2) is tension. From equilibrium, the force in rod (2) is |
|  |
| Use the given diameter to calculate the cross-sectional area of rod (2):    The normal stress in rod (2) is  **Ans.** |

|  |
| --- |
| Cut an FBD through rod (3). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (3) is tension. From equilibrium, the force in rod (3) is |
|  |
| Use the given diameter to calculate the cross-sectional area of rod (3):    The normal stress in rod (3) is  **Ans.** |

|  |
| --- |
| **P1.7** Axial loads are applied with rigid bearing plates to the solid cylindrical rods shown in Figure P1.6/7. One load of *P* = 30 kips is applied to the assembly at *A*, two loads of *Q* = 25 kips are applied at *B*, and two loads of *R* = 35 kips are applied at *C*. The normal stress magnitude in aluminum rod (1) must be limited to 20 ksi. The normal stress magnitude in steel rod (2) must be limited to 35 ksi. The normal stress magnitude in brass rod (3) must be limited to 25 ksi. Determine the minimum diameter required for each of the three rods. |
| FIGURE P1.6/7 |

### Solution

|  |
| --- |
| Cut an FBD through aluminum rod (1). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (1) is tension. From equilibrium, the force in rod (1) is |
|  |
| The normal stress magnitude in aluminum rod (1) must be limited to 20 ksi. Therefore, the minimum cross-sectional area of rod (1) must be    The diameter must be  **Ans.** |

|  |
| --- |
| Cut an FBD through steel rod (2). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (2) is tension. From equilibrium, the force in rod (2) is |
|  |

|  |
| --- |
| The normal stress magnitude in steel rod (2) must be limited to 35 ksi. Therefore, the minimum cross-sectional area of rod (2) must be    The diameter of rod (2) must be  **Ans.** |

|  |
| --- |
| Cut an FBD through brass rod (3). The FBD should include the free end of the assembly at *A*. We will assume that the internal force in rod (3) is tension. From equilibrium, the force in rod (3) is |
|  |
| The normal stress magnitude in brass rod (3) must be limited to 25 ksi. Therefore, the minimum cross-sectional area of rod (3) must be    The diameter of rod (3) must be  **Ans.** |

|  |  |
| --- | --- |
| **P1.8** Determine the normal stress in rod (1) for the mechanism shown in Figure P1.8. The diameter of rod (1) is 8 mm, and load *P* = 2,300 N. Use the following dimensions: *a* = 120 mm, *b* = 200 mm, *c* = 170 mm, and *d* = 90 mm. | FIGURE P1.8 |

### Solution

First, consider an FBD of the pulley to determine the reaction forces exerted on the pulley by the mechanism.



|  |  |
| --- | --- |
| FBD of pulley | FBD of mechanism |

Next, consider an FBD of the mechanism to determine the force in rod (1). Rod (1) is oriented at an angle of:



Rod (1) is a two-force member, and its axial force can be calculated from:



The area of rod (1) is



The normal stress in the rod is thus

 **Ans.**

|  |  |
| --- | --- |
| **P1.9** Determine the normal stress in bar (1) for the mechanism shown in Figure P1.9. The area of bar (1) is 2,600 mm2. The distributed load intensities are *wC* = 12 kN/m and *wD* = 30 kN/m. Use the following dimensions: *a* = 7.5 m and *b* = 3.0 m. | FIGURE P1.9 |

### Solution

|  |  |
| --- | --- |
| Consider an FBD of the mechanism. Determine the angle ** between rod (1) and the horizontal axis:    Write an equilibrium equation for the sum of moments about *C* to compute the force in bar (1). Note: Bar (1) is a two-force member. |  |



The normal stress in bar (1) is thus:

 **Ans.**

|  |  |
| --- | --- |
| **P1.10** The rigid beam *BC* shown in Figure P1.10 is supported by rods (1) and (2) that have diameters of 0.875 in. and 1.125 in., respectively. For a uniformly distributed load of *w* = 4,200 lb/ft, determine the normal stress in each rod. Assume *L* = 14 ft and *a* = 9 ft. | P01  FIGURE P1.10 |

### Solution

#### **Equilibrium:** Calculate the internal forces in rods (1) and (2).



**Areas:**



**Stresses:**

 **Ans.**

 **Ans.**

|  |  |
| --- | --- |
| **P1.11** The rigid beam *ABC* shown in Figure P1.11 is supported by a pin connection at *C* and by steel rod (1), which has a diameter of 10 mm. If the normal stress in rod (1) must not exceed 225 MPa, what is the maximum uniformly distributed load *w* that may be applied to beam *ABC*? Use dimensions of *a* = 340 mm, *b* = 760 mm, and *c* = 550 mm. |  |
|  | FIGURE P1.11 |

### Solution

The cross-sectional area of rod (1) is



Since the normal stress in rod (1) must not exceed 225 MPa, the allowable force that can be applied to rod (1) is:



Rod (1) is oriented at an angle of ** with respect to the horizontal direction:



|  |  |
| --- | --- |
| Consider an FBD of rigid beam *ABC*. From the moment equilibrium equation about joint *C*, the relationship between the force in rod (1) and the distributed load *w* is: |  |

Substitute the allowable force *F*1,allow into this relationship to obtain the maximum distributed load that may be applied to the structure:

 **Ans.**

|  |  |
| --- | --- |
| **P1.12** A simple pin-connected truss is loaded and supported as shown in Figure P1.12. The load *P* is 200 kN. All members of the truss are aluminum pipes that have an outside diameter of 115 mm and a wall thickness of 6 mm. Determine the normal stress in each truss member. Assume truss dimensions of *a* = 12.0 m, *b* = 7.5 m, and *c* = 6.0 m. | FIGURE P1.12 |

### Solution

|  |  |
| --- | --- |
| **Overall equilibrium:** Begin the solution by determining the external reaction forces acting on the truss at supports *B* and *D*. Write equilibrium equations that include all *external* forces. Note that only the external forces (i.e., loads and reaction forces) are considered at this time. The internal forces acting in the truss members will be considered after the external reactions have been computed. The free-body diagram (FBD) of the entire truss is shown. The following equilibrium equations can be written for this structure: |  |





#### **Method of joints:**

Before beginning the process of determining the internal forces in the axial members, the geometry of the truss will be used to determine the magnitude of the inclination angles of members *AC* and *BC*. Use the definition of the tangent function to determine *AC* and *BC*:



|  |  |
| --- | --- |
| Joint A: Begin the solution process by considering an FBD of joint *A*. Consider only those forces acting directly on joint *A*. In this instance, two axial members, *AB* and *AC*, are connected at joint *A*. Tension forces will be assumed in each truss member.  (a)  (b) |  |

Solve Eq. (b) for *FAC*:



and then compute *FAB* using Eq. (a):

**

|  |  |
| --- | --- |
| Joint D: Next, consider an FBD of joint *D*. As before, tension forces will be assumed in each truss member.  (c)  (d) |  |

Solve Eq. (c) for *FCD*:



and solve Eq. (d) for *FBD*:

**

|  |  |
| --- | --- |
| Joint C: Next, consider an FBD of joint *C*. As before, tension forces will be assumed in each truss member.  (e)  (f) |  |

Solve Eq. (e) for *FBC*:



Eq. (f) can be used as a check on our calculations:

 **Checks!**

#### **Section properties:**

For each of the five truss members:



**Normal stress in each truss member:**

** Ans.**

** Ans.**

** Ans.**

** Ans.**

** Ans.**

|  |  |
| --- | --- |
| **P1.13** A horizontal load *P* is applied to an assembly consisting of two inclined bars, as shown in Figure 1.13. The cross-sectional area of bar (1) is 1.5 in.2, and the cross-sectional area of bar (2) is 1.8 in.2. The normal stress in either bar may not exceed 24 ksi. Determine the maximum load *P* that may be applied to this assembly. Assume dimensions of *a* = 16 ft, *b* = 8 ft, and *c* = 13 ft. | FIGURE P1.13 |

### Solution

**Allowable member forces:**

Using the allowable stresses and the member areas, we can determine the allowable force for each member:

 (a)

 (b)

**Equilibrium:**

The geometry of the two-bar assembly will be used to determine the magnitude of the inclination angles for members *AB* and *BC*. We can use the definition of the tangent function to determine *AB* and *BC*:



|  |  |
| --- | --- |
| Consider a free-body diagram (FBD) of joint *B*. The following equilibrium equations can be written for this joint:  (c)  (d) |  |

**Erroneous approach for finding maximum load *P*:**

#### Since we are trying to calculate P, the temptation at this point in the solution is to substitute the values from Equations (a) and (b) into Eq. (c) and simply solve for P:

 (e)

However, if we use the values from Equations (a) and (b) in Eq. (d), we find that equilibrium is not satisfied:



Equilibrium must always be satisfied; therefore, we must conclude that *F*1 and *F*2 will not have the allowable values of Equations (a) and (b). **The answer obtained in Eq. (e) is incorrect** because equilibrium is not satisfied.

