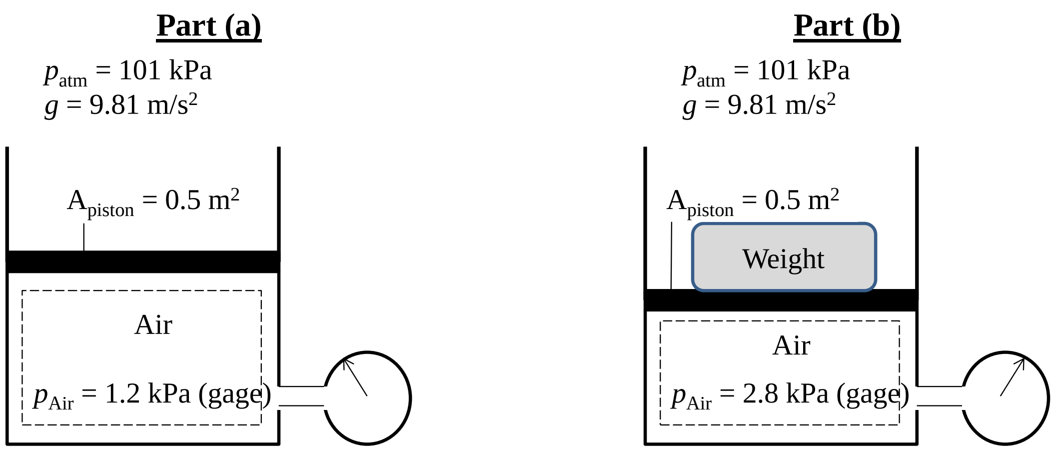
1.35 Air is contained in a vertical piston-cylinder assembly such that the piston is in static equilibrium. The atmosphere exerts a pressure of 101 kPa on top of the 0.5-meter-diameter piston. The gage pressure of the air inside the cylinder is 1.2 kPa. The local acceleration of gravity is $g = 9.81\ \text{m/s}^2$. Subsequently, a weight is placed on top of the piston causing the piston to fall until reaching a new static equilibrium position. At this position, the gage pressure of the air inside the cylinder is 2.8 kPa. Determine (a) the mass of the piston, in kg, and (b) the mass of the added weight, in kg.

KNOWN: A piston-cylinder assembly contains air such that the piston is in static equilibrium. Upon addition of a weight, the piston falls until reaching a new position of static equilibrium.

FIND: (a) The mass of the piston, in kg, and (b) the mass of the added weight, in kg.

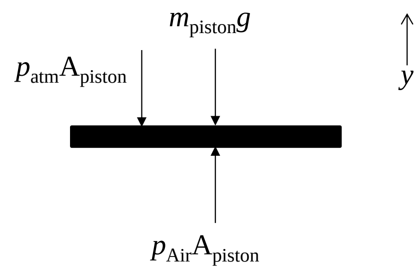
SCHEMATIC AND GIVEN DATA:



ENGINEERING MODEL:
1. The air is a closed system.
2. The piston is in static equilibrium for both part (a) and part (b).
3. Atmospheric pressure is exerted on the top of the piston.
4. Local gravitational acceleration is $9.81\ \text{m/s}^2$.

ANALYSIS:
(a) Draw a free body diagram indicating all forces acting on the piston. Taking upward as the positive y-direction, the sum of the forces acting on the piston in the y-direction must equal zero for static equilibrium of the piston.

Free Body Diagram



$$\uparrow \sum F_y = 0$$

$$p_{\text{Air}}A_{\text{piston}} - p_{\text{atm}}A_{\text{piston}} - m_{\text{piston}}g = 0$$

Solving for the mass of the piston,

$$m_{\text{piston}} = \frac{p_{\text{Air}}A_{\text{piston}} - p_{\text{atm}}A_{\text{piston}}}{g}$$

$$m_{\text{piston}} = \frac{(p_{\text{Air}} - p_{\text{atm}})A_{\text{piston}}}{g}$$

From Eq. 1.14, the quantity in parenthesis is the gage pressure of the air in the cylinder. Rewriting the equation above

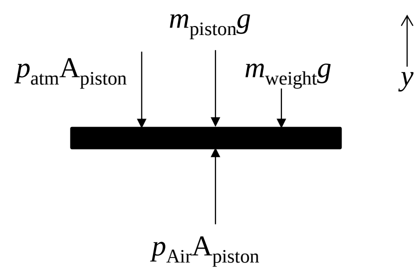
$$m_{\text{piston}} = \frac{p_{\text{Air(gage)}}A_{\text{piston}}}{g}$$

Substituting values and solving for the mass of the piston,

$$m_{\text{piston}} = \frac{(1.2\,\text{kPa})(0.5\,\text{m}^2)}{9.81\dfrac{\text{m}}{\text{s}^2}}\left|\frac{1000\dfrac{\text{N}}{\text{m}^2}}{1\,\text{kPa}}\right|\left|\frac{1\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}}{1\,\text{N}}\right| = \underline{\mathbf{61.2\ kg}}$$

(b) Draw a second free body diagram indicating all forces acting on the piston including the newly added weight expressed as the product of its mass and gravitational acceleration. Taking upward as the positive *y*-direction, the sum of the forces acting on the piston in the *y*-direction must equal zero for static equilibrium of the piston.

Free Body Diagram



$$\uparrow \sum F_y = 0$$

$$p_{Air}A_{piston} - p_{atm}A_{piston} - m_{piston}g - m_{weight}g = 0$$

Solving for the mass of the weight,

$$m_{weight} = \frac{p_{Air}A_{piston} - p_{atm}A_{piston}}{g} - m_{piston}$$

$$m_{weight} = \frac{(p_{Air} - p_{atm})A_{piston}}{g} - m_{piston}$$

From Eq. 1.14, the quantity in parenthesis is the gage pressure of the air in the cylinder. Rewriting the equation above

$$m_{weight} = \frac{p_{Air(gage)}A_{piston}}{g} - m_{piston}$$

Substituting values and solving for the mass of the weight,

$$m_{weight} = \frac{(2.8\,\text{kPa})(0.5\,\text{m}^2)}{9.81\,\frac{\text{m}}{\text{s}^2}} \left| \frac{1000\,\frac{\text{N}}{\text{m}^2}}{1\,\text{kPa}} \right| \left| \frac{1\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}}{1\,\text{N}} \right| - 61.2\,\text{kg} = \mathbf{\underline{81.5\,kg}}$$