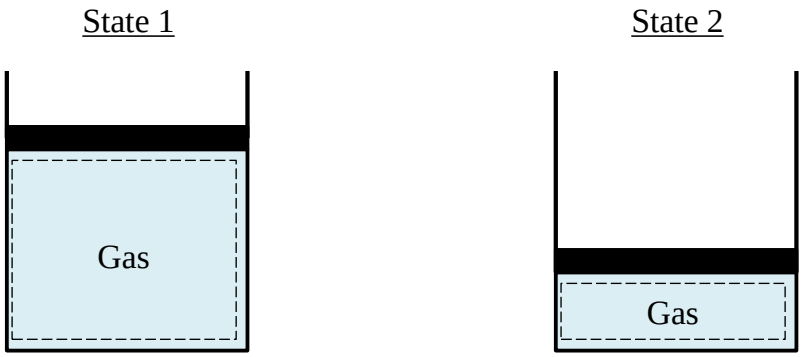
1.23 A closed system consisting of 4 lb of a gas undergoes a process during which the relation between pressure and volume is $pV^n = constant$. The process begins with $p_1 = 15$ lbf/in.^{2}, $v_1 = 1.25$ ft^3/lb and ends with $p_2 = 53$ lbf/in.^{2}, $v_2 = 0.5$ ft^3/lb. Determine (a) the volume, in ft^3, occupied by the gas at states 1 and 2 and (b) the value of n. (c) Sketch Process 1-2 on pressure-volume coordinates.

KNOWN: Gas undergoes a process from a known initial pressure and specific volume to a known final pressure and specific volume.

FIND: Determine (a) the volume, in ft^3, occupied by the gas at states 1 and 2 and (b) the value of n. (c) Sketch Process 1-2 on pressure-volume coordinates.

SCHEMATIC AND GIVEN DATA:



$m = 4$ lb
$p_1 = 15$ lbf/in.^{2}
$v_1 = 1.25$ ft^3/lb

$p_2 = 53$ lbf/in.^{2}
$v_2 = 0.5$ ft^3/lb

ENGINEERING MODEL:
1. The gas is a closed system.
2. The relation between pressure and volume is $pV^n = constant$ during process 1-2.

ANALYSIS:
(a) The specific volume is volume per unit mass. Thus, the volume occupied by the gas can be determined by multiplying its mass by its specific volume.

$$V = mv$$

For state 1

$$V_1 = mv_1 = (4\,\text{lb})\left(1.25\frac{\text{ft}^3}{\text{lb}}\right) = \underline{\textbf{5 ft}^3}$$

For state 2

$$V_2 = mv_2 = (4\,\text{lb})\left(0.5\frac{\text{ft}^3}{\text{lb}}\right) = \underline{\textbf{2 ft}^3}$$

(b) The value of n can be determined by substituting values into the relationship:

$$p_1(V_1)^n = constant = p_2(V_2)^n$$

Solving for n

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^n$$

$$\ln\left(\frac{p_1}{p_2}\right) = n\ln\left(\frac{V_2}{V_1}\right)$$

$$n = \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\frac{15\,\text{lbf/in.}^2}{53\,\text{lbf/in.}^2}\right)}{\ln\left(\frac{2\,\text{ft}^3}{5\,\text{ft}^3}\right)} = \underline{\mathbf{1.38}}$$

(c) Process 1-2 is shown on pressure-volume coordinates below: