1.36 Figure P1.36 shows a tank used to collect rainwater having a diameter of 4 m. As shown in the figure, the depth of the tank varies linearly from 3.5 m at its center to 3 m along the perimeter. The local atmospheric pressure is 1 bar, the acceleration of gravity is g = 9.8 m/s^2, and the density of the water is 987.1 kg/m^3. When the tank is filled with water, determine

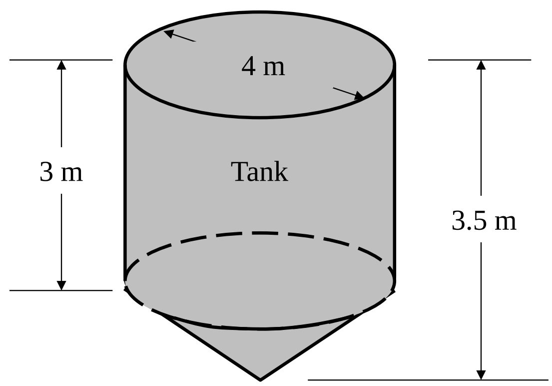
(a) The pressure, in kPa, at the bottom center of the tank.

(b) The total force, in kN, acting on the bottom of the tank.

KNOWN: Rainwater is collected in a tank that varies linearly from it center to its perimeter.

FIND: (a) the pressure at the bottom center of the tank and (b) the total force acting on the bottom of the tank.

SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. Water density is 987.1 kg/m^3.
2. Local atmospheric pressure is 1 bar.
3. Local gravitational acceleration is 9.8 m/s^2.

ANALYSIS:

(a) The depth at the center of the tank is 3.5 m and the corresponding pressure at the center (p_c) in kPa is as follows

$$p_c = p_{atm} + \rho g h = (1\,\text{bar})\left|\frac{100\,\text{kPa}}{1\,\text{bar}}\right| + \left(987.1\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\frac{\text{m}}{\text{s}^2}\right)(3.5\,\text{m})\left|\frac{1\,\text{N}}{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\,\text{kPa}}{10^3\,\frac{\text{N}}{\text{m}^2}}\right| = \underline{\mathbf{133.9\ kPa}}$$

(b) The force acting on the bottom (F_{tot}) of the tank is the sum of the weight of the water plus the force of the atmosphere. The force of the atmosphere (F_{atm}) in kN is

$$F_{atm} = p_{atm}\pi\frac{D^2}{4} = (1\,\text{bar})\left|\frac{10^5\,\frac{\text{N}}{\text{m}^2}}{1\,\text{bar}}\right|\pi\frac{(4\,\text{m})^2}{4}\left|\frac{1\,\text{kN}}{10^3\,\text{N}}\right| = 12.6\times10^2\ \text{kN}$$

The weight of the water is given by

$$\text{weight} = m_w g = \rho V g \tag{1}$$

where ρ is the density of the water and g is the acceleration of gravity which were both given. The total volume of the water in the tank (V) is equal to the volume of a cylinder having a diameter, D = 4 m, and a length, L= 3 m, plus the volume of a cone having D = 4 m and a height, H = 0.5 m. Thus,

$$V = V_{\text{cyl}} + V_{\text{cone}} = \pi L\left(\frac{D^2}{4}\right) + \left(\frac{1}{3}\right)\pi H\left(\frac{D^2}{4}\right) = \pi\left(\frac{D^2}{4}\right)\left(L + \frac{H}{3}\right)$$

$$V = \pi\left(\frac{(4\,\text{m})^2}{4}\right)\left(3 + \frac{0.5}{3}\right)\text{m} = 39.8\ \text{m}^3$$

Substituting values into Eq. (1)

$$\rho V g = \left(987.1\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)(39.8\,\text{m}^3)\left|\frac{1\,\text{N}}{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}\right|\left|\frac{1\,\text{kN}}{10^3\ \text{N}}\right| = 3.85 \times 10^2\ \text{kN}$$

Finally, the total force acting on the bottom of the tank is

$$F_{\text{tot}} = \text{weight} + F_{\text{atm}} = 3.85 \times 10^2\ \text{kN} + 12.6 \times 10^2\ \text{kN} = \underline{\mathbf{16.5 \times 10^2\ kN}}$$