**Correct method for calculating the capacity of the two-bar assembly:**

The allowable load that can be applied to this two-bar assembly will be the load *P* that produces the allowable load in either member (1) or member (2). Let’s return to Eq. (d), only this time, we are going to make an assumption. We will assume that the force in member (1) will control the capacity of the two-bar assembly. If this assumption is true, then the force in member (1) will equal its allowable force as given in Eq. (a), and the force in member (2) will be less than its allowable force as given in Eq. (b).

**N.G.**

This calculation shows that the force in member (2) will exceed its allowable force when the force in member (1) equals its allowable force. Therefore, our assumption is proved incorrect. This result shows us that the force in member (2) will control the capacity of the two-bar assembly. We’ll return to Eq. (d), only this time, we know that member (2) will control. Set the force in member (2) to its allowable force from Eq. (b) and solve for the force in member (1) that is required to satisfy equilibrium.

**O.K.**

We now know the forces in members (1) and (2) that will satisfy the equilibrium equations without exceeding the allowable force in either member. Finally, we use these values to determine the load *P* from Eq. (c):

 **Ans.**

|  |  |
| --- | --- |
| **P1.14** The rectangular bar shown in Figure P1.14 is subjected to a uniformly distributed axial loading of *w* = 13 kN/m and a concentrated force of *P* = 9 kN at *B*. Determine the magnitude of the maximum normal stress in the bar and its location *x*. Assume *a* = 0.5 m, *b* = 0.7 m, *c* = 15 mm, and *d* = 40 mm. | P01 |
|  | FIGURE P1.14 |

### Solution

**Equilibrium:**

|  |  |
| --- | --- |
| Draw an FBD for the interval between *A* and *B* where . Write the following equilibrium equation:  The largest force in this interval occurs at *x* = 0 where *F* = 6.6 kN. | S01 |
| In the interval between *B* and *C* where , and write the following equilibrium equation:    The largest force in this interval occurs at *x* = *a* where *F* = 9.1 kN. | S01 |

**Maximum Normal Stress:**

 **Ans.**

|  |  |
| --- | --- |
| **P1.15** The solid 1.25 in. diameter rod shown in Figure P1.15 is subjected to a uniform axial distributed loading along its length of *w* = 750 lb/ft. Two concentrated loads also act on the rod: *P* = 2,000 lb and *Q* = 1,000 lb. Assume *a* = 16 in. and *b* = 32 in. Determine the normal stress in the rod at the following locations:  (a) *x* = 10 in.  (b) *x* = 30 in. | P01 |
|  | FIGURE P1.15 |

### Solution

|  |  |
| --- | --- |
| **(a) *x* = 10 in.**  **Equilibrium:** Draw an FBD for the interval between *A* and *B* where , and write the following equilibrium equation:  At *x* = 10 in., *F* = 5,375 lb. | S01 |

**Stress:** The normal stress at this location can be calculated as follows.

 **Ans.**

|  |  |
| --- | --- |
| **(b) *x* = 30 in.**  **Equilibrium:** Draw an FBD for the interval between *B* and *C* where , and write the following equilibrium equation:  At *x* = 30 in., *F* = 2,125 lb. | S01 |

**Stress:** The normal stress at this location can be calculated as follows.

 **Ans.**

|  |  |
| --- | --- |
| **P1.16** A block of wood is tested in direct shear using the test fixture shown below. The dimensions of the test specimen are *a* = 3.75 in., *b* = 1.25 in., *c* = 2.50 in., and *d* = 6.0 in. During the test, a load of *P* = 1,590 lb produces a shear failure in the wood specimen. What is the magnitude of the average shear stress in the wood specimen at failure? | FIGURE P1.16 |

|  |  |
| --- | --- |
| Solution Visualize the surface that will be exposed when the specimen fails. The area of this surface will be    The average shear stress in the specimen at failure is thus  **Ans.** |  |

|  |  |
| --- | --- |
| **P1.17** A cylindrical rod of diameter *d* = 0.625 in. is attached to a plate by a cylindrical rubber grommet. The plate has a thickness of *t* = 0.875 in. If the axial load on the rod is *P* = 175 lb, what is the average shear stress on the cylindrical surface of contact between the rod and the grommet? | FIGURE P1.17 |

### Solution

Visualize the contact surface between the rod and the grommet. It will be a cylinder with a diameter of *d* and a height of *t*. The area of this cylinder will be



The average shear stress between the rod and the grommet is thus

 **Ans.**

|  |  |
| --- | --- |
| **P1.18** Two wood boards, each 19 mm thick, are joined by the glued finger joint shown in Figure P1.18. The finger joint will fail when the average shear stress in the glue reaches 940 kPa. Determine the shortest allowable length *d* of the cuts if the joint is to withstand an axial load of *P* = 5.5 kN. Use *a* = 23 mm and *b* = 184 mm. | FIGURE P1.18 |

### Solution

We are considering the shear strength of the glued joint. The minimum shear area that is required for this connection can be determined from the load *P* and the shear strength of the glue. Consequently, we will need at least this much area



to transmit the load *P* through the joint, based on the shear strength of the glue.

For this particular joint, there are seven surfaces that will be glued. Each of these surfaces has a length of *d* and a thickness of 19 mm. Accordingly, the minimum length *d* required for each of the finger joints is

 **Ans.**

|  |  |
| --- | --- |
| **P1.19** For the connection shown in Figure P1.19, determine the average shear stress produced in the 7/8 in. diameter bolts if the applied load is *P* = 32,000 lb. |  |
|  | FIGURE P1.19 |

### Solution

There are three bolts, and it is always assumed that each bolt supports an equal portion of the external load *P*. Therefore, the shear force *V* carried by each bolt is



The bolts in this connection act in single shear. The cross-sectional area of a single bolt is



Therefore, the average shear stress in each bolt is

 **Ans.**

|  |  |  |
| --- | --- | --- |
| **P1.20** For the clevis connection shown in Figure P1.20, determine the maximum applied load *P* that can be supported by the 15 mm diameter pin if the average shear stress in the pin must not exceed 130 MPa. |  | |
|  | | FIGURE P1.20 |

### Solution

|  |  |
| --- | --- |
| Consider an FBD of the bar that is connected by the clevis, including a portion of the pin. If the shear force acting on each exposed surface of the pin is denoted by *V*, then the shear force on each pin surface is related to the load *P* by: | S01 |

The area of the pin surface exposed by the FBD is simply the cross-sectional area of the pin:



If the average shear stress in the pin must be limited to 130 MPa, the maximum shear force *V* on a single cross-sectional surface must be limited to



Therefore, the maximum load *P* that may be applied to the connection is

 **Ans.**

|  |  |
| --- | --- |
| **P1.21** The five-bolt connection shown in Figure P1.21 must support an applied load of *P* = 160 kips. If the average shear stress in the bolts must be limited to 30 ksi, what is the minimum bolt diameter that may be used for this connection? |  |
|  | FIGURE P1.21 |

### Solution

There are five bolts, and it is assumed that each bolt supports an equal portion of the external load *P*. Therefore, the shear force carried by each bolt is



Since the average shear stress must be limited to 30 ksi, each bolt must provide a shear area of at least:



Each bolt in this connection acts in double shear; therefore, two cross-sectional bolt surfaces are available to transmit shear stress in each bolt.



The minimum bolt diameter must be

 **Ans.**

|  |  |
| --- | --- |
| **P1.22** The handle shown in Figure P1.22 is attached to a 40 mm diameter shaft with a square shear key. The forces applied to the lever are *P* = 1,300 N. If the average shear stress in the key must not exceed 150 MPa, determine the minimum dimension *a* that must be used if the key is 25 mm long. The overall length of the handle is *L* = 0.70 m. | P01 |
|  | FIGURE P1.22 |

### Solution

To determine the shear force *V* that must be resisted by the shear key, sum moments about the center of the shaft (which will be denoted *O*):



Since the average shear stress in the key must not exceed 150 MPa, the shear area required is



The shear area in the key is given by the product of its length *L* (i.e., 25 mm) and its width *a*. Therefore, the minimum key width *a* is

 **Ans.**

|  |  |
| --- | --- |
| **P1.23** An axial load *P* is supported by the short steel column shown in Figure P1.23. The column has a cross-sectional area of 14,500 mm2. If the average normal stress in the steel column must not exceed 75 MPa, determine the minimum required dimension *a* so that the bearing stress between the base plate and the concrete slab does not exceed 8 MPa. Assume *b* = 420 mm. | P01 |
|  | FIGURE P1.23 |

### Solution

Since the normal stress in the steel column must not exceed 75 MPa, the maximum column load is



