Chapter 2
Fundamentals of Solar Radiation

2.1
a.
Begin with Equation (2.3), neglecting refractive effects.

$$E_{b\lambda} = \frac{C_1}{\left(e^{\frac{C_2}{\lambda T}} - 1\right)\lambda^5}$$

Take $E_{b\lambda} d\lambda$ (with $\tilde{\nu} = 1/\lambda$). Hence:

$$E_{b\lambda} d\lambda = \frac{C_1 \tilde{\nu}^5 d\lambda}{\left(e^{\frac{C_2 \tilde{\nu}}{T}} - 1\right)} = -\frac{C_1 \tilde{\nu}^3 d\tilde{\nu}}{\left(e^{\frac{C_2 \tilde{\nu}}{T}} - 1\right)} = -E_{b\tilde{\nu}} d\tilde{\nu}$$

$$\therefore \; E_{b\tilde{\nu}} = \frac{C_1 \tilde{\nu}^3}{\left(e^{\frac{C_2 \tilde{\nu}}{T}} - 1\right)}$$

b.
Differentiate the expression from part (a) with respect to wave number. Then, set the expression equal to zero. The resulting equation is:

$$3\left(e^{\frac{C_2 \tilde{\nu}}{T}} - 1\right) = \frac{C_2 e^{\frac{C_2 \tilde{\nu}}{T}} \tilde{\nu}}{T}$$

This equation is transcendental in $\tilde{\nu}/T$. Solving numerically, we have:

$$\frac{\tilde{\nu}}{T} = 1.96 \text{ cm}^{-1}/\text{K}$$

2.2

From the problem statement, $L = 40.77°$, solar time is 2:00PM, on October 1st (n = 274). The declination angle, δ_s, is obtained from Equation (2.23).

$$\delta_s = 23.45° \sin\left[\frac{360(284 + n)°}{365}\right]$$
$$= -4.22° \; (-0.0736 \text{ rad})$$

To calculate the altitude angle, we need the hour angle, obtained from Equation (2.25).

$$h_s = \frac{15°}{\text{hr}}(\text{hours from solar noon}) = 30°$$

The altitude angle is obtained from Equation (2.28).

$$\sin\alpha = \sin L \sin\delta_s + \cos L \cos\delta_s \cos h_s$$
$$\alpha = 37.3° \; (0.651 \text{ rad})$$

And the zenith angle immediately follows, according to Equation (2.24).

$$z = 90° - \alpha = 52.7° \; (0.920 \text{ rad})$$

For this time / location, the sun will be south of the east-west line, so $|a_s| \leq 90°$. Hence, the azimuth angle follows directly from Equation (2.29).

$$\sin a_s = \frac{\cos\delta_s \sin h_s}{\cos\alpha}$$
$$a_s = 38.8° \; (0.678 \text{ rad})$$

2.3

(1) First, find the minimum normalized distance, d, for placement of the second collector. At solar noon, the profile angle is equal to the solar altitude angle, α_1. From the geometry, we have the following relationships.

$$\tan\alpha_1 = \frac{h}{d}$$
$$\sin\beta = \frac{h}{w}$$

Here, h is the vertical height of the collector, and w is the arbitrary width. The normalized distance, d/w, is desired.

$$\frac{d}{w} = \frac{\sin\beta}{\tan\alpha_1}$$

The collector tilt angle, β, is known. The altitude angle follows from Equation (2.28). For Tampa, Florida, we have $L = 27.96°$N (Tampa International Airport); for December 21st, $\delta_s = -23.45°$.

$$\sin\alpha_1 = \sin L \sin\delta_s + \cos L \cos\delta_s \cos h_s$$
$$\alpha_1 = 38.6° \; (0.673 \text{ rad})$$

Normalized distance:

$$\frac{d}{w} = \frac{\sin\beta}{\tan\alpha_1}$$
$$= 0.627 \text{ (meter separation per meter width)}$$

(2) Second, the percent shading at 9:00AM solar time is desired; this quantity would be the width shaded divided by the total collector width.

$$\% \text{ shading} = \frac{w_s}{w}$$

In this case, the sun is not due south, so the profile angle, γ_2, is needed, and it can be obtained from Equation 2.31. First, we need the new altitude angle ($h_s = -45°$).

$$\sin\alpha_2 = \sin L \sin\delta_s + \cos L \cos\delta_s \cos h_s$$
$$\alpha_2 = 22.7° \; (0.397 \text{ rad})$$

Next, the solar azimuth angle:

$$\sin a_s = \frac{\cos\delta_s \sin h_s}{\cos\alpha_2}$$
$$a_s = -44.7° \; (-0.780 \text{ rad})$$

Finally, the profile angle is obtained.

$$\tan\gamma_2 = \sec a_s \tan\alpha$$
$$\gamma_2 = 30.5° \; (0.532 \text{ rad})$$

From the geometry and the law of sines, we arrive at the following relation.

$$\frac{\sin(\beta + \gamma_2)}{h/\sin\alpha_1} = \frac{\sin(\alpha_1 - \gamma_2)}{w_s}$$

Simplifying:

$$\frac{w_s}{w} = \frac{\sin(\alpha_1 - \gamma_2)}{\sin(\beta + \gamma_2)} \frac{\sin\beta}{\sin\alpha_1}$$
$$= 0.129; \text{i.e., } 12.9\% \text{ of the collector is shaded.}$$

2.4

The location is not specified; the date (September 1st) gives $n = 244$. The declination angle is obtained from Equation (2.23).

$$\delta_s = 7.72° \text{ (0.135 rad)}$$

The sunrise / sunset times are obtained from Equation (2.30).

$$h_{ss}, h_{sr} = \pm\cos^{-1}(-\tan L \tan\delta_s)$$

Solar sunrise and sunset times are found as follows [see Equation (2.25)].

$$\text{Solar sunrise time} = 12{:}00\text{PM} + h_{sr}\left(\frac{4 \text{ min}}{°}\right)$$

$$\text{Solar sunset time} = 12{:}00\text{PM} + h_{ss}\left(\frac{4 \text{ min}}{°}\right)$$

To convert to local time, Equation (2.26) is needed.

$$LST = \text{Solar time} - ET - (l_{st} - l_{local})\left(\frac{4 \text{ min}}{°}\right)$$

Here, the equation of time, ET, is computed with Equation (2.27):

$$ET \text{ (in minutes)} = 9.87\sin 2B - 7.53\cos B - 1.5\sin B$$

$$B = \frac{360(n-81)°}{364}$$

For this date, September 1st, ET is determined as follows.

$$B = 161.2°$$

$$ET = 0.626 \text{ min}$$

Given the above information and the latitude of the specific location, sunrise / sunset times can be determined with Equations (2.30), (2.25), and (2.26).

2.5

The day numbers are set by the month (e.g., for January 15th, $n = 15$); from the characteristic n for each month, a declination angle is obtained from Equation (2.23). The sunrise and sunset times are computed as in Problem 2.4. Given h_{ss} and h_{sr}, the bounds of the day in solar time are known. Data for hours in between these bounds are computed by first determining the hour angle [Equation (2.25)], then the altitude angle [Equation (2.28), with latitude angle, L, set by the location], and finally the zenith and azimuth angles [Equations (2.24) and (2.29), respectively]. If desired, the solar time for sunrise / sunset can be converted to local time using the procedure outlined in Problem 2.4.

2.6

The unit directional for the sun can be written in terms of an East-North-Vertical coordinate system.

$$\hat{s} = \cos\alpha\sin a_s\,\hat{E} + \cos\alpha\cos a_s\,\hat{N} - \sin\alpha\,\hat{V}$$

Similarly for the panel normal,

$$\hat{p} = \cos(90-\beta)\sin(-a_w)\,\hat{E} - \cos(90-\beta)\cos(-a_w)\,\hat{N} + \sin(90-\beta)\,\hat{V}$$

The scalar product of the two is

$$\cos i = -\hat{s}\cdot\hat{p} = \cos\alpha\sin a_s\sin\beta\sin a_w + \cos\alpha\cos a_s\sin\beta\cos a_w + \sin\alpha\cos\beta$$

Combining terms and using a trigonometric identity:

$$\cos i = \cos\alpha\sin\beta\cos(a_s - a_w) + \sin\alpha\cos\beta$$

2.7

In the case of the tubular surface, the incidence angle is found as the angle between the sun's rays and a plane perpendicular to the cylinder's long axis. This is equivalent to modeling the incidence angle on a flat plate collector rotating about a titled axis. Using a procedure similar to that used in Problem 2.6:

$$\cos i = \sqrt{1 - \{\cos(\alpha+\beta) - \cos\alpha\cos\beta\,[1-\cos(a_s - a_w)]\}^2}$$

In the case of a titled axis in the north-south plane,

$$\cos i = \sqrt{1 - [\cos(\alpha+\beta) - \cos\alpha\cos\beta\,(1-\cos a_s)]^2}$$

Applying a trigonometric identity, we arrive at the following equation.

$$\cos i = (1 - [\cos\alpha\sin\beta - \sin\alpha\cos\beta - \cos\alpha\cos\beta\,(1-\cos a_s)]^2)^{0.5}$$

From Figures 2.8 and 2.9:

$$\cos\alpha\cos a_s = \cos\delta_s\sin L\cos h_s - \sin\delta_s\cos L$$

Using this expression in conjunction with Equation (2.28), and further applying a trigonometric identity, we arrive at the desired equation.

$$\cos i = \sqrt{1 - [\sin(\beta - L)\cos\delta_s\cos h_s + \cos(\beta - L)\sin\delta_s]^2}$$

2.8

On September 21st, the declination angle, δ_s, is zero (autumnal equinox). For solar noon, both the hour angle, h_s, and the solar azimuth angle, a_s, are zero. From Equation (2.28):

$$\alpha = 90 - L$$

For Tampa, Florida, $L = 27.96°\text{N}$; hence, $\alpha = 62.0°$ (1.08 rad).

The zenith angle follows immediately [Equation (2.24)].

$$z = 90 - \alpha = 28.0° \text{ (0.488 rad)}$$

From Equation (2.48), the incidence angle is calculated ($\beta = 30°$).

$$\cos i = \cos\alpha\sin\beta + \sin\alpha\cos\beta$$

$$i = 2.04° \text{ (0.0356 rad)}$$

From the 2009 ASHRAE Handbook for Tampa International Airport [either taken directly or calculated according to Equations (2.43) and (2.44)]:

$$I_{b,N} = 836\,\frac{W}{m^2}$$

$$I_{d,h} = 143\,\frac{W}{m^2}$$

The beam radiation on the tilted surface is found as follows.

$$I_{b,c} = I_{b,N}\cos i = 835\,\frac{W}{m^2}$$

The diffuse radiation on the tilted surface is then calculated.

$$I_{d,c} = I_{d,h}\cos^2\frac{\beta}{2} = 133\,\frac{W}{m^2}$$

Finally, the reflected radiation incident on the surface is calculated, using Equation (2.51) (assume a ground reflectance, ρ, of 0.2).

$$I_{r,c} = \rho\left(I_{b,N}\sin\alpha + I_{d,h}\right)\sin^2\frac{\beta}{2} = 11.8\,\frac{W}{m^2}$$

Summing:

$$I_c = 981\,\frac{W}{m^2}$$

From Equation (2.27):

$$ET = 7.90\text{ min}$$

The local standard time would therefore be $LST = 12{:}21\text{PM}$ [Equation (2.26)]. Accounting for daylight savings (in effect in Tampa on this date), local daylight time would be $LDT = 1{:}21PM$.

2.9

Horizontal extraterrestrial radiation is given as

$$I_h = I\sin\alpha$$

The average value of this over one hour is

$$I_{o,h} = \frac{\int_{t-0.5\text{hr}}^{t+0.5\text{hr}} I\sin\alpha\,dt}{(t+0.5\text{hr}) - (t-0.5\text{hr})}$$

or in terms of hour angles (rad),

$$I_{o,h} = \frac{12}{\pi}\int_{h_s-\pi/24}^{h_s+\pi/24} I\sin\alpha\,dh_s$$

I is approximated as constant for the day number according to Eq. (2.35), and so can be taken out of the integral. From Eq. (2.28) for the solar altitude,

$$\sin\alpha = \sin L\sin\delta_s + \cos L\cos\delta_s\cos h_s$$

where the latitude is constant for the location and the solar declination is approximated as constant for the day number. Therefore the integral becomes

$$I_{o,h} = \frac{12}{\pi}I\left(\sin L\sin\delta_s\int_{h_s-\pi/24}^{h_s+\pi/24} dh_s + \cos L\cos\delta_s\int_{h_s-\pi/24}^{h_s+\pi/24}\cos h_s\,dh_s\right)$$

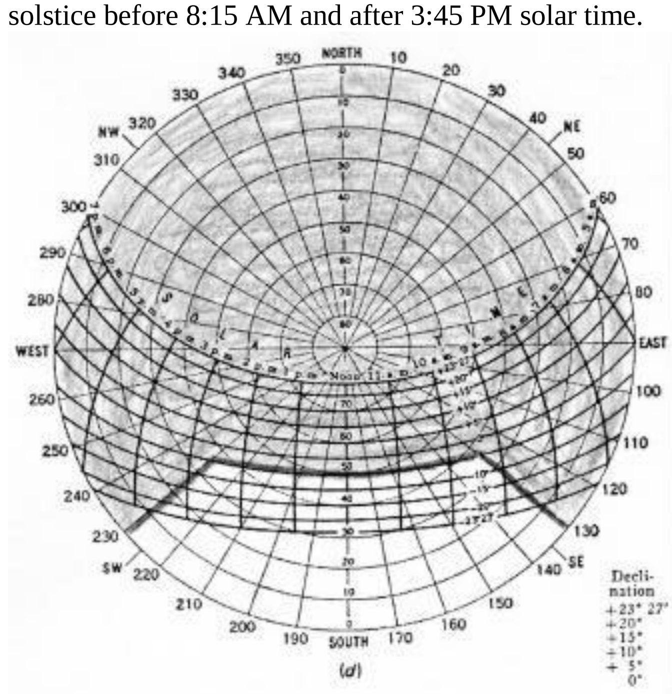
Solving the integral,

$$I_{o,h} = I(\sin L\sin\delta_s + 0.9971\ \cos L\cos\delta_s\cos h_s) \approx I\sin\alpha$$

The last equality holds to within less than one percent, depending on the magnitude of $\sin L\sin\delta_s$.

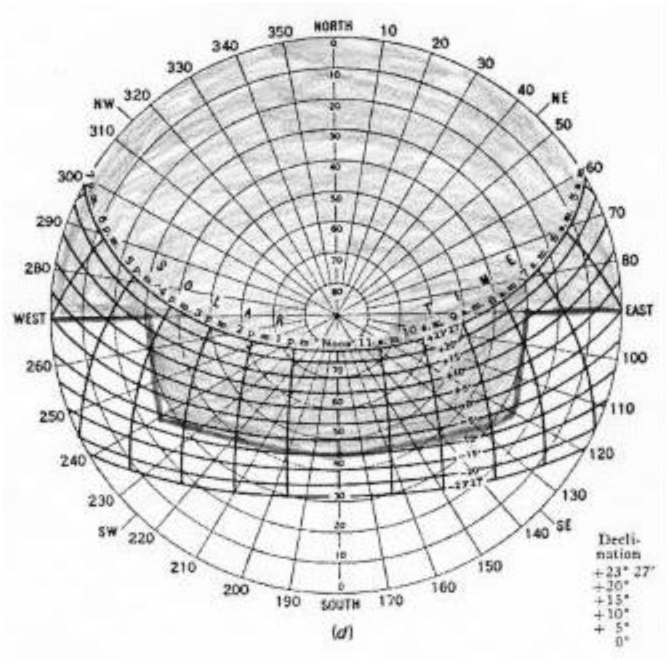
2.10

Sun-path diagrams for the two latitudes are found in Appendix 2. For geometry (a), point C will be shaded when the altitude is given according to Eq. (2.31) as $\tan 50° = \sec a_s\tan\alpha$. For the limiting case of the sun at 40º east or west of south, the altitude angle is then 37.45º. For a noon sun, the altitude angle is 50º. The shadow map is plotted on the sun-path diagram for the 35º location. As shown, point C is shaded when the solar declination is greater than -5º, which occurs between early March and early October. For other times of the year, the map shows, for example, shading on winter solstice before 8:15 AM and after 3:45 PM solar time.



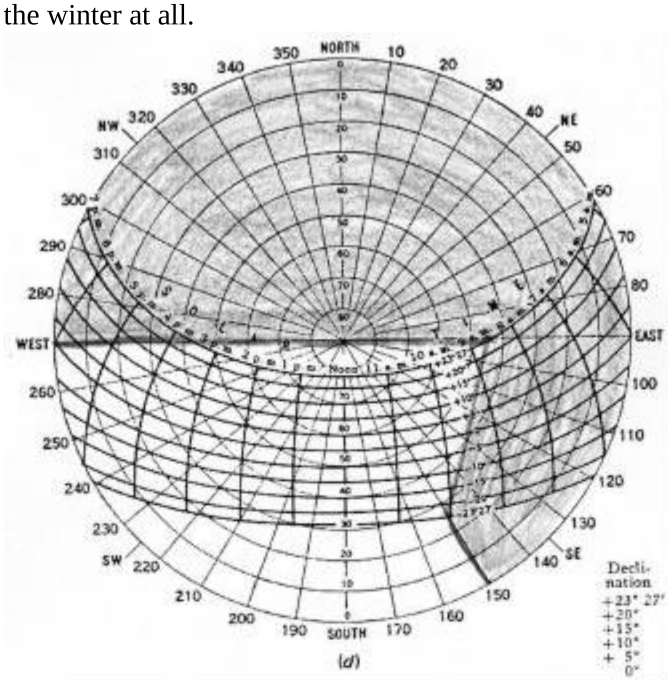
Shadow map for geometry (a), 35ºN latitude.

For geometry (b), point C will be shaded at noon with the altitude angle greater than 45º. With the solar azimuth ± 60º of south, the corner of the overhang is in line with the sun and point C. The altitude angle of interest here is 26.57º, from looking at the geometry. Finally at ± 90º of south, the critical altitude angle is 30º.



Shadow map for geometry (b), 35ºN latitude.

For geometry (b), it is noted that during April, for example, point C is shaded in the morning before the sun reaches due east, then is in sunshine for about an hour until the overhang blocks the sun. The sun reappears on point C when it dips below the overhang in the afternoon, but disappears as it moves north of due west and behind the back wall. Point C is not shaded during the winter at all.



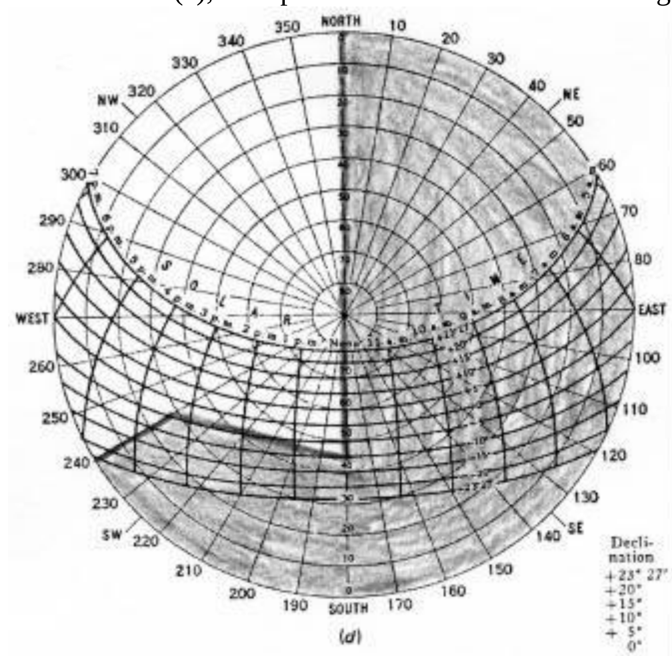
Shadow map for geometry (c), 35ºN latitude.

For geometry (c), point C will not be shaded after the sun rises to within 30º of south, provided it remains south of west in the late afternoon also. The critical altitude angle right before -30º azimuth is 26.57º. At due east, shading will occur below 45º.

Shadow maps for the 40º latitude are similar and are not shown.

2.11

There will be no sunlight on point P until the solar noon. Then the altitude angle must be above 45º. At 60º west of south, the altitude angle of interest is 26.57º. Moving further west, the sun will shine on P provided it does not yet set. From the geometry, it will shine on P until it reaches due north. Note that the shadow map shown below is simply a rotation of the shadow map for Problem 2.10 (c), with point P in shadow in the morning.



2.12

The solar declinations are found with n = 172 and n = 355, respectively. Assuming solar time, the hour angles are found from Eq. (2.25). The solar altitude angles are found from Eq. (2.28). The solar azimuth angles are found from Eq. (2.29). For the south-facing collector, $a_w = 0$. Plug all the above angles into Eq. (2.48).

	Jun 21 9 AM	Jun 21 noon	Dec 21 9 AM	Dec 21 noon
Declination	23.45º	23.45º	-23.45º	-23.45º
Hour angle	-45º	0º	-45º	0º
Solar altitude	48.8º	73.4º	14.0º	26.6º
Solar azimuth	-80.2º	0º	-41.9º	0º
Panel azimuth	0º	0º	0º	0º
Solar incidence	68.7º	53.4º	40.5º	6.6º

2.13

a.

Determine the angles with the same procedure as in 2.12, except $a_w = -45°$.

	Jun 21 9 AM	Jun 21 noon	Dec 21 9 AM	Dec 21 noon
Declination	23.45º	23.45º	-23.45º	-23.45º
Hour angle	-45º	0º	-45º	0º
Solar altitude	48.8º	73.4º	14.0º	26.6º
Solar azimuth	-80.2º	0º	-41.9º	0º
Panel azimuth	-45º	-45º	-45º	-45º
Solar incidence	40.3º	58.9º	6.7º	41.6º

b.

The procedure is similar to that in 2.12, but with the orientation N-S the incidence angle is determined by the equation derived in Problem 2.7.

$$\cos i = \{1 - [\sin(\beta - L)\cos\delta_s \cos h_s + \cos(\beta - L)\sin\delta_s]^2\}^{0.5}$$

	Jun 21 9 AM	Jun 21 noon	Dec 21 9 AM	Dec 21 noon
Declination	23.45º	23.45º	-23.45º	-23.45º
Hour angle	-45º	0º	-45º	0º
Solar altitude	48.8º	73.4º	14.0º	26.6º
Solar azimuth	-80.2º	0º	-41.9º	0º
Panel azimuth	0º	0º	0º	0º
Solar incidence	42.0º	53.4º	1.2º	6.6º

2.14

For a one term Fourier cosine series, we want the declination to be of the form

$$\delta(n) = a\cos\left(\frac{\pi}{L}n + \varphi\right) = a\cos\left(\frac{360}{365}n + \varphi\right)$$

where n is the day number and the period is $2L =$ 365 days. From the given data, the maximum declination occurs on June 21 as

$$\delta_{max} = \delta(172) = 23.45° = a$$

This must correspond to where the cosine term is equal to 1, such that

$$\frac{360}{365}n_{max} + \varphi = 0 \rightarrow \varphi = -n_{max}\frac{360}{365}$$

Thus the declination becomes

$$\delta(n) = 23.45\cos\left(\frac{360}{365}n - 172\right)$$

or equivalently,

$$\delta(n) = -23.45\cos\left(\frac{360}{365}n + 10.5\right)$$

2.15

In order to plot lines of constant declination on a plot similar to Fig. 2.10, we must write the declination in terms of two polar coordinates, namely the solar azimuth and altitude angles. The solar altitude angle is from Eq. (2.28), and can be written in terms of the hour angle as

$$\cos h_s = \frac{\sin\alpha - \sin L\sin\delta_s}{\cos L\cos\delta_s}$$

The solar azimuth is given by Eq. (2.29), which can be rearranged as

$$\sin h_s = \frac{\sin a_s \cos\alpha}{\cos\delta_s}$$

Using the identity, $\sin^2\theta + \cos^2\theta = 1$, the hour angle is removed as a variable. The answer is then written as

$$\sin a_s = \pm\sqrt{\frac{(\cos L\cos\delta_s)^2 - (\sin\alpha - \sin L\sin\delta_s)^2}{(\cos L\cos\alpha)^2}}$$

There will be a unique plot for each latitude. For each declination, two solar azimuth angles will result for each solar altitude angle—i.e., one before (-) and one after (+) solar noon.

