Chapter 1 Equations and Inequalities

Section 1.1 Linear Equations and Rational Equations

1. linear

2. first

3. solution

4. conditional

5. identity

6. contradiction

7. rational

8. empty (or null); $\{\ \}$ or ϕ

9. a. Linear; $-2x = 8$

$$\frac{-2x}{-2} = \frac{8}{-2}$$
$$x = -4$$
$$\{-4\}$$

b. Nonlinear

c. Linear; $-\frac{1}{2}x = 8$

$$-2\left(-\frac{1}{2}x\right) = -2(8)$$
$$x = -16$$
$$\{-16\}$$

d. Nonlinear

e. Linear; $x - 2 = 8$

$$x - 2 + 2 = 8 + 2$$
$$x = 10$$
$$\{10\}$$

10. a. Linear; $12 = 4x$

$$\frac{12}{4} = \frac{4x}{4}$$
$$3 = x$$
$$\{3\}$$

b. Nonlinear

c. Linear; $12 = \frac{1}{4}x$

$$4(12) = 4\left(\frac{1}{4}x\right)$$
$$48 = x$$
$$\{48\}$$

d. Nonlinear

e. Linear; $12 = 4 + x$

$$12 - 4 = 4 + x - 4$$
$$8 = x$$
$$\{8\}$$

11. $-6x - 4 = 20$

$$-6x = 24$$
$$x = -4$$
$$\{-4\}$$

12. $-8y + 6 = 22$

$$-8y = 16$$
$$y = -2$$
$$\{-2\}$$

13. $4 = 7 - 3(4t + 1)$

$$4 = 7 - 12t - 3$$
$$4 = 4 - 12t$$
$$0 = -12t$$
$$0 = t$$
$$\{0\}$$

14. $11 = 7 - 2(5p - 2)$

$$11 = 7 - 10p + 4$$
$$11 = 11 - 10p$$
$$0 = -10p$$
$$0 = p$$
$$\{0\}$$

15. $-6(v-2)+3=9-(v+4)$

$\qquad -6v+12+3=9-v-4$

$\qquad\quad -6v+15=5-v$

$\qquad\qquad\quad -5v=-10$

$\qquad\qquad\qquad v=2$

$\quad \{2\}$

16. $-5(u-4)+2=11-(u-3)$

$\qquad -5u+20+2=11-u+3$

$\qquad\quad -5u+22=14-u$

$\qquad\qquad\quad -4u=-8$

$\qquad\qquad\qquad u=2$

$\quad \{2\}$

17. $\quad 2.3=4.5x+30.2$

$\qquad -27.9=4.5x$

$\qquad\quad -6.2=x$

$\quad \{-6.2\}$

18. $9.4=3.5p-0.4$

$\qquad 9.8=3.5p$

$\qquad 2.8=p$

$\quad \{2.8\}$

19. $0.05y+0.02(6000-y)=270$

$\qquad 0.05y+120-0.02y=270$

$\qquad\qquad 0.03y+120=270$

$\qquad\qquad\qquad 0.03y=150$

$\qquad\qquad\qquad\quad y=5000$

$\quad \{5000\}$

20. $0.06x+0.04(10{,}000-x)=520$

$\qquad 0.06x+400-0.04x=520$

$\qquad\qquad\qquad 0.02x=120$

$\qquad\qquad\qquad\quad x=6000$

$\quad \{6000\}$

21. $2(5x-6)=4\left[x-3(x-10)\right]$

$\qquad 10x-12=4(x-3x+30)$

$\qquad 10x-12=4(-2x+30)$

$\qquad 10x-12=-8x+120$

$\qquad\quad 18x=132$

$\qquad\qquad x=\dfrac{132}{18}=\dfrac{22}{3}$

$\left\{\dfrac{22}{3}\right\}$

22. $4(y-3)=3\left[y+2(y-2)\right]$

$\qquad 4y-12=3(y+2y-4)$

$\qquad 4y-12=3(3y-4)$

$\qquad 4y-12=9y-12$

$\qquad\quad -5y=0$

$\qquad\qquad y=0$

$\quad \{0\}$

23. $\qquad \dfrac{1}{4}x-\dfrac{3}{2}=2$

$\qquad 4\left(\dfrac{1}{4}x-\dfrac{3}{2}\right)=4(2)$

$\qquad\qquad x-6=8$

$\qquad\qquad\quad x=14$

$\quad \{14\}$

24. $\qquad \dfrac{1}{6}x-\dfrac{5}{3}=1$

$\qquad 6\left(\dfrac{1}{6}x-\dfrac{5}{3}\right)=6(1)$

$\qquad\qquad x-10=6$

$\qquad\qquad\quad x=16$

$\quad \{16\}$

25. $\qquad \dfrac{1}{2}w-\dfrac{3}{4}=\dfrac{2}{3}w+2$

$\qquad 12\left(\dfrac{1}{2}w-\dfrac{3}{4}\right)=12\left(\dfrac{2}{3}w+2\right)$

$\qquad\qquad 6w-9=8w+24$

$\qquad\qquad\quad -2w=33$

$\qquad\qquad\qquad w=-\dfrac{33}{2}$

$\left\{-\dfrac{33}{2}\right\}$

26.
$$\frac{2}{5}p - \frac{3}{10} = \frac{7}{15}p - 1$$
$$30\left(\frac{2}{5}p - \frac{3}{10}\right) = 30\left(\frac{7}{15}p - 1\right)$$
$$12p - 9 = 14p - 30$$
$$-2p = -21$$
$$p = \frac{21}{2}$$
$$\left\{\frac{21}{2}\right\}$$

27.
$$\frac{y-1}{5} + \frac{y}{4} = \frac{y+3}{2} + 1$$
$$20\left(\frac{y-1}{5} + \frac{y}{4}\right) = 20\left(\frac{y+3}{2} + 1\right)$$
$$4(y-1) + 5y = 10(y+3) + 20$$
$$4y - 4 + 5y = 10y + 30 + 20$$
$$9y - 4 = 10y + 50$$
$$-y = 54$$
$$y = -54$$
$$\{-54\}$$

28.
$$\frac{x-6}{3} + \frac{x}{7} = \frac{x+1}{3} + 2$$
$$21\left(\frac{x-6}{3} + \frac{x}{7}\right) = 21\left(\frac{x+1}{3} + 2\right)$$
$$7(x-6) + 3x = 7(x+1) + 42$$
$$7x - 42 + 3x = 7x + 7 + 42$$
$$10x - 42 = 7x + 49$$
$$3x = 91$$
$$x = \frac{91}{3}$$
$$\left\{\frac{91}{3}\right\}$$

29.
$$\frac{n+3}{4} - \frac{n-2}{5} = \frac{n+1}{10} - 1$$
$$20\left(\frac{n+3}{4} - \frac{n-2}{5}\right) = 20\left(\frac{n+1}{10} - 1\right)$$
$$5(n+3) - 4(n-2) = 2(n+1) - 20$$
$$5n + 15 - 4n + 8 = 2n + 2 - 20$$
$$n + 23 = 2n - 18$$
$$-n = -41$$
$$n = 41$$
$$\{41\}$$

30.
$$\frac{t-2}{3} - \frac{t+7}{5} = \frac{t-4}{10} + 2$$
$$30\left(\frac{t-2}{3} - \frac{t+7}{5}\right) = 30\left(\frac{t-4}{10} + 2\right)$$
$$10(t-2) - 6(t+7) = 3(t-4) + 60$$
$$10t - 20 - 6t - 42 = 3t - 12 + 60$$
$$4t - 62 = 3t + 48$$
$$t = 110$$
$$\{110\}$$

31. a. $T = -1.83a + 212$
$$T = -1.83(4) + 212$$
$$= 204.68°F$$
$$\approx 205°F$$

b.
$$T = -1.83a + 212$$
$$193 = -1.83a + 212$$
$$-19 = -1.83a$$
$$10.4 \approx a$$
$$10.4 \times 10^3 = 10{,}400$$
Approximately 10, 400 ft

32. a. $C = 167.95x + 94$
$$C = 167.95(9) + 94$$
$$= \$1605.55$$

b.
$$C = 167.95x + 94$$
$$2445.30 = 167.95x + 94$$
$$2351.30 = 167.95x$$
$$14 = x$$
14 credit-hours

33.
$$S = 14.2t + 149$$
$$362 = 14.2t + 149$$
$$213 = 14.2t$$
$$15 = t$$
$$2004 + 15 = 2019$$
In 2019

34.
$$S = 18t + 232$$
$$628 = 18t + 232$$
$$396 = 18t$$
$$22 = t$$
$$2000 + 22 = 2022$$
In 2022

35. a. $C = 7x$

   b. $C = 105$
$$7x = 105$$
$$x = 15$$
The motorist will save money beginning on the 16th working day.

36. a. $C = 2.25x$

   b. $C = 89$
$$2.25x = 89$$
$$x \approx 39.6$$
The commuter will save money on the 40th ride.

37. a. $S_1 = 45,000 + 2250x$

   b. $S_2 = 48,000 + 2000x$

   c.
$$S_1 = S_2$$
$$45,000 + 2250x = 48,000 + 2000x$$
$$250x = 3000$$
$$x = 12 \text{ yr}$$

38. a. $S_1 = 25,000 + 0.16x$

   b. $S_2 = 30,000 + 0.15x$

   c.
$$S_1 = S_2$$
$$25,000 + 0.16x = 30,000 + 0.15x$$
$$0.01x = 5000$$
$$x = \$500,000$$

39. $2x - 3 = 4(x - 1) - 1 - 2x$
$$2x - 3 = 4x - 4 - 1 - 2x$$
$$2x - 3 = 2x - 5$$
$$-3 = -5$$
Contradiction

40. $4(3 - 5n) + 1 = -4n - 8 - 16n$
$$12 - 20n + 1 = -4n - 8 - 16n$$
$$-20n + 13 = -20n - 8$$
$$13 = -8$$
Contradiction

41. $-(6 - 2w) = 4(w + 1) - 2w - 10$
$$-6 + 2w = 4w + 4 - 2w - 10$$
$$-6 + 2w = 2w - 6$$
$$0 = 0$$
Identity; $\mathbb{R}$

42. $-5 + 3x = 3(x - 1) - 2$
$$-5 + 3x = 3x - 3 - 2$$
$$-5 + 3x = 3x - 5$$
$$0 = 0$$
Identity; $\mathbb{R}$

43.
$$\frac{1}{2}x + 3 = \frac{1}{4}x + 1$$
$$4\left(\frac{1}{2}x + 3\right) = 4\left(\frac{1}{4}x + 1\right)$$
$$2x + 12 = x + 4$$
$$x = -8$$
Conditional equation; $\{-8\}$

44.
$$\frac{2}{3}y - 5 = \frac{1}{6}y - 4$$
$$6\left(\frac{2}{3}y - 5\right) = 6\left(\frac{1}{6}y - 4\right)$$
$$4y - 30 = y - 24$$
$$3y = 6$$
$$y = 2$$
Conditional equation; $\{2\}$

45.
$$\frac{3}{x - 5} + \frac{2}{x + 4} = \frac{5}{7}$$
$$x \neq 5, x \neq -4$$

46.
$$\frac{2}{x + 1} - \frac{5}{x - 7} = \frac{2}{3}$$
$$x \neq -1, x \neq 7$$

47.
$$\frac{5}{2x - 3} - \frac{3}{5x} = \frac{1}{3 - x}$$
$$\frac{5}{2\left(x - \frac{3}{2}\right)} - \frac{3}{5x} = \frac{1}{3 - x}$$
$$x \neq \frac{3}{2}, x \neq 0, x \neq 3$$

48. $\dfrac{1}{2x} - \dfrac{3}{6-x} = \dfrac{2}{4x-5}$

$\dfrac{1}{2x} - \dfrac{3}{6-x} = \dfrac{2}{4\left(x - \dfrac{5}{4}\right)}$

$x \neq 0, x \neq 6, x \neq \dfrac{5}{4}$

49. $\dfrac{1}{2} - \dfrac{7}{2y} = \dfrac{5}{y}$

$2y\left(\dfrac{1}{2} - \dfrac{7}{2y}\right) = 2y\left(\dfrac{5}{y}\right)$

$y - 7 = 10$

$y = 17$

$\{17\}$

50. $\dfrac{1}{3} - \dfrac{4}{3t} = \dfrac{7}{t}$

$3t\left(\dfrac{1}{3} - \dfrac{4}{3t}\right) = 3t\left(\dfrac{7}{t}\right)$

$t - 4 = 21$

$t = 25$

$\{25\}$

51. $\dfrac{w+3}{4w} + 1 = \dfrac{w-5}{w}$

$4w\left(\dfrac{w+3}{4w} + 1\right) = 4w\left(\dfrac{w-5}{w}\right)$

$w + 3 + 4w = 4(w - 5)$

$5w + 3 = 4w - 20$

$w = -23$

$\{-23\}$

52. $\dfrac{x+2}{6x} + 1 = \dfrac{x-7}{x}$

$6x\left(\dfrac{x+2}{6x} + 1\right) = 6x\left(\dfrac{x-7}{x}\right)$

$x + 2 + 6x = 6(x - 7)$

$7x + 2 = 6x - 42$

$x = -44$

$\{-44\}$

53.

$\dfrac{c}{c-3} = \dfrac{3}{c-3} - \dfrac{3}{4}$

$4(c-3)\left(\dfrac{c}{c-3}\right) = 4(c-3)\left(\dfrac{3}{c-3} - \dfrac{3}{4}\right)$

$4c = 12 - 3(c - 3)$

$4c = 12 - 3c + 9$

$7c = 21$

$c = 3$

$\{\ \}$; The value 3 does not check.

54. $\dfrac{7}{d-7} - \dfrac{7}{8} = \dfrac{d}{d-7}$

$8(d-7)\left(\dfrac{7}{d-7} - \dfrac{7}{8}\right) = 8(d-7)\left(\dfrac{d}{d-7}\right)$

$56 - 7(d - 7) = 8d$

$56 - 7d + 49 = 8d$

$-15d = -105$

$d = 7$

$\{\ \}$; The value 7 does not check.

55.

$\dfrac{1}{t-1} = \dfrac{3}{t^2 - 1}$

$\dfrac{1}{t-1} = \dfrac{3}{(t+1)(t-1)}$

$(t+1)(t-1)\left(\dfrac{1}{t-1}\right) = (t+1)(t-1)\left[\dfrac{3}{(t+1)(t-1)}\right]$

$t + 1 = 3$

$t = 2$

$\{2\}$

56.
$$\frac{1}{w+2} = \frac{5}{w^2-4}$$
$$\frac{1}{w+2} = \frac{5}{(w+2)(w-2)}$$
$$(w+2)(w-2)\left(\frac{1}{w+2}\right) = (w+2)(w-2)\left[\frac{5}{(w+2)(w-2)}\right]$$
$$w-2=5$$
$$w=7$$

$\{7\}$

57.
$$\frac{2}{x-5} - \frac{1}{x+5} = \frac{11}{x^2-25}$$
$$\frac{2}{x-5} - \frac{1}{x+5} = \frac{11}{(x+5)(x-5)}$$
$$(x+5)(x-5)\left(\frac{2}{x-5} - \frac{1}{x+5}\right) = (x+5)(x-5)\left[\frac{11}{(x+5)(x-5)}\right]$$
$$2(x+5)-1(x-5)=11$$
$$2x+10-x+5=11$$
$$x+15=11$$
$$x=-4$$

$\{-4\}$

58.
$$\frac{2}{c+3} - \frac{1}{c-3} = \frac{10}{c^2-9}$$
$$\frac{2}{c+3} - \frac{1}{c-3} = \frac{10}{(c+3)(c-3)}$$
$$(c+3)(c-3)\left(\frac{2}{c+3} - \frac{1}{c-3}\right) = (c+3)(c-3)\left[\frac{10}{(c+3)(c-3)}\right]$$
$$2(c-3)-1(c+3)=10$$
$$2c-6-c-3=10$$
$$c-9=10$$
$$c=19$$

$\{19\}$

59.
$$\frac{-14}{x^2-x-12}-\frac{1}{x-4}=\frac{4}{x+3}$$

$$\frac{-14}{(x-4)(x+3)}-\frac{1}{(x-4)}=\frac{2}{(x+3)}$$

$$(x-4)(x+3)\left[\frac{-14}{(x-4)(x+3)}-\frac{1}{(x-4)}\right]=(x-4)(x+3)\left[\frac{2}{(x+3)}\right]$$

$$-14-(x+3)=2(x-4)$$

$$-14-x-3=2x-8$$

$$-17-x=2x-8$$

$$-3=x$$

$\{\ \}$; The value -3 does not check.

60.
$$\frac{2}{x^2+5x+6}-\frac{2}{x+2}=\frac{1}{x+3}$$

$$\frac{2}{(x+2)(x+3)}-\frac{2}{(x+2)}=\frac{1}{(x+3)}$$

$$(x+2)(x+3)\left[\frac{2}{(x+2)(x+3)}-\frac{2}{(x+2)}\right]=(x+2)(x+3)\left[\frac{1}{(x+3)}\right]$$

$$2-2(x+3)=(x+2)$$

$$2-2x-6=x+2$$

$$-4-2x=x+2$$

$$-2=x$$

$\{\ \}$; The value -2 does not check.

61.
$$\frac{5}{x^2-x-2}-\frac{2}{x^2-4}=\frac{4}{x^2+3x+2}$$

$$\frac{5}{(x-2)(x+1)}-\frac{2}{(x-2)(x+2)}=\frac{4}{(x+2)(x+1)}$$

$$(x+2)(x-2)(x+1)\left[\frac{5}{(x-2)(x+1)}-\frac{2}{(x-2)(x+2)}\right]=(x+2)(x-2)(x+1)\left[\frac{4}{(x+2)(x+1)}\right]$$

$$5(x+2)-2(x+1)=4(x-2)$$

$$5x+10-2x-2=4x-8$$

$$3x+8=4x-8$$

$$16=x$$

$\{16\}$

62.
$$\frac{4}{x^2-2x-8}-\frac{1}{x^2-16}=\frac{2}{x^2+6x+8}$$

$$\frac{4}{(x-4)(x+2)}-\frac{1}{(x-4)(x+4)}=\frac{2}{(x+4)(x+2)}$$

$$(x+4)(x-4)(x+2)\left[\frac{4}{(x-4)(x+2)}-\frac{1}{(x-4)(x+4)}\right]=(x+4)(x-4)(x+2)\left[\frac{2}{(x+4)(x+2)}\right]$$

$$4(x+4)-1(x+2)=2(x-4)$$

$$4x+16-x-2=2x-8$$

$$3x+14=2x-8$$

$$x=-22$$

$$\{-22\}$$

63.
$$\frac{5}{m-2}=\frac{3m}{m^2+2m-8}-\frac{2}{m+4}$$

$$\frac{5}{m-2}=\frac{3m}{(m+4)(m-2)}-\frac{2}{m+4}$$

$$(m+4)(m-2)\left(\frac{5}{m-2}\right)=(m+4)(m-2)\left[\frac{3m}{(m+4)(m-2)}-\frac{2}{m+4}\right]$$

$$5(m+4)=3m-2(m-2)$$

$$5m+20=3m-2m+4$$

$$5m+20=m+4$$

$$4m=-16$$

$$m=-4$$

$\{\ \}$; The value -4 does not check.

64.
$$\frac{10}{n-6}=\frac{15n}{n^2-2n-24}-\frac{6}{n+4}$$

$$\frac{10}{n-6}=\frac{15n}{(n-6)(n+4)}-\frac{6}{n+4}$$

$$(n-6)(n+4)\left(\frac{10}{n-6}\right)=(n-6)(n+4)\left[\frac{15n}{(n-6)(n+4)}-\frac{6}{n+4}\right]$$

$$10(n+4)=15n-6(n-6)$$

$$10n+40=15n-6n+36$$

$$10n+40=9n+36$$

$$n=-4$$

$\{\ \}$; The value -4 does not check.

65.

$$\frac{5x}{3x^2-5x-2}-\frac{1}{3x+1}=\frac{3}{2-x}$$

$$\frac{5x}{(3x+1)(x-2)}-\frac{1}{3x+1}=\frac{-3}{x-2}$$

$$(3x+1)(x-2)\left[\frac{5x}{(3x+1)(x-2)}-\frac{1}{3x+1}\right]=(3x+1)(x-2)\left(\frac{-3}{x-2}\right)$$

$$5x-1(x-2)=-3(3x+1)$$

$$5x-x+2=-9x-3$$

$$4x+2=-9x-3$$

$$13x=-5$$

$$x=-\frac{5}{13}$$

$$\left\{-\frac{5}{13}\right\}$$

66.

$$\frac{3x}{2x^2+x-3}-\frac{2}{2x+3}=\frac{4}{1-x}$$

$$\frac{3x}{(2x+3)(x-1)}-\frac{2}{2x+3}=\frac{-4}{x-1}$$

$$(2x+3)(x-1)\left[\frac{3x}{(2x+3)(x-1)}-\frac{2}{2x+3}\right]=(2x+3)(x-1)\left(\frac{-4}{x-1}\right)$$

$$3x-2(x-1)=-4(2x+3)$$

$$3x-2x+2=-8x-12$$

$$x+2=-8x-12$$

$$9x=-14$$

$$x=-\frac{14}{9}$$

$$\left\{-\frac{14}{9}\right\}$$

67. $A = lw$ for l

$$\frac{A}{w}=\frac{lw}{w}$$

$$\frac{A}{w}=l \text{ or } l=\frac{A}{w}$$

68. $E = IR$ for R

$$\frac{E}{I}=\frac{IR}{I}$$

$$\frac{E}{I}=R \text{ or } R=\frac{E}{I}$$

69. $P = a+b+c$ for c

$$P-a-b=c \text{ or } c=P-a-b$$

70. $W = K - T$ for K

$$W +T= K \text{ or } K = W +T$$

71. $\Delta s= s_2 - s_1$ for s_1

$$\Delta s - s_2 = -s_1$$

$$s_1 = s_2 - \Delta s$$

72. $\Delta t= t_f - t_i$ for t_i

$$\Delta t - t_f = -t_i$$

$$t_i = t_f - \Delta t$$

73. $7x + 2y = 8$ for y

$$2y = -7x + 8$$

$$\frac{2y}{2} = \frac{-7x + 8}{2}$$

$$y = \frac{-7x + 8}{2} \quad \text{or} \quad y = -\frac{7}{2}x + 4$$

74. $3x + 5y = 15$ for y

$$5y = -3x + 15$$

$$\frac{5y}{5} = \frac{-3x + 15}{5}$$

$$y = \frac{-3x + 15}{5} \quad \text{or} \quad y = -\frac{3}{5}x + 3$$

75. $5x - 4y = 2$ for y

$$-4y = -5x + 2$$

$$\frac{-4y}{-4} = \frac{-5x + 2}{-4}$$

$$y = \frac{5x - 2}{4} \quad \text{or} \quad y = \frac{5}{4}x - \frac{1}{2}$$

76. $7x - 2y = 5$ for y

$$-2y = -7x + 5$$

$$\frac{-2y}{-2} = \frac{-7x + 5}{-2}$$

$$y = \frac{7x - 5}{2} \quad \text{or} \quad y = \frac{7}{2}x - \frac{5}{2}$$

77. $\frac{1}{2}x + \frac{1}{3}y = 1$ for y

$$6\left(\frac{1}{2}x + \frac{1}{3}y\right) = 6(1)$$

$$3x + 2y = 6$$

$$2y = -3x + 6$$

$$\frac{2y}{2} = \frac{-3x + 6}{2}$$

$$y = \frac{-3x + 6}{2} \quad \text{or} \quad y = -\frac{3}{2}x + 3$$

78. $\frac{1}{4}x - \frac{2}{3}y = 2$ for y

$$12\left(\frac{1}{4}x - \frac{2}{3}y\right) = 12(2)$$

$$3x - 8y = 24$$

$$-8y = -3x + 24$$

$$\frac{-8y}{-8} = \frac{-3x + 24}{-8}$$

$$y = \frac{3x - 24}{8} \quad \text{or} \quad y = \frac{3}{8}x - 3$$

79. $S = \frac{n}{2}(a + d)$ for d

$$2(S) = 2\left[\frac{n}{2}(a + d)\right]$$

$$2S = n(a + d)$$

$$2S = na + nd$$

$$2S - na = nd$$

$$\frac{2S - na}{n} = \frac{nd}{n}$$

$$\frac{2S - na}{n} = d$$

$$d = \frac{2S - na}{n} \quad \text{or} \quad d = \frac{2S}{n} - a$$

80. $S = \frac{n}{2}\left[2a + (n-1)d\right]$ for a

$$2(S) = 2\left\{\frac{n}{2}\left[2a + (n-1)d\right]\right\}$$

$$2S = n\left[2a + (n-1)d\right]$$

$$2S = n(2a + nd - d)$$

$$2S = 2an + n^2d - nd$$

$$2S - n^2d + nd = 2an$$

$$\frac{2S - n^2d + nd}{2n} = \frac{2an}{2n}$$

$$\frac{2S - n^2d + nd}{2n} = a \quad \text{or} \quad a = \frac{2S - n^2d + nd}{2n}$$

81. $V = \dfrac{1}{3}\pi r^2 h$ for h

$3(V) = 3\left(\dfrac{1}{3}\pi r^2 h\right)$

$3V = \pi r^2 h$

$\dfrac{3V}{\pi r^2} = \dfrac{\pi r^2 h}{\pi r^2}$

$\dfrac{3V}{\pi r^2} = h$ or $h = \dfrac{3V}{\pi r^2}$

82. $V = \dfrac{1}{3} Bh$ for B

$3(V) = 3\left(\dfrac{1}{3} Bh\right)$

$3V = Bh$

$\dfrac{3V}{h} = \dfrac{Bh}{h}$

$\dfrac{3V}{h} = B$ or $B = \dfrac{3V}{h}$

83. $6 = 4x + tx$ for x

$6 = x(4+t)$

$\dfrac{6}{4+t} = \dfrac{x(4+t)}{4+t}$

$\dfrac{6}{4+t} = x$ or $x = \dfrac{6}{4+t}$

84. $8 = 3x + kx$ for x

$8 = x(3+k)$

$\dfrac{8}{3+k} = \dfrac{x(3+k)}{3+k}$

$\dfrac{8}{3+k} = x$ or $x = \dfrac{8}{3+k}$

85. $6x + ay = bx + 5$ for x

$6x - bx = 5 - ay$

$x(6-b) = 5 - ay$

$\dfrac{x(6-b)}{6-b} = \dfrac{5-ay}{6-b}$

$x = \dfrac{5-ay}{6-b}$ or $x = \dfrac{ay-5}{b-6}$

86. $3x + 2y = cx + d$ for x

$3x - cx = d - 2y$

$x(3-c) = d - 2y$

$\dfrac{x(3-c)}{3-c} = \dfrac{d-2y}{3-c}$

$x = \dfrac{d-2y}{3-c}$ or $x = \dfrac{2y-d}{c-3}$

87. $A = P + Prt$ for P

$A = P(1+rt)$

$\dfrac{A}{1+rt} = \dfrac{P(1+rt)}{1+rt}$

$\dfrac{A}{1+rt} = P$ or $P = \dfrac{A}{1+rt}$

88. $C = A + Ar$ for A

$C = A(1+r)$

$\dfrac{C}{1+r} = \dfrac{A(1+r)}{1+r}$

$\dfrac{C}{1+r} = A$ or $A = \dfrac{C}{1+r}$

89.
$$\dfrac{5}{2n+1} = \dfrac{-2}{3n-4}$$

$$(2n+1)(3n-4)\left(\dfrac{5}{2n+1}\right) = (2n+1)(3n-4)\left(\dfrac{-2}{3n-4}\right)$$

$$5(3n-4) = -2(2n+1)$$

$$15n - 20 = -4n - 2$$

$$19n = 18$$

$$n = \dfrac{18}{19}$$

$$\left\{\dfrac{18}{19}\right\}$$

90

90.
$$\frac{4}{5z-3} = \frac{-2}{4z+7}$$

$$(5z-3)(4z+7)\left(\frac{4}{5z-3}\right) = (5z-3)(4z+7)\left(\frac{-2}{4z+7}\right)$$

$$4(4z+7) = -2(5z-3)$$

$$16z+28 = -10z+6$$

$$26z = -22$$

$$z = -\frac{22}{26} = -\frac{11}{13}$$

$$\left\{-\frac{11}{13}\right\}$$

91. $5-2\left\{3-\left[5v+3(v-7)\right]\right\} = 8v+6(3-4v)-61$

$$5-2\left[3-(5v+3v-21)\right] = 8v+18-24v-61$$

$$5-2\left[3-(8v-21)\right] = -16v-43$$

$$5-2(3-8v+21) = -16v-43$$

$$5-2(24-8v) = -16v-43$$

$$5-48+16v = -16v-43$$

$$16v-43 = -16v-43$$

$$32v = 0$$

$$v = 0$$

$$\{0\}$$

92. $6-\left\{4-2\left[8u-2(u-3)\right]\right\} = -4u+3(2-u)+8$

$$6-\left[4-2(8u-2u+6)\right] = -4u+6-3u+8$$

$$6-\left[4-2(6u+6)\right] = -7u+14$$

$$6-(4-12u-12) = -7u+14$$

$$6-(12u-8) = -7u+14$$

$$6-12u+8 = -7u+14$$

$$-5u = 0$$

$$u = 0$$

$$\{0\}$$

93. $(x-7)(x+2) = x^2 + 4x+13$

$$x^2 + 2x-7x-14 = x^2 + 4x+13$$

$$x^2 - 5x-14 = x^2 + 4x+13$$

$$-9x = 27$$

$$x = -3$$

$$\{-3\}$$

94. $(m+3)(2m-5) = 2m^2 + 4m - 3$

$2m^2 + 6m - 5m - 15 = 2m^2 + 4m - 3$

$2m^2 + m - 15 = 2m^2 + 4m - 3$

$-3m = 12$

$m = -4$

$\{-4\}$

95.
$$\frac{3}{c^2 - 4c} - \frac{9}{2c^2 + 3c} = \frac{2}{2c^2 - 5c - 12}$$

$$\frac{3}{c(c-4)} - \frac{9}{c(2c+3)} = \frac{2}{(2c+3)(c-4)}$$

$$c(2c+3)(c-4)\left[\frac{3}{c(c-4)} - \frac{9}{c(2c+3)}\right] = c(2c+3)(c-4)\left[\frac{2}{(2c+3)(c-4)}\right]$$

$$3(2c+3) - 9(c-4) = 2c$$

$$6c + 9 - 9c + 36 = 2c$$

$$-3c + 45 = 2c$$

$$-5c = -45$$

$$c = 9$$

$\{9\}$

96.
$$\frac{4}{d^2 - d} - \frac{5}{2d^2 + 5d} = \frac{2}{2d^2 + 3d - 5}$$

$$\frac{4}{d(d-1)} - \frac{5}{d(2d+5)} = \frac{2}{(2d+5)(d-1)}$$

$$d(2d+5)(d-1)\left[\frac{4}{d(d-1)} - \frac{5}{d(2d+5)}\right] = d(2d+5)(d-1)\left[\frac{2}{(2d+5)(d-1)}\right]$$

$$4(2d+5) - 5(d-1) = 2d$$

$$8d + 20 - 5d + 5 = 2d$$

$$3d + 25 = 2d$$

$$d = -25$$

$\{-25\}$

97. $\dfrac{1}{3}x + \dfrac{1}{2} = \dfrac{1}{2}(x+1) - \dfrac{1}{6}x$

$6\left(\dfrac{1}{3}x + \dfrac{1}{2}\right) = 6\left[\dfrac{1}{2}(x+1) - \dfrac{1}{6}x\right]$

$2x + 3 = 3(x+1) - x$

$2x + 3 = 3x + 3 - x$

$2x + 3 = 2x + 3$

$0 = 0$

$\mathbb{R}$

98. $\dfrac{1}{2}x + \dfrac{2}{5} = \dfrac{2}{5}(x+1) + \dfrac{1}{10}x$

$10\left(\dfrac{1}{2}x + \dfrac{2}{5}\right) = 10\left[\dfrac{2}{5}(x+1) + \dfrac{1}{10}x\right]$

$5x + 4 = 4(x+1) + x$

$5x + 4 = 4x + 4 + x$

$5x + 4 = 5x + 4$

$0 = 0$

$\mathbb{R}$

99. $(t+2)^2 = (t-4)^2$

$t^2 + 4t + 4 = t^2 - 8t + 16$

$12t = 12$

$t = 1$

$\{1\}$

100. $(y-3)^2 = (y+1)^2$

$y^2 - 6y + 9 = y^2 + 2y + 1$

$-8y = -8$

$y = 1$

$\{1\}$

101. $\dfrac{3}{3a+4} = \dfrac{5}{5a-1}$

$\left[(3a+4)(5a-1) \left(\dfrac{3}{3a+4} \right) \right] = \left[(3a+4)(5a-1) \left(\dfrac{5}{5a-1} \right) \right]$

$3(5a-1) = 5(3a+4)$

$15a - 3 = 15a + 20$

$-3 = 20$

$\{ \ \}$

102. $\dfrac{8}{8x-3} = \dfrac{2}{2x+5}$

$\left[(8x-3)(2x+5) \left(\dfrac{8}{8x-3} \right) \right] = \left[(8x-3)(2x+5) \left(\dfrac{2}{2x+5} \right) \right]$

$8(2x+5) = 2(8x-3)$

$16x + 40 = 16x - 6$

$40 = -6$

$\{ \ \}$

103. $P = \dfrac{40 + 20x}{1 + 0.05x}$

$200 = \dfrac{40 + 20x}{1 + 0.05x}$

$(1 + 0.05x)(200) = (1 + 0.05x) \left(\dfrac{40 + 20x}{1 + 0.05x} \right)$

$200 + 10x = 40 + 20x$

$-10x = -160$

$x = 16 \text{ yr}$

104. $v = \dfrac{180t}{2t + 10}$

$60 = \dfrac{180t}{2t + 10}$

$(2t + 10)(60) = (2t + 10) \left(\dfrac{180t}{2t + 10} \right)$

$120t + 600 = 180t$

$-60t = -600$

$t = 10 \text{ sec}$

105. $A = \dfrac{1}{22}c + \dfrac{1}{30}h$

$7 = \dfrac{1}{22}c + \dfrac{1}{30}(165)$

$7 = \dfrac{1}{22}c + \dfrac{11}{2}$

$22(7) = 22 \left(\dfrac{1}{22}c + \dfrac{11}{2} \right)$

$154 = c + 121$

$c = 33 \text{ mi}$

106. $A = \dfrac{1}{24}c + \dfrac{1}{32}h$

$9 = \dfrac{1}{24}(60) + \dfrac{1}{32}h$

$9 = \dfrac{5}{2} + \dfrac{1}{32}h$

$32(9) = 32 \left(\dfrac{5}{2} + \dfrac{1}{32}h \right)$

$288 = 80 + h$

$h = 208 \text{ mi}$

107. The value 5 is not defined within the expressions in the equation. Substituting 5 into the equation would result in division by 0.

108. The equation is an identity. The solution set is all real numbers.

109. The equation cannot be written in the form $ax + b = 0$. The term $\dfrac{3}{x} = 3x^{-1}$. Therefore, the term $\dfrac{3}{x}$ is not first degree and the equation is not a first-degree equation.

110. The equation cannot be written in the form $ax + b = 0$. The term $2\sqrt{x} = 2x^{1/2}$. Therefore, the term $2\sqrt{x}$ is not first degree and the equation is not a first-degree equation.

111. The equation is a contradiction. There is no real number x to which we add 1 that will equal the same real number x to which we add 2.

112. In each case, we can clear fractions by multiplying both sides of the equation by the LCD. For the equation $\dfrac{x}{3} + \dfrac{1}{2} = 1$, the LCD is 6, whereas for $\dfrac{3}{x} + \dfrac{1}{2} = 1$, the LCD is $2x$.

113.
$$ax + 6 = 4x + 14$$
$$a(4) + 6 = 4(4) + 14$$
$$4a + 6 = 16 + 14$$
$$4a + 6 = 30$$
$$4a = 24$$
$$a = 6$$

114.
$$ax - 3 = 2x + 9$$
$$a(3) - 3 = 2(3) + 9$$
$$3a - 3 = 6 + 9$$
$$3a - 3 = 15$$
$$3a = 18$$
$$a = 6$$

115.
$$a(2x - 5) + 6 = 5x + 7$$
$$a[2(16) - 5] + 6 = 5(16) + 7$$
$$a(32 - 5) + 6 = 80 + 7$$
$$27a + 6 = 87$$
$$27a = 81$$
$$a = 3$$

116.
$$a(2x + 4) + 12x = 3(2 - x)$$
$$a[2(34) + 4] + 12(34) = 3[2 - (34)]$$
$$a(68 + 4) + 408 = 3(-32)$$
$$72a + 408 = -96$$
$$72a = -504$$
$$a = -7$$

Section 1.2 Applications with Linear and Rational Equations

1. $900

2. $0.08(2) = 0.16$ L

3. $\dfrac{d}{r}$

4. $\dfrac{d}{t}$

5. $P = 2l + 2w$

6. $180°$

7. Let x represent the amount borrowed at 3%. Then, $(5000 - x)$ is the amount borrowed at 2.5%.

	3% Interest Loan	2.5% Interest Loan	Total
Principal	x	$5000 - x$	
Interest ($I = Prt$)	$x(0.03)(1)$	$(5000 - x)(0.025)(1)$	132.50

$$x(0.03)+(5000-x)(0.025)=132.50$$
$$0.03x+125-0.025x=132.50$$
$$0.005x+125=132.50$$
$$0.005x=7.50$$
$$x=1500$$
$$5000-x=5000-1500$$
$$=3500$$

Rocco borrowed $1500 at 3% and $3500 at 2.5%.

8. Let x represent the amount borrowed at 4%. Then, $(22,000-x)$ is the amount borrowed
at 5.5%.

	4% Interest Loan	5.5% Interest Loan	Total
Principal	x	$22,000-x$	
Interest ($I = Prt$)	$x(0.04)(1)$	$(22,000-x)(0.055)(1)$	910

$$x(0.04)+(22,000-x)(0.055)=910$$
$$0.04x+1210-0.055x=910$$
$$-0.015x+1210=910$$
$$-0.015x=-300$$
$$x=20,000$$
$$22,000-x=22,000-20,000$$
$$=2000$$

Laura borrowed $20,000 from the bank charging 4% interest, and $2000 from the bank
charging 5.5% interest.

9. Let x represent the amount invested in the 3-yr CD. Then, $x-2000$ is the amount invested
in the 18-month CD.

	3-yr CD	18-month (1.5-yr) CD	Total
Principal	x	$x-2000$	
Interest ($I = Prt$)	$x(0.044)(3)$	$(x-2000)(0.03)(1.5)$	706.50

$$x(0.044)(3)+(x-2000)(0.03)(1.5)=706.50$$
$$0.132x+0.045x-90=706.50$$
$$0.177x-90=706.50$$
$$0.177x=796.50$$
$$x=4500$$
$$x-2000=4500-2000$$
$$=2500$$

Fernando invested $4500 in the 3-yr CD and $2500 in the 18-month CD.

10. Let x represent the amount invested in the 5-yr Treasury note. Then, $(x+5000)$ is the
amount invested in the 10-yr bond.

	5-yr Note	10-yr Bond	Total
Principal	x	$x+5000$	
Interest ($I = Prt$)	$x(0.028)(5)$	$(x+5000)(0.036)(10)$	5300

$$x(0.028)(5)+(x+5000)(0.036)(10)=5300$$
$$0.14x+0.36x+1800=5300$$
$$0.5x+1800=5300$$
$$0.5x=3500$$
$$x=7000$$
$$x+5000=7000+5000$$
$$=12{,}000$$

Ebony invested $7000 in the Treasury note and $12,000 in the bond.

11. Let x represent the amount of the 5% solution (in gallons). 5000 gal is the amount of the
10% solution. Therefore, $x+5000$ is the amount of the resulting 9% solution.

	5% Solution	10% Solution	9% Solution
Amount of Solution	x	5000	$x+5000$
Pure Ethanol	0.05x	$0.1(5000)$	$0.09(x+5000)$

$$0.05x+0.1(5000)=0.09(x+5000)$$
$$0.05x+500=0.09x+450$$
$$50=0.04x$$
$$1250=x$$

1250 gal of E5 should be mixed with the E10.

12. Let x represent the amount of the 10% solution (in cubic centimeters). 60 cc is the amount of the 50% solution. Therefore, $x+60$ is the amount of the resulting 25% solution.

	10% Solution	50% Solution	25% Solution
Amount of Solution	x	60	$x+60$
Pure Saline	0.1x	$0.5(60)$	$0.25(x+60)$

$$0.1x+0.5(60)=0.25(x+60)$$
$$0.1x+30=0.25x+15$$
$$15=0.15x$$
$$100=x$$

100 cc of 10% saline solution should be mixed with the 50% saline solution.

13. Let x represent the amount of the pure sand (in cubic feet). 480 ft^2 is the amount of the
concrete mix that is 70% sand. Therefore, $(x+480)$ is the amount of the resulting 75%
sand mixture.

	100% Sand	70% Sand	75% Sand

Amount of Mixture	x	480	$x+480$
Pure Sand	x	$0.7(480)$	$0.75(x+480)$

$$x+0.7(480)=0.75(x+480)$$
$$x+336=0.75x+360$$
$$0.25x=24$$
$$x=96$$

96 ft² of sand should be mixed with the 70% sand mixture.

14. Let x represent the amount of 50% antifreeze solution (in gallons) to be drained (and
therefore the amount of 100% antifreeze solution to be added). 4 gal is the amount of the
resulting 65% antifreeze solution. Therefore, $(4-x)$ is the amount of 50% antifreeze
solution that is not drained.

	100% Solution	50% Solution	65% Solution
Amount of Solution	x	$4-x$	4
Pure Antifreeze	x	$0.5(4-x)$	$0.65(4)$

$$x+0.5(4-x)=0.65(4)$$
$$x+2-0.5x=2.6$$
$$0.5x=0.6$$
$$x=1.2$$

1.2 gal of 50% antifreeze solution should be drained and replaced with 100% antifreeze.

15. Let x represent the speed of the plane flying to Los Angeles. Then, $(x+60)$ is the speed
of the plane flying to New York City.

	Distance	Rate	Time
Los Angeles Flight	$3.4x$	x	3.4
New York City Flight	$2.4(x+60)$	$x+60$	2.4

$$3.4x+2.4(x+60)=2464$$
$$3.4x+2.4x+144=2464$$
$$5.8x=2320$$
$$x=400$$
$$x+60=400+60$$
$$=460$$

The plane to Los Angeles travels 400 mph and the plane to New York City travels 460
mph.

16. Let x represent the speed of the plane flying to Seattle. Then, $(x - 44)$ is the speed of the
  plane flying to Boston.

	Distance	Rate	Time
Seattle Flight	$5.2x$	x	5.2
Boston Flight	$2.5(x - 44)$	$x - 44$	2.5

$$5.2x + 2.5(x - 44) = 3124$$
$$5.2x + 2.5x - 110 = 3124$$
$$7.7x = 3234$$
$$x = 420$$
$$x - 44 = 420 - 44$$
$$= 376$$

  The plane to Seattle travels 420 mph, and the plane to Boston travels 376 mph.

17. Let x represent the distance from Darren's home to his school.

	Distance	Rate	Time
To School	x	32	$\dfrac{x}{32}$
To Home	x	48	$\dfrac{x}{48}$

$$\frac{x}{32} + \frac{x}{48} = 1.25$$
$$96\left(\frac{x}{32} + \frac{x}{48}\right) = 96(1.25)$$
$$3x + 2x = 120$$
$$5x = 120$$
$$x = 24$$

  The distance is 24 mi.

18. Let x represent the distance of the loop.

	Distance	Rate	Time
Running	x	8	$\dfrac{x}{8}$
Riding	$5x$	16	$\dfrac{5x}{16}$

$$\frac{x}{8} + \frac{5x}{16} = 1.75$$
$$16\left(\frac{x}{8} + \frac{5x}{16}\right) = 16(1.75)$$
$$2x + 5x = 28$$
$$7x = 28$$
$$x = 4$$

  The loop is 4 mi.

19. Let t represent the time it takes for the runners to cover $\dfrac{1}{4}$ mile.

$$\frac{1\ \text{lap}}{66\ \text{sec}}+\frac{1\ \text{lap}}{60\ \text{sec}}=\frac{1\ \text{lap}}{t\ \text{sec}}$$

$$660t\left(\frac{1}{66}+\frac{1}{60}\right)=660t\left(\frac{1}{t}\right)$$

$$10t+11t=660$$

$$21t=660$$

$$t=\frac{220}{7}\approx31.4\ \text{sec}$$

20. Let t represent the time it takes Marta and her daughter to vacuum the house together.

$$\frac{1\ \text{job}}{40\ \text{min}}+\frac{1\ \text{job}}{60\ \text{min}}=\frac{1\ \text{job}}{t\ \text{min}}$$

$$120t\left(\frac{1}{40}+\frac{1}{60}\right)=120t\left(\frac{1}{t}\right)$$

$$3t+2t=120$$

$$5t=120$$

$$t=24\ \text{min}$$

21. Let t represent the time it takes the second pump to fill the pool by itself.

$$\frac{1\ \text{job}}{10\ \text{hr}}+\frac{1\ \text{job}}{t\ \text{hr}}=\frac{1\ \text{job}}{6\ \text{hr}}$$

$$30t\left(\frac{1}{10}+\frac{1}{t}\right)=30t\left(\frac{1}{6}\right)$$

$$3t+30=5t$$

$$30=2t$$

$$t=15\ \text{hr}$$

22. Let t represent the time it takes Angelina to mow the lawn by herself.

$$\frac{1\ \text{job}}{50\ \text{min}}+\frac{1\ \text{job}}{t\ \text{min}}=\frac{1\ \text{job}}{30\ \text{min}}$$

$$150t\left(\frac{1}{50}+\frac{1}{t}\right)=150t\left(\frac{1}{30}\right)$$

$$3t+150=5t$$

$$150=2t$$

$$t=75\ \text{min or }1\ \text{hr }15\ \text{min}$$

23. Let x represent the amount of cement and y represent the amount of gravel.

$$\frac{1}{2.4}=\frac{x}{150}$$

$$2.4x=150$$

$$x=62.5$$

$$\frac{2.4}{3.6}=\frac{150}{y}$$

$$2.4y=540$$

$$y=225$$

62.5 lb of cement and 225 lb of gravel

24. Let x represent the property tax on a house that is \$240,000.

$$\frac{180,000}{1296}=\frac{240,000}{x}$$

$$180,000x=1296(240,000)$$

$$x=\frac{1296(240,000)}{180,000}$$

$$x=\$1728$$

25. Let x represent the patient's LDL cholesterol level. The HDL cholesterol level is 60 g/dL, and the total cholesterol is $(x+60)$.

$$\frac{x+60}{60}=3.4$$

$$60\left(\frac{x+60}{60}\right)=60(3.4)$$

$$x+60=204$$

$$x=144$$

$$x+60=144+60$$

$$=204$$

LDL is 144 mg/dL and the total cholesterol is 204 mg/dL.

26. Let x represent the number of Democrats. Then, $(x-10)$ represents the number of Republicans.

$$\frac{x}{x-10} = \frac{11}{9}$$

$$9(x-10)\left(\frac{x}{x-10}\right) = 9(x-10)\left(\frac{11}{9}\right)$$

$$9x = 11(x-10)$$

$$9x = 11x - 110$$

$$110 = 2x$$

$$55 = x$$

$$x - 10 = 55 - 10 = 45$$

There were 55 Democrat and 45 Republican senators.

27. Let x represent the number of deer in the population.

$$\frac{30}{x} = \frac{5}{80}$$

$$5x = 2400$$

$$x = 480 \text{ deer}$$

28. Let x represent the number of bass in the lake.

$$\frac{24}{x} = \frac{4}{40}$$

$$4x = 960$$

$$x = 240 \text{ bass}$$

29. Let x represent the distance from the epicenter to the station.

	Distanc	Rat	Tim
P Wave s	x	5	$\frac{x}{5}$
S Wave s	x	3	$\frac{x}{3}$

$$\frac{x}{3} - \frac{x}{5} = 40$$

$$15\left(\frac{x}{3} - \frac{x}{5}\right) = 15(40)$$

$$5x - 3x = 600$$

$$2x = 600$$

$$x = 300 \text{ km}$$

30. Let x represent the distance from the epicentre to the station.

	Distanc	Rat	Tim
P Wave s	x	8	$\frac{x}{8}$

S Wave s	x	4.8	$\frac{x}{4.8}$

$$\frac{x}{4.8} - \frac{x}{8} = 20$$

$$24\left(\frac{x}{4.8} - \frac{x}{8}\right) = 24(20)$$

$$5x - 3x = 480$$

$$2x = 480$$

$$x = 240 \text{ km}$$

31. Let x represent the price set by the merchant.

$$x - 0.25x = 180 + 0.40(180)$$

$$0.75x = 180 + 72$$

$$0.75x = 252$$

$$x = \$336$$

32. Let x represent the price set by the bookstore.

$$x - 0.10x = 80 + 0.35(80)$$

$$0.90x = 80 + 28$$

$$0.90x = 108$$

$$x = \$120$$

33. a. $C = 110 + 60x$

b. $C = 350$

$$110 + 60x = 350$$

$$60x = 240$$

$$x = 4 \text{ hr}$$

34. a. $C = 2400 + 80x$

b. $C = 5520$

$$2400 + 80x = 5520$$
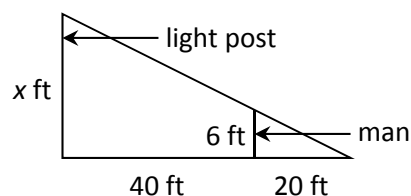
$$80x = 3120$$

$$x = 39 \text{ hr}$$

35. Let x represent the height of the Washington Monument.

$$\frac{5}{4} = \frac{x}{444}$$

$$4x = 2220$$

$$x = 555 \text{ ft}$$

36. Let x represent the height of the light post.

$$\frac{6}{20} = \frac{x}{40+20}$$

$$\frac{6}{20} = \frac{x}{60}$$

$$20x = 360$$

$$x = 18 \text{ ft}$$

37. Let x represent the height of the pole. Then, $\frac{1}{8}x$ is the length of the pole that is in the ground, and $\frac{2}{3}x$ is the length of the pole that is in the snow.

$$x = 15 + \frac{2}{3}x + \frac{1}{8}x$$

$$x - \frac{2}{3}x - \frac{1}{8}x = 15$$

$$24\left(x - \frac{2}{3}x - \frac{1}{8}x\right) = 24(15)$$

$$24x - 16x - 3x = 36$$

$$5x = 36$$

$$x = 7.2$$

The pole is 7.2 ft long, and the snow is 4.8 ft deep.

38.

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{5}{9}(F - 32)$$

$$9(F) = 9\left[\frac{5}{9}(F - 32)\right]$$

$$9F = 5F - 160$$

$$4F = -160$$

$$F = -40$$

$$-40°C = -40°F$$

39. Let x represent the amount of 20% fertilizer solution (in litres) to be drained (and therefore the amount of water to be added). 40 L is the amount of the resulting 15% fertilizer solution. Therefore, $(40 - x)$ is the amount of 20% fertilizer solution that is not drained.

	0% Solution	20% Solution	15% Solution
Amount of Solution	x	$40 - x$	40
Pure fertilizer	$0(x)$	$0.20(40 - x)$	$0.15(40)$

$$0(x) + 0.20(40 - x) = 0.15(40)$$

$$8 - 0.20x = 6$$

$$-0.20x = -2$$

$$x = 10$$

10 L should be drained and replaced by water.

40. Let x represent the amount of water (in litres) to be evaporated. Therefore, $(200 - x)$ is the amount of the final 25% salt solution.

	0% Solution	5% Solution	25% Solution
Amount of Solution	x	200	$200 - x$
Pure salt	$0(x)$	$0.05(200)$	$0.25(200 - x)$

$$0(x) + 0.05(200) = 0.25(200 - x)$$
$$10 = 50 - 0.25x$$
$$-40 = -0.25x$$
$$160 = x$$

160 mL should be evaporated.

41. The length of the lot is $l = 128 + 2x$. The width of the lot is $w = 60 + 2x$.

$$P = 2l + 2w$$
$$440 = 2(128 + 2x) + 2(60 + 2x)$$
$$440 = 256 + 4x + 120 + 4x$$
$$440 = 8x + 376$$
$$64 = 8x$$
$$8 = x$$

The width of the easement is 8 ft.

42. The length of the play area is $l = 78 + 2x$. The width of the play area is $w = 36 + 2x$.

$$P = 2l + 2w$$
$$396 = 2(78 + 2x) + 2(36 + 2x)$$
$$396 = 156 + 4x + 72 + 4x$$
$$396 = 8x + 228$$
$$168 = 8x$$
$$21 = x$$

The width of the border is 21 ft.

43. a. The width of the kitchen is w. The length of the kitchen is $l = w + 4$.

$$P = 2l + 2w$$
$$48 = 2(w + 4) + 2w$$
$$48 = 2w + 8 + 2w$$
$$48 = 4w + 8$$
$$40 = 4w$$
$$10 = w$$

The kitchen is 14 ft by 10 ft.

b. $A = lw + 0.1lw$
$$= 1.1lw$$
$$A = 1.1(14)(10)$$
$$= 154 \text{ ft}^2$$

c. $C = 1.06(12)(154)$
$$= \$1958.88$$

44. a. The width of the porch is w. The length of the porch is $l = 2w + 2$.

$$P = 2l + 2w$$
$$64 = 2(2w + 2) + 2w$$
$$64 = 4w + 4 + 2w$$
$$64 = 6w + 4$$
$$60 = 6w$$
$$10 = w$$
$$l = 2w + 2$$
$$= 2(10) + 2$$
$$= 22$$

The porch is 22 ft by 10 ft.

b. $A = lw + 0.1lw$
$$= 1.1lw$$
$$A = 1.1(22)(10)$$
$$= 242 \text{ ft}^2$$

c. $C = 1.075(5.85)(242)$
$$\approx \$1521.88$$

45. Aliyah had $8000 - 0.28(8000) = 8000 - 2240 = 5760$ to invest. Let x represent the amount she invested at 11%. Then, $(5760 - x)$ is the amount she invested at 5%.

	11% Investment	5% Investment	Total
Principal	x	$5760 - x$	
Interest ($I = Prt$)	$x(0.11)(1)$	$(5760 - x)(0.05)(1)$	453.60

$$x(0.11)+(5760-x)(0.05)=453.60$$
$$0.11x+288-0.05x=453.60$$
$$0.06x+288=453.60$$
$$0.06x=165.60$$
$$x=2760$$
$$5760-x=5760-2760$$
$$=3000$$

Aliyah invested \$2760 in the stock returning 11% and \$3000 in the stock returning 5%.

46. Let x represent the amount Caitlin invested in the balanced fund. Then, $(2x)$ is the

amount she invested in the stock fund.

	Balanced Fund (3.5%)	Stock Fund (17%)	Total
Principal	x	$2x$	
Interest ($I = Prt$)	$x(0.035)(1)$	$(2x)(0.17)(1)$	1125

$$x(0.035)+(2x)(0.17)=1125$$
$$0.035x+0.34x=1125$$
$$0.375x=1125$$
$$x=3000$$
$$2x=2(3000)$$
$$=6000$$

Caitlin invested \$3000 in the balanced fund and \$6000 in the stock fund.

47. $\dfrac{7}{8} = \dfrac{x}{12.8}$

$8x = 89.6$

$x = 11.2$

$\dfrac{8}{y} = \dfrac{12.8}{12}$

$12.8y = 96$

$y = 7.5$

x = 11.2 ft and y = 7.5 cm

48. $\dfrac{12}{0.96} = \dfrac{x}{1.04}$

$0.96x = 12.48$

$x = 13$

$\dfrac{0.5}{y} = \dfrac{12}{0.96}$

$12y = 0.48$

$y = 0.4$

x = 1.3 m and y = 0.4 in.

49. No. If x represents the measure of the smallest angle, then the equation $x + (x+2) + (x+4) = 180$ does not result in an odd integer value for x. Instead the measures of the angles would be even integers.

50. No. If x represents the number of each type of bill, then the solution to the equation $20x + 10x + 5x = 100$ is not a whole number.

51. Let x represent the smaller number. Then, $(x+16)$ is the larger number.

$\dfrac{x+16}{x} = 3 + \dfrac{2}{x}$

$x\left(\dfrac{x+16}{x}\right) = x\left(3 + \dfrac{2}{x}\right)$

$x + 16 = 3x + 2$

$14 = 2x$

$7 = x$

$x + 16 = 7 + 16$

$= 23$

The numbers are 7 and 23.

52. Let x represent the smaller number. Then, $(x+25)$ is the larger number.

$\dfrac{x+25}{x} = 4 + \dfrac{1}{x}$

$x\left(\dfrac{x+25}{x}\right) = x\left(4 + \dfrac{1}{x}\right)$

$x + 25 = 4x + 1$

$24 = 3x$

$8 = x$

$x + 25 = 8 + 25$

$= 33$

The numbers are 8 and 33.

53. Let x represent the tens digit of the number. Then, $(14-x)$ is the ones digit.

$10(14-x) + 1(x) = 10(x) + 1(14-x) + 18$

$140 - 10x + x = 10x + 14 - x + 18$

$140 - 9x = 9x + 32$

$108 = 18x$

$6 = x$

$14 - x = 14 - 6$

$= 8$

The original number is 68.

54. Let x represent the tens digit of the number. Then, $(9-x)$ is the ones digit.

$10(9-x) + 1(x) = 10(x) + 1(9-x) - 45$

$90 - 10x + x = 10x + 9 - x - 45$

$90 - 9x = 9x - 36$

$126 = 18x$

$7 = x$

$9 - x = 9 - 7$

$= 2$

The original number is 72.

55. $m_1 x_1 + m_2 x_2 = 0$

$(30)(-1.2) + (20)x_2 = 0$

$20x_2 = 36$

$x_2 = 1.8$ m

56. $m_1 x_1 + m_2 x_2 = 0$

$(64)x_1 + (80)(2) = 0$

$64x_1 = -160$

$x_1 = -2.5$ m

57.
$$m_1 x_1 + m_2 x_2 = 0$$
$$(10)(-3.2) + m_2 (8) = 0$$
$$8m_2 = -32$$
$$m_2 = 4 \text{ kg}$$

58.
$$m_1 x_1 + m_2 x_2 = 0$$
$$m_1(-10) + (6)(7) = 0$$
$$-10m_1 = -42$$
$$m_1 = 4.2 \text{ kg}$$

Section 1.3 Complex Numbers

1. -1

2. $i\sqrt{b}$

3. real; imaginary

4. conjugate

5. $\sqrt{-121} = i\sqrt{121} = 11i$

6. $\sqrt{-100} = i\sqrt{100} = 10i$

7. $\sqrt{-98} = i\sqrt{98} = 7i\sqrt{2}$

8. $\sqrt{-63} - i\sqrt{63} = 3i\sqrt{7}$

9. $\sqrt{-19} = i\sqrt{19}$

10. $\sqrt{-23} = i\sqrt{23}$

11. $-\sqrt{-16} = -i\sqrt{16} = -4i$

12. $-\sqrt{-25} = -i\sqrt{25} = -5i$

13. $\sqrt{-4}\sqrt{-9} = i\sqrt{4} \cdot i\sqrt{9}$
$$= 2i \cdot 3i = 6i^2$$
$$= 6(-1) = -6$$

14. $\sqrt{-1}\sqrt{-36} = i\sqrt{1} \cdot i\sqrt{36}$
$$= 1i \cdot 6i = 6i^2$$
$$= 6(-1) = -6$$

15. $\sqrt{-10}\sqrt{-5} = i\sqrt{10} \cdot i\sqrt{5}$
$$= i^2\sqrt{50}$$
$$= (-1)\sqrt{5^2 \cdot 2}$$
$$= -5\sqrt{2}$$

16. $\sqrt{-6}\sqrt{-15} = i\sqrt{6} \cdot i\sqrt{15}$
$$= i^2\sqrt{90}$$
$$= (-1)\sqrt{3^2 \cdot 10}$$
$$= -3\sqrt{10}$$

17. $\sqrt{-6}\sqrt{-14} = i\sqrt{6} \cdot i\sqrt{14}$
$$= i^2\sqrt{84}$$
$$= (-1)\sqrt{2^2 \cdot 21}$$
$$= -2\sqrt{21}$$

18. $\sqrt{-10}\sqrt{-15} = i\sqrt{10} \cdot i\sqrt{15}$
$$= i^2\sqrt{150}$$
$$= (-1)\sqrt{5^2 \cdot 6}$$
$$= -5\sqrt{6}$$

19. $\dfrac{\sqrt{-98}}{\sqrt{-2}} = \dfrac{i\sqrt{98}}{i\sqrt{2}}$
$$= \sqrt{\frac{98}{2}}$$
$$= \sqrt{49} = 7$$

20. $\dfrac{\sqrt{-45}}{\sqrt{-5}} = \dfrac{i\sqrt{45}}{i\sqrt{5}}$
$$= \sqrt{\frac{45}{5}}$$
$$= \sqrt{9} = 3$$

21. $\dfrac{\sqrt{-63}}{\sqrt{7}} = \dfrac{i\sqrt{63}}{\sqrt{7}}$
$$= i\sqrt{\frac{63}{7}}$$
$$= i\sqrt{9} = 3i$$

22. $\dfrac{\sqrt{-80}}{\sqrt{5}} = \dfrac{i\sqrt{80}}{\sqrt{5}}$
$$= i\sqrt{\frac{80}{5}}$$
$$= i\sqrt{16}$$
$$= 4i$$

23. Real part: 3; Imaginary part: -7

24. Real part: 2; Imaginary part: -4

25. Real part: 0; Imaginary part: 19

26. Real part: 0; Imaginary part: 40

27. Real part: $-\dfrac{1}{4}$; Imaginary part: 0

28. Real part: $-\dfrac{4}{7}$; Imaginary part: 0

29. $4\sqrt{-4} = 4 \cdot 2i$
$$= 8i = 0 + 8i$$

30. $2\sqrt{-144} = 2 \cdot 12i$

$$= 24i = 0 + 24i$$

31. $2 + \sqrt{-12} = 2 + 2\sqrt{3}i \text{ or } 2 + 2i\sqrt{3}$

32. $6 - \sqrt{-24} = 6 + \left(-2\sqrt{6}\right)i \text{ or } 6 - 2i\sqrt{6}$

33. $\dfrac{8 + 3i}{14} = \dfrac{8}{14} + \dfrac{3}{14}i$

$$= \dfrac{4}{7} + \dfrac{3}{14}i$$

34. $\dfrac{4 + 5i}{6} = \dfrac{4}{6} + \dfrac{5}{6}i$

$$= \dfrac{2}{3} + \dfrac{5}{6}i$$

35. $\dfrac{-4 - 6i}{-2} = \dfrac{-4}{-2} + \dfrac{-6}{-2}i$

$$= 2 + 3i$$

36. $\dfrac{9 - 15i}{-3} = \dfrac{9}{-3} - \dfrac{15}{-3}i$

$$= -3 + 5i$$

37.

$$\dfrac{-18 + \sqrt{-48}}{4} = \dfrac{-18 + 4\sqrt{3}i}{4}$$

$$= -\dfrac{18}{4} + \dfrac{4\sqrt{3}i}{4}$$

$$= -\dfrac{9}{2} + \sqrt{3}i \text{ or } -\dfrac{9}{2} + i\sqrt{3}$$

38. $\dfrac{-20 + \sqrt{-50}}{-10} = \dfrac{-20 + 5\sqrt{2}i}{-10}$

$$= \dfrac{-20}{-10} + \dfrac{5\sqrt{2}i}{-10}$$

$$= 2 - \dfrac{\sqrt{2}}{2}i \text{ or } 2 - i\dfrac{\sqrt{2}}{2}$$

39. $\dfrac{14 - \sqrt{-98}}{-7} = \dfrac{14 - 7\sqrt{2}i}{-7}$

$$= -\dfrac{14}{7} + \dfrac{7\sqrt{2}i}{7}$$

$$= -2 + \sqrt{2}i \text{ or } -2 + i\sqrt{2}$$

40.

$$\dfrac{-10 + \sqrt{-125}}{5} = \dfrac{-10 + 5\sqrt{5}i}{5}$$

$$= -\dfrac{10}{5} + \dfrac{5\sqrt{5}i}{5}$$

$$= -2 + \sqrt{5}i \text{ or } -2 + i\sqrt{5}$$

41. a. $i^{20} = 1$

b. $i^{29} = i^{28} \cdot i^{1}$

$$= \left(1\right) \cdot i^{1} = i$$

c. $i^{50} = i^{48} \cdot i^{2}$

$$= \left(1\right) \cdot i^{2} = -1$$

d. $i^{-41} = i^{-44} \cdot i^{3}$

$$= \left(1\right) \cdot i^{3} = -i$$

42. a. $i^{32} = 1$

b. $i^{47} = i^{44} \cdot i^{3}$

$$= \left(1\right) \cdot i^{3} = -i$$

c. $i^{66} = i^{64} \cdot i^{2}$

$$= \left(1\right) \cdot i^{2} = -1$$

d. $i^{-27} = i^{-28} \cdot i^{1}$

$$= \left(1\right) \cdot i^{1} = i$$

43. a. $i^{37} = i^{36} \cdot i^{1}$

$$= i$$

b. $i^{-37} = i^{-40} \cdot i^{3}$

$$= \left(1\right) \cdot i^{3} = -i$$

c. $i^{82} = i^{80} \cdot i^{2}$

$$= \left(1\right) \cdot i^{2} = -1$$

d. $i^{-82} = i^{-84} \cdot i^{2}$

$$= \left(1\right) \cdot i^{2} = -1$$

44. a. $i^{103} = i^{100} \cdot i^{3}$

$$= \left(1\right) \cdot i^{3} = -i$$

b. $i^{-103} = i^{-104} \cdot i^{1}$

$$= \left(1\right) \cdot i^{1} = i$$

c. $i^{52} = 1$

d. $i^{-52} = 1$

45. $\left(2 - 7i\right) + \left(8 - 3i\right)$

$$= \left(2 + 8\right) + \left(-7 - 3\right)i$$

$$= 10 - 10i$$

46. $\left(6 - 10i\right) + \left(8 + 4i\right)$

$$= (6+8) + (-10+4)i$$
$$= 14-6i$$

47. $(15+21i) - (18-40i)$

$$= (15-18) + [21-(-40)]i$$
$$= -3+61i$$

48. $(250+100i) - (80+25i)$

$$= (250-80) + (100-25)i$$
$$= 170+75i$$

49. $\left(\dfrac{1}{2}+\dfrac{2}{3}i\right) - \left(\dfrac{5}{6}+\dfrac{1}{12}i\right)$

$$= \left(\dfrac{1}{2}-\dfrac{5}{6}\right) + \left(\dfrac{2}{3}-\dfrac{1}{12}\right)i$$
$$= \left(\dfrac{3}{6}-\dfrac{5}{6}\right) + \left(\dfrac{8}{12}-\dfrac{1}{12}\right)i$$
$$= -\dfrac{2}{6}+\dfrac{7}{12}i$$
$$= -\dfrac{1}{3}+\dfrac{7}{12}i$$

50. $\left(\dfrac{3}{5}-\dfrac{1}{8}i\right) - \left(\dfrac{7}{10}+\dfrac{1}{6}i\right)$

$$= \left(\dfrac{3}{5}-\dfrac{7}{10}\right) + \left(-\dfrac{1}{8}-\dfrac{1}{6}\right)i$$
$$= \left(\dfrac{6}{10}-\dfrac{7}{10}\right) + \left(-\dfrac{3}{24}-\dfrac{4}{24}\right)i$$
$$= -\dfrac{1}{10}-\dfrac{7}{24}i$$

51. $(2.3+4i) - (8.1-2.7i) + (4.6-6.7i)$

$$= (2.3-8.1+4.6) + (4+2.7-6.7)i$$
$$= -1.2+0i$$

52. $\begin{pmatrix} 0.05 \\ -0.03i \end{pmatrix} + \begin{pmatrix} -0.12 \\ +0.08i \end{pmatrix} - \begin{pmatrix} 0.07 \\ +0.05i \end{pmatrix}$

$$= (0.05-0.12-0.07)$$
$$+ (-0.03+0.08-0.05)i$$
$$= -0.14+0i$$

53. $-\dfrac{1}{8}(16+24i) = -2-3i$

54. $-\dfrac{1}{6}(60-30i) = -10+5i$

55. $2i(5+i) = 10i+2i^2$

$$= 10i+2(-1)$$
$$= -2+10i$$

56. $4i(6+5i) = 24i+20i^2$

$$= 24i+20(-1)$$
$$= -20+24i$$

57. $\sqrt{-3}\left(\sqrt{11}-\sqrt{-7}\right) = i\sqrt{3}\left(\sqrt{11}-i\sqrt{7}\right)$

$$= i\sqrt{33} - i^2\sqrt{21}$$
$$= i\sqrt{33} - (-1)\sqrt{21}$$
$$= \sqrt{21} + i\sqrt{33}$$

58. $\sqrt{-2}\left(\sqrt{13}+\sqrt{-5}\right) = i\sqrt{2}\left(\sqrt{13}+i\sqrt{5}\right)$

$$= i\sqrt{26} + i^2\sqrt{10}$$
$$= i\sqrt{26} + (-1)\sqrt{10}$$
$$= -\sqrt{10} + i\sqrt{26}$$

59. $(3-6i)(10+i)$

$$= 3(10) + 3(i) + (-6i)(10) + (-6i)(i)$$
$$= 30+3i-60i-6i^2$$
$$= 30-57i-6(-1)$$
$$= 36-57i$$

60. $(2-5i)(8+2i)$

$$= 2(8) + 2(2i) + (-5i)(8) + (-5i)(2i)$$
$$= 16+4i-40i-10i^2$$
$$= 16-36i-10(-1)$$
$$= 26-36i$$

61. $(3-7i)^2 = (3)^2 - 2(3)(7i) + (7i)^2$

$$= 9-42i+49i^2$$
$$= 9-42i+49(-1)$$
$$= 9-42i-49$$
$$= -40-42i$$

62. $(10-3i)^2 = (10)^2 - 2(10)(3i) + (3i)^2$

$$= 100-60i+9i^2$$
$$= 100-60i+9(-1)$$
$$= 100-60i-9$$
$$= 91-60i$$

63. $\left(3-\sqrt{-5}\right)\left(4+\sqrt{-5}\right)$

$$= \left(3 - i\sqrt{5}\right)\left(4 + i\sqrt{5}\right)$$
$$= 3(4) + 3\left(i\sqrt{5}\right) + \left(-i\sqrt{5}\right)(4)$$
$$\qquad\qquad + \left(-i\sqrt{5}\right)\left(i\sqrt{5}\right)$$
$$= 12 + 3i\sqrt{5} - 4i\sqrt{5} - 5i^2$$
$$= 12 - i\sqrt{5} - 5(-1)$$
$$= 17 - i\sqrt{5}$$

64. $\left(2 + \sqrt{-7}\right)\left(10 + \sqrt{-7}\right)$
$$= \left(2 + i\sqrt{7}\right)\left(10 + i\sqrt{7}\right)$$
$$= 2(10) + 2\left(i\sqrt{7}\right) + i\sqrt{7}(10)$$
$$\qquad\qquad + i\sqrt{7}\left(i\sqrt{7}\right)$$
$$= 20 + 2i\sqrt{7} + 10i\sqrt{7} + 7i^2$$
$$= 20 + 12i\sqrt{7} + 7(-1)$$
$$= 13 + 12i\sqrt{7}$$

65. $4(6 + 2i) - 5i(3 - 7i)$
$$= 24 + 8i - 15i + 35i^2$$
$$= 24 - 7i + 35(-1)$$
$$= -11 - 7i$$

66. $-3(8 - 3i) - 6i(2 + i)$
$$= -24 + 9i - 12i - 6i^2$$
$$= -24 - 3i - 6(-1)$$
$$= -18 - 3i$$

67. $(2 - i)^2 + (2 + i)^2$
$$= (2)^2 - 2(2)(i) + i^2 + (2)^2$$
$$\qquad\qquad + 2(2)(i) + i^2$$
$$= 4 - 4i + i^2 + 4 + 4i + i^2$$
$$= 8 + 2i^2$$
$$= 8 + 2(-1) = 6$$

68. $(3 - 2i)^2 + (3 + 2i)^2$
$$= (3)^2 - 2(3)(2i) + (2i)^2 + (3)^2$$
$$\qquad\qquad + 2(3)(2i) + (2i)^2$$
$$= 9 - 12i + 4i^2 + 9 + 12i + 4i^2$$
$$= 18 + 8i^2$$
$$= 18 + 8(-1) = 10$$

69. a. $3 + 6i$

   b. $(3 - 6i)(3 + 6i) = (3)^2 + (6)^2$
$$= 9 + 36$$
$$= 45$$

70. a. $4 + 5i$

   b. $(4 - 5i)(4 + 5i) = (4)^2 + (5)^2$
$$= 16 + 25$$
$$= 41$$

71. a. $0 - 8i$

   b. $(0 - 8i)(0 + 8i) = (0)^2 + (8)^2$
$$= 0 + 64$$
$$= 64$$

72. a. $0 - 9i$

   b. $(0 - 9i)(0 + i) = (0)^2 + (9)^2$
$$= 0 + 81$$
$$= 81$$

73. $(10 - 4i)(10 + 4i) = (10)^2 + (4)^2$
$$= 100 + 16$$
$$= 116$$

74. $(3 - 9i)(3 + 9i) = (3)^2 + (9)^2$
$$= 9 + 81$$
$$= 90$$

75. $(7i)(-7i) = 7^2 = 49$

76. $(-5i)(5i) = (5)^2 = 25$

77. $\left(\sqrt{2} + \sqrt{3}i\right)\left(\sqrt{2} + \sqrt{3}i\right)$
$$= \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2$$
$$= 2 + 3 = 5$$

78. $\left(\sqrt{5} + \sqrt{7}i\right)\left(\sqrt{5} - \sqrt{7}i\right)$
$$= \left(\sqrt{5}\right)^2 + \left(\sqrt{7}\right)^2$$
$$= 5 + 7 = 12$$

79. $\dfrac{6+2i}{3-i} = \dfrac{(6+2i)(3+i)}{(3-i)(3+i)}$

$= \dfrac{18+6i+6i+2i^2}{(3)^2+(1)^2}$

$= \dfrac{18+12i+2(-1)}{9+1}$

$= \dfrac{16+12i}{10}$

$= \dfrac{16}{10} + \dfrac{12}{10}i$

$= \dfrac{8}{5} + \dfrac{6}{5}i$

80. $\dfrac{5+i}{4-i} = \dfrac{(5+i)(4+i)}{(4-i)(4+i)}$

$= \dfrac{20+5i+4i+i^2}{(4)^2+(1)^2}$

$= \dfrac{20+9i+1(-1)}{16+1}$

$= \dfrac{19+9i}{17}$

$= \dfrac{19}{17} + \dfrac{9}{17}i$

81. $\dfrac{8-5i}{13+2i} = \dfrac{(8-5i)(13-2i)}{(13+2i)(13-2i)}$

$= \dfrac{104-16i-65i+10i^2}{(13)^2+(2)^2}$

$= \dfrac{104-81i+10(-1)}{169+4}$

$= \dfrac{94-81i}{173}$

$= \dfrac{94}{173} - \dfrac{81}{173}i$

82. $\dfrac{10-3i}{11+4i} = \dfrac{(10-3i)(11-4i)}{(11+4i)(11-4i)}$

$= \dfrac{110-40i-33i+12i^2}{(11)^2+(4)^2}$

$= \dfrac{110-73i+12(-1)}{121+16}$

$= \dfrac{98-73i}{137}$

$= \dfrac{98}{137} - \dfrac{73}{137}i$

83. $\left(6+\sqrt{5}i\right)^{-1} = \dfrac{1}{6+\sqrt{5}i}$

$= \dfrac{1\left(6-\sqrt{5}i\right)}{\left(6+\sqrt{5}i\right)\left(6-\sqrt{5}i\right)}$

$= \dfrac{6-\sqrt{5}i}{(6)^2+\left(\sqrt{5}\right)^2}$

$= \dfrac{6-\sqrt{5}i}{36+5}$

$= \dfrac{6-\sqrt{5}i}{41}$

$= \dfrac{6}{41} - \dfrac{\sqrt{5}}{41}i$

84. $\left(4-\sqrt{3}i\right)^{-1} = \dfrac{1}{4-\sqrt{3}i}$

$= \dfrac{1\left(4+\sqrt{3}i\right)}{\left(4-\sqrt{3}i\right)\left(4+\sqrt{3}i\right)}$

$= \dfrac{4+\sqrt{3}i}{(4)^2+\left(\sqrt{3}\right)^2}$

$= \dfrac{4+\sqrt{3}i}{16+3}$

$= \dfrac{4+\sqrt{3}i}{19}$

$= \dfrac{4}{19} + \dfrac{\sqrt{3}}{19}i$

85. $\dfrac{5}{13i} = \dfrac{5 \cdot i}{13i \cdot i}$

$\qquad = \dfrac{5i}{13i^2} = \dfrac{5i}{13(-1)}$

$\qquad = \dfrac{5i}{-13} = -\dfrac{5}{13}i$

$\qquad = 0 - \dfrac{5}{13}i$

86. $\dfrac{6}{7i} = \dfrac{6(-i)}{7i(-i)}$

$\qquad = \dfrac{-6i}{-7i^2} = \dfrac{-6i}{-7(-1)}$

$\qquad = -\dfrac{6}{7}i = 0 - \dfrac{6}{7}i$

87. $\dfrac{-1}{\sqrt{-3}} = \dfrac{-1}{\sqrt{3}i}$

$\qquad = \dfrac{-1 \cdot \sqrt{3}i}{\sqrt{3}i \cdot \sqrt{3}i} = \dfrac{-\sqrt{3}i}{3i^2}$

$\qquad = \dfrac{-\sqrt{3}i}{3(-1)} = \dfrac{-\sqrt{3}i}{-3}$

$\qquad = \dfrac{\sqrt{3}i}{3} = 0 + \dfrac{\sqrt{3}i}{3}$

88. $\dfrac{-2}{\sqrt{-11}} = \dfrac{-2}{\sqrt{11}i}$

$\qquad = \dfrac{-2 \cdot \sqrt{11}i}{\sqrt{11}i \cdot \sqrt{11}i}$

$\qquad = \dfrac{-2\sqrt{11}i}{11i^2} = \dfrac{-2\sqrt{11}i}{11(-1)}$

$\qquad = \dfrac{2\sqrt{11}}{11}i = 0 + \dfrac{2\sqrt{11}}{11}i$

89. $\sqrt{b^2 - 4ac} = \sqrt{(4)^2 - 4(2)(6)}$

$\qquad = \sqrt{16 - 48}$

$\qquad = \sqrt{-32} = i\sqrt{32}$

$\qquad = 4i\sqrt{2}$

90. $\sqrt{b^2 - 4ac} = \sqrt{(-5)^2 - 4(5)(10)}$

$\qquad = \sqrt{25 - 200}$

$\qquad = \sqrt{-175} = i\sqrt{175}$

$\qquad = 5i\sqrt{7}$

91. $\sqrt{b^2 - 4ac} = \sqrt{(-6)^2 - 4(2)(5)}$

$\qquad = \sqrt{36 - 40}$

$\qquad = \sqrt{-4} = i\sqrt{4}$

$\qquad = 2i$

92. $\sqrt{b^2 - 4ac} = \sqrt{(4)^2 - 4(2)(4)}$

$\qquad = \sqrt{16 - 32}$

$\qquad = \sqrt{-16} = i\sqrt{16}$

$\qquad = 4i$

93. a. $\qquad x^2 + 25 = 0$

$\qquad (5i)^2 + 25 = 0$

$\qquad 25(-1) + 25 = 0$

$\qquad -25 + 25 = 0$

$\qquad 0 = 0 \checkmark$

b. $\qquad x^2 + 25 = 0$

$\qquad (-5i)^2 + 25 = 0$

$\qquad 25(-1) + 25 = 0$

$\qquad -25 + 25 = 0$

$\qquad 0 = 0 \checkmark$

94. a. $\qquad x^2 + 49 = 0$

$\qquad (7i)^2 + 49 = 0$

$\qquad 49(-1) + 49 = 0$

$\qquad -49 + 49 = 0$

$\qquad 0 = 0 \checkmark$

b. $\qquad x^2 + 49 = 0$

$\qquad (-7i)^2 + 49 = 0$

$\qquad 49(-1) + 49 = 0$

$\qquad -49 + 49 = 0$

$\qquad 0 = 0 \checkmark$

95. a. $\qquad x^2 - 4x + 7 = 0$

$\qquad (2 + i\sqrt{3})^2 - 4(2 + i\sqrt{3}) + 7 = 0$

$\qquad 4 + 4i\sqrt{3} + 3i^2 - 8 - 4i\sqrt{3} + 7 = 0$

$\qquad 4 + 3(-1) - 8 + 7 = 0$

$\qquad 4 - 3 - 1 = 0$

$\qquad 0 = 0 \checkmark$

b.
$$x^2 - 4x + 7 = 0$$
$$\left(2 - i\sqrt{3}\right)^2 - 4\left(2 - i\sqrt{3}\right) + 7 = 0$$
$$4 - 4i\sqrt{3} + 3i^2 - 8 + 4i\sqrt{3} + 7 = 0$$
$$4 + 3(-1) - 8 + 7 = 0$$
$$4 - 3 - 1 = 0$$
$$0 = 0 \checkmark$$

96. a.
$$x^2 - 6x + 11 = 0$$
$$\left(3 + i\sqrt{2}\right)^2 - 6\left(3 + i\sqrt{2}\right) + 11 = 0$$
$$9 + 6i\sqrt{3} + 2i^2 - 18 - 6i\sqrt{2} + 11 = 0$$
$$9 + 2(-1) - 18 + 11 = 0$$
$$9 - 2 - 7 = 0$$
$$0 = 0 \checkmark$$

b.
$$x^2 - 6x + 11 = 0$$
$$\left(3 - i\sqrt{2}\right)^2 - 6\left(3 - i\sqrt{2}\right) + 11 = 0$$
$$9 - 6i\sqrt{3} + 2i^2 - 18 + 6i\sqrt{2} + 11 = 0$$
$$9 + 2(-1) - 18 + 11 = 0$$
$$9 - 2 - 7 = 0$$
$$0 = 0 \checkmark$$

97. $\left(a + bi\right)\left(c + di\right)$
$$= ac + adi + bci + bdi^2$$
$$= ac + \left(ad + bc\right)i + bd(-1)$$
$$= \left(ac - bd\right) + \left(ad + bc\right)i$$

98. $\left(a + bi\right)^2 = \left(a\right)^2 + 2\left(a\right)\left(bi\right) + \left(bi\right)^2$
$$= a^2 + \left(2ab\right)i + b^2 i^2$$
$$= a^2 + \left(2ab\right)i + b^2(-1)$$
$$= \left(a^2 - b^2\right) + \left(2ab\right)i$$

99. The second step does not follow because the multiplication property of radicals can be applied only if the individual radicals are real numbers. Because $\sqrt{-9}$ and $\sqrt{-4}$ are imaginary numbers, the correct

logic for simplification would be
$$\sqrt{-9} \cdot \sqrt{-4} = i\sqrt{9} \cdot i\sqrt{4}$$
$$= i^2\sqrt{36}$$
$$= -1 \cdot 6 = -6$$

100. The product $\left(a + b\right)\left(a - b\right)$ simplifies to $a^2 - b^2$. The product $\left(a + bi\right)\left(a - bi\right)$ simplifies to $a^2 - \left(bi\right)^2$, which simplifies to $a^2 + b^2$.

