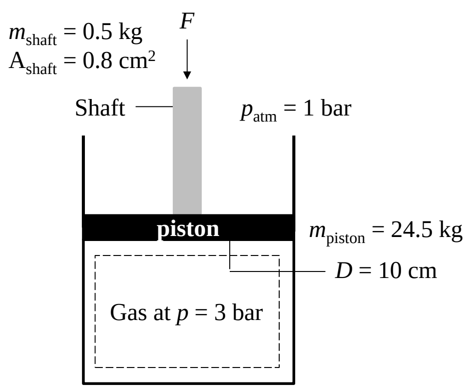
1.24 Figure P1.24 shows a gas contained in a vertical piston-cylinder assembly. A vertical shaft whose cross-sectional area is 0.8 cm^2 is attached to the top of the piston. Determine the magnitude, F, of the force acting on the shaft, in N, required if the gas pressure is 3 bar. The masses of the piston and attached shaft are 24.5 kg and 0.5 kg, respectively. The piston diameter is 10 cm. The local atmospheric pressure is 1 bar. The piston moves smoothly in the cylinder and g = 9.81 m/s^2.

KNOWN: A piston-cylinder assembly with a vertical shaft attached to the piston contains gas.

FIND: The required magnitude of the force acting on the shaft if the gas is at a specified pressure.

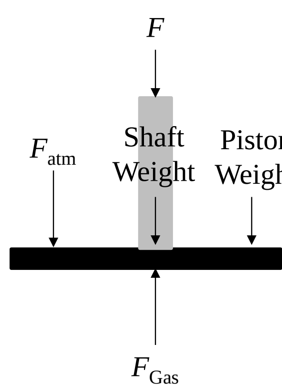
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The gas is a closed system.
2. The piston is in static equilibrium.
3. Atmospheric pressure is exerted on the top of the piston.
4. Local gravitational acceleration is 9.81 m/s^2.

ANALYSIS:

Since the piston moves smoothly within the cylinder, the force exerted by the gas equals the resisting force comprised of the piston weight, shaft weight, the force exerted by the atmospheric pressure, and the force acting on the shaft, F. That is, the sum of the forces acting vertically is zero, giving



F_{gas} = Piston Weight + Shaft Weight + F_{atm} + F

Solving,

$F = F_{gas}$ − Piston Weight − Shaft Weight − F_{atm} (*)

In this expression,

$F_{gas} = p_{gas}A_{piston}$, where A_{piston} is the piston force area:

$$A_{piston} = \frac{\pi D^2_{piston}}{4} = \frac{\pi(10\text{ cm})^2}{4} = 78.54\text{ cm}^2$$

Therefore, $F_{gas} = (3\text{ bar})\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|(78.54\text{ cm}^2)\left|\frac{1\text{ m}}{10^2\text{ cm}}\right|^2 = 2356.2\text{ N}$

The pressure of the atmosphere acts only on the <u>net</u> area at the top of the piston – namely, the piston face area less the area occupied by the shaft. The force is then

$F_{atm} = p_{atm}(A_{piston} - A_{shaft})$

$$F_{atm} = (1\text{ bar})(78.54\text{ cm}^2 - 0.8\text{ cm}^2)\left|\frac{10^5\ \frac{\text{N}}{\text{m}^2}}{1\text{ bar}}\right|\left|\frac{1\text{ m}}{10^2\text{ cm}}\right|^2 = 777.4\text{ N}$$

The total weight of the piston and shaft is

Total Weight = $(m_{piston} + m_{shaft})g$

$$\text{Total Weight} = (24.5\text{ kg} + 0.5\text{ kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left|\frac{1\text{ N}}{1\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right| = 245.3\text{ N}$$

Collecting results, Eq. (*) gives

$$F = 2356.2\text{ N} - 245.3\text{ N} - 777.4\text{ N} = \underline{\mathbf{1333.5\ N}}$$