The maximum column load must be distributed over a large enough area so that the bearing stress between the base plate and the concrete slab does not exceed 8 MPa; therefore, the minimum plate area is



The area of the plate is *a* ×*b*. Since *b* = 420, the minimum length of *a* must be

 **Ans.**

|  |  |
| --- | --- |
| **P1.24** The two wooden boards shown in Figure P1.24 are connected by a 0.5 in. diameter bolt. Washers are installed under the head of the bolt and under the nut. The washer dimensions are *D* = 2 in. and *d* = 5/8 in. The nut is tightened to cause a tensile stress of 9,000 psi in the bolt. Determine the bearing stress between the washer and the wood. | **P01** |
|  | FIGURE P1.24 |

### Solution

The tensile stress in the bolt is 9,000 psi; therefore, the tension force that acts in the bolt is



The contact area between the washer and the wood is



Thus, the bearing stress between the washer and the wood is

 **Ans.**

|  |  |  |
| --- | --- | --- |
| **P1.25** For the beam shown in Figure P1.25, the allowable bearing stress for the material under the supports at *A* and *B* is *b* = 800 psi. Assume *w* = 2,100 lb/ft, *P* = 4,600 lb, *a* = 20 ft, and *b* = 8 ft. Determine the size of *square* bearing plates required to support the loading shown. Dimension the plates to the nearest ½ in. | P01 | |
|  | FIGURE P1.25 | |
| Solution **Equilibrium:** Using the FBD shown, calculate the beam reaction forces. | P01 |





**Bearing plate at *A*:** The area of the bearing plate required for support *A* is



Since the plate is to be square, its dimensions must be

 **Ans.**

**Bearing plate at *B*:** The area of the bearing plate required for support *B* is



Since the plate is to be square, its dimensions must be

 **Ans.**

|  |  |
| --- | --- |
| **P1.26** A wood beam rests on a square post. The vertical reaction force of the beam at the post is *P* = 1,300 lb. The square post has cross-sectional dimensions of *a* = 6.25 in. The beam has a width of *b* = 1.50 in. and a depth of *d* = 7.50 in. What is the average bearing stress in the wood beam? |  |
|  | FIGURE P1.26 |

### Solution

**Contact area:** Visualize the contact area between the beam and the post. The contact area is



**Bearing stress:** The average bearing stress in the wood beam is

 **Ans.**

|  |  |
| --- | --- |
| **P1.27** The pulley shown in Figure P1.27 is connected to a bracket with a circular pin of diameter *d* = 6 mm. Each vertical side of the bracket has a width of *b* = 25 mm and a thickness of *t* = 4 mm. If the pulley belt tension is *P* = 570 N, what is the average bearing stress produced in the bracket by the pin? |  |
|  | FIGURE P1.27 |

### Solution

|  |  |
| --- | --- |
| **Pulley FBD:** Consider an FBD of the pulley with the belt tensions. From equilibrium, the bracket exerts horizontal and vertical reaction forces *Rx* and *Ry*, respectively, on the pulley. |  |

The resultant force exerted on the pulley by the bracket is thus



**Bearing stress in the bracket:** From Newton’s Third Law, the pulley pin exerts an equal force *R* on the bracket. The bracket has two vertical pieces (i.e., a plate on each side of the pulley). The resultant force *R* is divided equally between these two vertical pieces. Therefore, the force exerted by the pin on one of the vertical bracket pieces is 493.635 N. The average bearing stress in the bracket is based on the projected area of the pin. Therefore, the average bearing stress produced in the bracket by the pin is

 **Ans.**

|  |  |
| --- | --- |
| **P1.28** The *d* = 15 mm diameter solid rod shown in Figure P1.28 passes through a *D* = 20 mm diameter hole in the support plate. When a load *P* is applied to the rod, the rod head rests on the support plate. The support plate has a thickness of *b* = 12 mm. The rod head has a diameter of *a* = 30 mm and the head has a thickness of *t* = 10 mm. If the normal stress produced in the rod by load *P* is 225 MPa, determine:  (a) the average bearing stress acting between the support plate and the rod head.  (b) the average shear stress produced in the rod head.  (c) the punching shear stress produced in the support plate by the rod head. | P01 |
|  | FIGURE P1.28 |

### Solution

The cross-sectional area of the rod is:



The tensile stress in the rod is 225 MPa; therefore, the tension force in the rod is