101. Any real number. For example: 5.

102. Any complex number and its conjugate. For example: $2 + 5i$ and $2 - 5i$. In general, for real numbers, a and b, $\left(a + bi\right)\left(a - bi\right) = a^2 + b^2$, which is a real number.

103. $z \cdot \overline{z} = \left(a + bi\right)\left(a - bi\right) = a^2 + b^2$

104. $z^2 - \overline{z}^2$
$$= \left(a + bi\right)^2 - \left(a - bi\right)^2$$
$$= a^2 + 2abi + \left(bi\right)^2 - \left[a^2 - 2abi + \left(bi\right)^2\right]$$
$$= a^2 + 2abi + b^2 i^2 - a^2 + 2abi - b^2 i^2$$
$$= \left(4ab\right)i$$

105. a. $x^2 - 9 = \left(x + 3\right)\left(x - 3\right)$
b. $x^2 + 9 = \left(x + 3i\right)\left(x - 3i\right)$

106. a. $x^2 - 100 = \left(x + 10\right)\left(x - 10\right)$
b. $x^2 + 100 = \left(x + 10i\right)\left(x - 10i\right)$

107. a. $x^2 - 64 = \left(x + 8\right)\left(x - 8\right)$
b. $x^2 + 64 = \left(x + 8i\right)\left(x - 8i\right)$

108. a. $x^2 - 25 = \left(x + 5\right)\left(x - 5\right)$
b. $x^2 + 25 = \left(x + 5i\right)\left(x - 5i\right)$

109. a. $x^2 - 3 = \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$
b. $x^2 + 3 = \left(x + i\sqrt{3}\right)\left(x - i\sqrt{3}\right)$

110. a. $x^2 - 11 = \left(x + \sqrt{11}\right)\left(x - \sqrt{11}\right)$
b. $x^2 + 11 = \left(x + i\sqrt{11}\right)\left(x - i\sqrt{11}\right)$

111.
```
√(-16)
                4i
(4-5i)-(2+3i)
             2-8i
(12-15i)(-2+9i)
           111+138i
```

112.
```
√(-169)
              13i
(-11-2i)+(-4+9i)
            -15+7i
(8+12i)(-3-7i)
            60-92i
```

113.
```
(4-9i)²
            -65-72i
7/(2i)►Frac
             -7/2i
(14+8i)/(3-i)►Fr
ac
        17/5+19/5i
```

114.
```
(11+4i)²
          105+88i
11/(10i)►Frac
            -11/10i
(5+7i)/(6+8i)►Fr
ac
       43/50+1/50i
```

Section 1.4 Quadratic Equation

1. quadratic

2. linear

3. $\pm\sqrt{k}$

4. 100

5. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

6. $b^2 - 4ac$

7. $(x-3)(x+7) = 0$

$\quad\quad x - 3 = 0 \quad$ or $\quad x + 7 = 0$

$\quad\quad\quad x = 3 \quad\quad\quad\quad x = -7$

$\quad\quad\quad\quad \{3, -7\}$

8. $(t+4)(t-1) = 0$

$\quad\quad t + 4 = 0 \quad$ or $\quad t - 1 = 0$

$\quad\quad\quad t = -4 \quad\quad\quad\quad t = 1$

$\quad\quad\quad\quad \{-4, 1\}$

9. $\quad n^2 + 5n = 24$

$\quad n^2 + 5n - 24 = 0$

$\quad (n+8)(n-3) = 0$

$\quad\quad n + 8 = 0 \quad$ or $\quad n - 3 = 0$

$\quad\quad\quad n = -8 \quad\quad\quad n = 3$

$\quad\quad\quad\quad \{-8, 3\}$

10. $\quad\quad y^2 = 18 - 7y$

$\quad y^2 + 7y - 18 = 0$

$\quad (y+9)(y-2) = 0$

$\quad\quad y + 9 = 0 \quad$ or $\quad y - 2 = 0$

$\quad\quad\quad y = -9 \quad\quad\quad y = 2$

$\quad\quad\quad\quad \{-9, 2\}$

11. $\quad 8t(t+3) = 2t - 5$

$\quad 8t^2 + 24t = 2t - 5$

$\quad 8t^2 + 22t + 5 = 0$

$\quad (2t+5)(4t+1) = 0$

$\quad\quad 2t + 5 = 0 \quad$ or $\quad 4t + 1 = 0$

$\quad\quad\quad 2t = -5 \quad\quad\quad 4t = -1$

$\quad\quad\quad t = -\dfrac{5}{2} \quad\quad\quad t = -\dfrac{1}{4}$

$\quad\quad\quad \left\{-\dfrac{5}{2}, -\dfrac{1}{4}\right\}$

12. $\quad 6m(m+4) = m - 15$

$\quad 6m^2 + 24m = m - 15$

$\quad 6m^2 + 23m + 15 = 0$

$\quad (6m+5)(m+3) = 0$

$\quad\quad 6m + 5 = 0 \quad$ or $\quad m + 3 = 0$

$\quad\quad\quad 6m = -5 \quad\quad\quad m = -3$

$\quad\quad\quad m = -\dfrac{5}{6}$

$\quad\quad\quad \left\{-\dfrac{5}{6}, -3\right\}$

13. $\quad 40p^2 - 90 = 0$

$$10\left(4p^2-9\right)=0$$
$$10\left(2p-3\right)\left(2p+3\right)=0$$
$$2p-3=0 \quad \text{or} \quad 2p+3=0$$
$$2p=3 \qquad\qquad 2p=-3$$
$$p=\frac{3}{2} \qquad\qquad p=-\frac{3}{2}$$
$$\left\{\frac{3}{2},-\frac{3}{2}\right\}$$

14. $\qquad 32n^2-162=0$

$$2\left(16n^2-81\right)=0$$
$$2\left(4n-9\right)\left(4n+9\right)=0$$
$$4n-9=0 \quad \text{or} \quad 4n+9=0$$
$$4n=9 \qquad\qquad 4n=-9$$
$$n=\frac{9}{4} \qquad\qquad n=-\frac{9}{4}$$
$$\left\{\frac{9}{4},-\frac{9}{4}\right\}$$

15. $\qquad 3x^2=12x$

$$3x^2-12x=0$$
$$3x\left(x-4\right)=0$$
$$3x=0 \quad \text{or} \quad x-4=0$$
$$x=0 \qquad\qquad x=4$$
$$\left\{0,4\right\}$$

16. $\qquad z^2=25z$

$$z^2-25z=0$$
$$z\left(z-25\right)=0$$
$$z=0 \quad \text{or} \quad z-25=0$$
$$z=25$$
$$\left\{0,25\right\}$$

17. $\qquad \left(m+4\right)\left(m-5\right)=-8$

$$m^2+4m-5m-20=-8$$
$$m^2-m-12=0$$
$$\left(m+3\right)\left(m-4\right)=0$$
$$m+3=0 \text{ or } m-4=0$$
$$m=-3 \qquad\qquad m=4$$
$$\left\{-3,4\right\}$$

18. $\quad \left(n+2\right)\left(n-4\right)=27$

$$n^2-4n+2n-8=27$$
$$n^2-2n-35=0$$
$$\left(n+5\right)\left(n-7\right)=0$$
$$n+5=0 \quad \text{or} \quad n-7=0$$
$$n=-5 \qquad\qquad n=7$$
$$\left\{-5,7\right\}$$

19. $x^2=81$

$$x=\pm\sqrt{81}$$
$$=\pm 9$$
$$\left\{9,-9\right\}$$

20. $w^2=121$

$$w=\pm\sqrt{121}$$
$$=\pm 11$$
$$\left\{11,-11\right\}$$

21. $5y^2-35=0$

$$5y^2=35$$
$$y^2=7$$
$$y=\pm\sqrt{7}$$
$$\left\{\sqrt{7},-\sqrt{7}\right\}$$

22. $6v^2-30=0$

$$6v^2=30$$
$$v^2=5$$
$$v=\pm\sqrt{5}$$
$$\left\{\sqrt{5},-\sqrt{5}\right\}$$

23. $4u^2+64=0$

$$4u^2=-64$$
$$u^2=-16$$
$$u=\pm\sqrt{-16}=\pm 4i$$
$$\left\{4i,-4i\right\}$$

24. $8p^2+72=0$

$$8p^2=-72$$
$$p^2=-9$$
$$p=\pm\sqrt{-9}=\pm 3i$$
$$\left\{3i,-3i\right\}$$

25. $(k+2)^2 = 28$

$\quad k+2 = \pm\sqrt{28}$

$\quad\quad k = -2 \pm \sqrt{28}$

$\quad\quad\quad = -2 \pm 2\sqrt{7}$

$\quad \left\{-2 \pm 2\sqrt{7}\right\}$

26. $3(z+11)^2 - 10 = 110$

$\quad 3(z+11)^2 = 120$

$\quad (z+11)^2 = 40$

$\quad\quad z+11 = \pm\sqrt{40}$

$\quad\quad\quad z = -11 \pm \sqrt{40}$

$\quad\quad\quad\quad = -11 \pm 2\sqrt{10}$

$\quad \left\{-11 \pm 2\sqrt{10}\right\}$

27. $2(w-5)^2 + 5 = 23$

$\quad 2(w-5)^2 = 18$

$\quad\quad w-5 = \pm\sqrt{9}$

$\quad\quad\quad w = 5 \pm \sqrt{9}$

$\quad\quad\quad w = 5 \pm 3$

$\quad\quad\quad w = 5+3 \ \text{ or } \ w = 5-3$

$\quad\quad\quad\quad w = 8 \quad\quad\quad w = 2$

$\quad \{8, 2\}$

28. $(c-3)^2 = 49$

$\quad c-3 = \pm\sqrt{49}$

$\quad\quad c = 3 \pm \sqrt{49}$

$\quad\quad c = 3 \pm 7$

$\quad\quad c = 3+7 \ \text{ or } \ c = 3-7$

$\quad\quad\quad c = 10 \quad\quad c = -4$

$\quad \{10, -4\}$

29. $\left(t-\dfrac{1}{2}\right)^2 = -\dfrac{17}{4}$

$t-\dfrac{1}{2} = \pm\sqrt{-\dfrac{17}{4}}$

$\quad t = \dfrac{1}{2} \pm \sqrt{-\dfrac{17}{4}}$

$\quad\quad = \dfrac{1}{2} \pm \dfrac{i\sqrt{17}}{2}$

$\quad\quad = \dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}i$

$\quad \left\{\dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}i\right\}$

30. $\left(a-\dfrac{1}{3}\right)^2 = -\dfrac{47}{9}$

$a-\dfrac{1}{3} = \pm\sqrt{-\dfrac{47}{9}}$

$\quad a = \dfrac{1}{3} \pm \sqrt{-\dfrac{47}{9}}$

$\quad\quad = \dfrac{1}{3} \pm \dfrac{i\sqrt{47}}{3}$

$\quad\quad = \dfrac{1}{3} \pm \dfrac{\sqrt{47}}{3}i$

$\quad \left\{\dfrac{1}{3} \pm \dfrac{\sqrt{47}}{3}i\right\}$

31. $x^2 + 14x + n = x^2 + 14x + \left[\dfrac{1}{2}(14)\right]^2$

$\quad\quad = x^2 + 14x + (7)^2$

$\quad\quad = x^2 + 14x + 49$

$\quad\quad = (x+7)^2$

$\quad n = 49; (x+7)^2$

32. $y^2 + 22y + n = y^2 + 22y + \left[\dfrac{1}{2}(22)\right]^2$

$\quad\quad = y^2 + 22y + (11)^2$

$\quad\quad = y^2 + 22y + 121$

$\quad\quad = (y+11)^2$

$\quad n = 121; (y+11)^2$