(a) The contact area between the support plate and the rod head is



Thus, the bearing stress between the support plate and the rod head is

 **Ans.**

(b) In the rod head, the area subjected to shear stress is equal to the perimeter of the rod times the thickness of the head.



and therefore, the average shear stress in the rod head is

 **Ans.**

(c) In the support plate, the area subjected to shear stress is equal to the product of the rod head perimeter and the thickness of the plate.



and therefore, the average punching shear stress in the support plate is

 **Ans.**

|  |
| --- |
| **P1.29** A hollow box beam *ABCD* is supported at *A* by a pin that passes through the beam as shown in Figure P1.29. The box beam is also supported by a roller that is located at *B*. The beam dimensions are *a* = 2.5 ft, *b* = 5.5 ft, and *c* = 3.5 ft. Two equal concentrated loads of *P* = 2,750 lb are placed on the box beam at points *C* and *D*. The box beam has a wall thickness of *t* = 0.375 in., and the pin at *A* has a diameter of 0.750 in. Determine:  (a) the average shear stress in the pin at *A*.  (b) the average bearing stress in the box beam at *A*. |
| FIGURE P1.29 |

### Solution

**Equilibrium:** Determine the reaction force exerted on the beam by the pin at *A*.

|  |  |
| --- | --- |
|  |  |

**Average shear stress in the pin at *A*:**  The pin diameter is 0.750 in. The cross-sectional area of the pin is



From the support detail figure, we observe that this pin acts in double shear; therefore, the shear area of the pin is



The average shear stress in the pin at *A* is thus

 **Ans.**

**Average bearing stress in the box beam at *A*:** The average bearing stress produced in the box beam by the pin is based on the **projected area** of the pin. The projected area is equal to the pin diameter times the wall thickness of the box beam, taking into account that there are two walls that contact the pin. Therefore, the average bearing stress in the box beam is

 **Ans.**

|  |  |  |
| --- | --- | --- |
| **P1.30** Rigid bar *ABC* shown in Figure P1.30 is supported by a pin at bracket *A* and by tie rod (1). Tie rod (1) has a diameter of 5 mm, and it is supported by double-shear pin connections at *B* and *D*. The pin at bracket *A* is a single-shear connection. All pins are 7 mm in diameter. Assume *a* = 600 mm, *b* = 300 mm, *h* = 450 mm, *P* = 900 N, and ** = 55°. Determine the following:  (a) the normal stress in rod (1)  (b) the average shear stress in pin *B*  (c) the average shear stress in pin *A* | P01 | |
|  | FIGURE P1.30 | |
| Solution **Equilibrium:** Using the FBD shown, calculate the reaction forces that act on rigid bar *ABC*. | P01 |





The resultant force at *A* is



**(a) Normal stress in rod (1).**

 **Ans.**

**(b) Shear stress in pin *B*.** The cross-sectional area of a 7 mm diameter pin is:



Pin *B* is a double shear connection; therefore, its average shear stress is

 **Ans.**

**(c) Shear stress in pin *A*.**

Pin *A* is a single shear connection; therefore, its average shear stress is

 **Ans.**

|  |  |
| --- | --- |
| **P1.31** The bell crank shown in Figure P1.31 is in equilibrium for the forces acting in rods (1) and (2). The bell crank is supported by a 10 mm diameter pin at *B* that acts in single shear. The thickness of the bell crank is 5 mm. Assume *a* = 65 mm, *b* = 150 mm, *F*1 = 1,100 N, and ** = 50°. Determine the following:  (a) the average shear stress in pin *B*  (b) the average bearing stress in the bell crank at *B* | P01 |
|  | FIGURE P1.31 |

### Solution

|  |  |
| --- | --- |
| **Equilibrium:** Using the FBD shown, calculate the reaction forces that act on the bell crank. | P01 |

The resultant force at *B* is



**(a) Shear stress in pin *B*.** The cross-sectional area of the 10 mm diameter pin is:



Pin *B* is a single shear connection; therefore, its average shear stress is

 **Ans.**

**(b) Bearing stress in the bell crank at *B*.** The average bearing stress produced in the bell crank by the pin is based on the **projected area** of the pin. The projected area is equal to the pin diameter times the bell crank thickness. Therefore, the average bearing stress in the bell crank is

 **Ans.**

|  |  |
| --- | --- |
| **P1.32** The beam shown in Figure P1.32 is supported by a pin at *C* and by a short link *AB*. If *w* = 30 kN/m, determine the average shear stress in the pins at *A* and *C*. Each pin has a diameter of 25 mm. Assume *L* = 1.8 m and ** = 35°. | P01 |
|  | FIGURE P1.32 |
| Solution **Equilibrium:** Using the FBD shown, calculate the reaction forces that act on the beam. | | P01 | |







The resultant force at *C* is



**Shear stress in pin *A*.** The cross-sectional area of a 25 mm diameter pin is:



Pin *A* is a single shear connection; therefore, its average shear stress is

 **Ans.**

**Shear stress in pin *C*.**

Pin *C* is a double shear connection; therefore, its average shear stress is

 **Ans.**

|  |  |
| --- | --- |
| **P1.33** The bell-crank mechanism shown in Figure P1.33 is in equilibrium for an applied load of *P* = 7 kN applied at *A*. Assume *a* = 200 mm, *b* = 150 mm, and ** = 65°. Determine the minimum diameter *d* required for pin *B* for each of the following conditions:  (a) The average shear stress in the pin may not exceed 40 MPa.  (b) The bearing stress in the bell crank may not exceed 100 MPa.  (c) The bearing stress in the support bracket may not exceed 165 MPa. | P01 |
|  | FIGURE P1.33 |

### Solution

|  |  |
| --- | --- |
| **Equilibrium:** Using the FBD shown, calculate the reaction forces that act on the bell crank. | P01 |

The resultant force at *B* is



**(a) The average shear stress in the pin may not exceed 40 MPa.** The shear area required for the pin at *B* is



Since the pin at *B* is supported in a double shear connection, the required cross-sectional area for the pin is



and therefore, the pin must have a diameter of

 **Ans.**

**(b) The bearing stress in the bell crank may not exceed 100 MPa.** The projected area of pin *B* on the bell crank must equal or exceed



The bell crank thickness is 8 mm; therefore, the projected area of the pin is *Ab* = (8 mm)*d*. Calculate the required pin diameter *d*:

 **Ans.**

(c) The bearing stress in the support bracket may not exceed 165 MPa. The pin at *B* bears on two 6 mm thick support brackets. Thus, the minimum pin diameter required to satisfy the bearing stress limit on the support bracket is



 **Ans.**

|  |  |
| --- | --- |
| **P1.34** A structural steel bar with a 4.0 in. × 0.875 in. rectangular cross section is subjected to an axial load of 45 kips. Determine the maximum normal and shear stresses in the bar. |  |

### Solution

The maximum normal stress in the steel bar is

 **Ans.**

The maximum shear stress is one-half of the maximum normal stress

 **Ans.**

|  |  |
| --- | --- |
| **P1.35** A stainless steel rod of circular cross section will be used to carry an axial load of 30 kN. The maximum stresses in the rod must be limited to 100 MPa in tension and 60 MPa in shear. Determine the required minimum diameter for the rod. |  |

### Solution

Based on the allowable 100 MPa tension stress limit, the minimum cross-sectional area of the rod must equal or exceed



For the 60 MPa shear stress limit, the minimum cross-sectional area of the rod must be equal or exceed



Therefore, the rod must have a cross-sectional area of at least 300 mm2 to satisfy both the normal and shear stress limits.

The minimum rod diameter *D* is therefore

 **Ans.**

|  |  |
| --- | --- |
| **P1.36** Two wooden members, each having a width of *b* = 1.50 in. and a depth of *d* = 0.5 in., are joined by the simple glued scarf joint shown in Figure P1.36/37. Assume *β* = 40°. If the allowable shear stress for the glue used in the joint is 90 psi, what is the largest axial load *P* that may be applied? |  |
|  | FIGURE P1.36 |

### Solution

|  |  |
| --- | --- |
| The angle ** shown for the scarf joint is 40°. The normal force *N* perpendicular to the scarf joint can be expressed as    and the shear force *V* parallel to the scarf joint can be expressed as |  |

The cross-sectional area of the bar is



but the area along the inclined scarf joint is



Consequently, the shear stress *nt* parallel to the scarf joint can be expressed as



Given that the shear stress *nt* must be limited to 90 psi, solve for the maximum load *P* as:

 **Ans.**

|  |  |
| --- | --- |
| **P1.37** Two wooden members, each having a width of *b* = 4.50 in. and a depth of *d* = 1.75 in., are joined by the simple glued scarf joint shown in Figure P1.36/37. Assume *β* = 35°. Given that the compressive axial load is *P* = 900 lb, what are the normal stress and shear stress magnitudes in the glued joint? |  |
|  | FIGURE P1.37 |

### Solution

|  |  |
| --- | --- |
| The angle ** shown for the scarf joint is 35°. The normal force *N* perpendicular to the scarf joint can be expressed as    and the shear force *V* parallel to the scarf joint can be expressed as |  |

The cross-sectional area of the bar is



but the area along the inclined scarf joint is



Consequently, the normal stress *n* magnitude perpendicular to the inclined scarf joint can be expressed as

 **Ans.**

and the shear stress *nt* magnitude parallel to the scarf joint can be expressed as

 **Ans.**

|  |
| --- |
| **P1.38** Two aluminum plates, each having a width of *b* = 7.0 in. and a thickness of *t* = 0.625 in., are welded together as shown in Figure P1.38/39. Assume *a* = 4.0 in. For a load of *P* = 115 kips, determine (a) the normal stress that acts perpendicular to the weld and (b) the shear stress that acts parallel to the weld. |
| FIGURE P1.38/39 |

### Solution

|  |  |
| --- | --- |
| Begin by calculating the angle ** for the weld joint.    The normal force *N* perpendicular to the weld joint can be expressed as    and the shear force *V* parallel to the weld joint can be expressed as |  |

The cross-sectional area of the bar is



but the area along the inclined weld joint is



**(a) Normal stress perpendicular to the weld:** The normal stress *n* magnitude perpendicular to the inclined weld joint can be expressed as

 **Ans.**

**(b) Shear stress parallel to the weld:** The shear stress *nt* magnitude parallel to the weld joint can be expressed as

 **Ans.**

|  |
| --- |
| **P1.39** Two aluminum plates, each having a width of *b* = 5.0 in. and a thickness of *t* = 0.75 in., are welded together as shown in Figure P1.38/39. Assume *a* = 2.0 in. Specifications require that the normal and shear stress magnitudes acting in the weld material may not exceed 35 ksi and 24 ksi, respectively. Determine the largest axial load *P* that can be applied to the aluminum plates. |
| FIGURE P1.38/39 |

### Solution

|  |  |
| --- | --- |
| Begin by calculating the angle ** for the weld joint.    The normal force *N* perpendicular to the weld joint can be expressed as    and the shear force *V* parallel to the weld joint can be expressed as |  |

The cross-sectional area of the bar is



but the area along the inclined weld joint is



**Normal stress perpendicular to the weld:** The normal stress *n* magnitude perpendicular to the inclined weld joint can be expressed as



The normal stress perpendicular to the weld joint may not exceed 35 ksi. The allowable load *P* that satisfies this constraint is



**Shear stress parallel to the weld:** The shear stress *nt* magnitude parallel to the weld joint can be expressed as



The shear stress parallel to the weld joint may not exceed 24 ksi. The allowable load *P* that satisfies this requirement is



**Allowable load *P*:** The largest axial load *P* that can be applied to the aluminum plates is thus

 **Ans.**

|  |  |
| --- | --- |
| **P1.40** Two wooden member are glued together as shown in Figure P1.40. Each member has a width of *b* = 1.50 in. and a depth of *d* = 3.50 in. Use *β* = 75°. Determine the average shear stress magnitude in the glue joint if *P* = 1,300 lb. |  |
|  | FIGURE P1.40 |

### Solution

|  |  |
| --- | --- |
| Using the notion of symmetry, we will consider an FBD for only the upper half of the left-hand wood piece. The central angle ** for the joint is 75°.  The shear force *V* parallel to the upper half joint can be expressed as |  |

The cross-sectional area of the upper half member is



but the area along the inclined upper half joint is



Consequently, the shear stress *nt* magnitude parallel to the joint can be calculated as

 **Ans.**

|  |  |
| --- | --- |
| **P1.41** Two bars are connected with a welded butt joint as shown in Figure P1.41. The bar dimensions are *b* = 200 mm and *t* = 50 mm, and the angle of the weld is ** = 35°. The bars transmit a force of *P* = 250 kN. What is the magnitude of the average shear stress that acts on plane *AB*? |  |
|  | FIGURE P1.41 |

### Solution

|  |  |
| --- | --- |
| The angle ** shown for the weld joint is 35°.  The normal force *N* perpendicular to the weld joint can be expressed as    and the shear force *V* parallel to the weld joint can be expressed as |  |

The cross-sectional area of the bar is



but the area along the inclined weld joint is



**Shear stress parallel to the weld:** The shear stress *nt* magnitude parallel to the weld joint can be expressed as

 **Ans.**