2.16

Referring to Fig. 2.11a,

$$\cos a = x/r$$
$$\tan\alpha = z/r$$
$$\tan\gamma = z/x$$

Solving for *z/x*,

$$\tan\gamma = \sec a\tan\alpha$$

2.17

Eq. (2.48), through some effort, can be written in terms of the hour angle and declination as

$$\cos i = \sin(L-\beta)\sin\delta_s + \cos(L-\beta)\cos\delta_s\cos h_s$$

To get the average value of the function *cos i*, we integrate over the year (declination) and over the entire day (hour angle).

$$\cos i|_{avg} = \frac{2\int_{\delta_{s,min}}^{\delta_{s,max}}\int_{h_{s,min}}^{h_{s,max}}\cos i\, dh_s d\delta_s}{2(\delta_{s,max} - \delta_{s,min})(h_{s,max} - h_{s,min})}$$

The factor of 2 is there as the declination range is seen twice in the yearly movement. Recognizing that the minimum values are simply the negative of the maximum values, integration with respect to the hour angle yields

$$\cos i|_{avg} = \frac{1}{4\delta_{s,max}h_{s,max}}\int_{\delta_{s,min}}^{\delta_{s,max}}\left[\sin(L-\beta)\sin\delta_s + 2\cos(L-\beta)\cos\delta_s\sin h_{s,max}\right]d\delta_s$$

Completing the second integration,

$$\cos i|_{avg} = \frac{\cos(L-\beta)\sin\delta_{s,max}\sin h_{s,max}}{\delta_{s,max}h_{s,max}}$$

It should be noted that the average of the cosine of i does not yield the average i, but this is acceptable as we don't need the average i. We only want to find β for the minimum i, which coincides with the β for the maximum cosine of i. Since the maximum occurs at $\cos(L-\beta)=1$,

$$\beta_{optimum} = L$$

Notes:

1. Integration in the range where the sun is not in view of the collector, or is below the horizon, would yield a meaningless average for the angle of incidence. However, the choice of range is not significant here as it cancels in finding the optimum β.

2.18

From Eq. (2.23),

$$\text{May 1}, n = 121, \delta_s = 14.9°$$

$$\text{Dec 1}, n = 335, \delta_s = -22.1°$$

From Eq. (2.30),

$$h_{ss} = \cos^{-1}[-\tan L \tan \delta_s]$$

$$\text{May 1}, h_{ss} = 96.49°$$

$$\text{Dec 1}, h_{ss} = 80.07°$$

With 15º per hour, sunsets are at

May 1, Sunset time = 6.43*hrs* = 6:26 *pm*

Dec 1, Sunset time = 5.34*hrs* = 5:20 *pm*

2.19

From Eq. (2.27),

$$\text{Jun 10}, n = 161, ET = 0.76 \text{ min}$$

$$\text{Jan 10}, n = 10, ET = -7.42 \text{ min}$$

From Eq. (2.26), on Jun 10,

$$\text{Solar Time} = 9am + 0.76\text{min} + (105-107)\cdot 4\text{min}$$

$$\text{Solar Time} = 8{:}53am$$

Similarly for Jan 10,

$$\text{Solar Time} = 10am - 7.42\text{min} + (105-107)\cdot 4\text{min}$$

$$\text{Solar Time} = 9{:}45am$$

Notes:

1. Daylight savings time has the clocks ahead by an hour, such that LST is one hour behind Local Daylight Time.

2.20

Miami is at latitude 25.79 ºN. For each month, Table A2.1 gives the average daily extraterrestrial horizontal insolation as

$$\text{May}, \overline{H}_{o,h} = 11.04\frac{kWh}{m^2day} = 39.74\frac{MJ}{m^2day}$$

$$\text{Oct}, \overline{H}_{o,h} = 8.125\frac{kWh}{m^2day} = 29.25\frac{MJ}{m^2day}$$

Using the Angström-Page method, Table 2.4 gives a = 0.42 and b = 0.22 for Miami. Then from Eq. (2.52),

$$\text{May}, \overline{H}_h = 37.51\left(0.22 + 0.57\frac{60}{100}\right)\frac{MJ}{m^2day}$$

$$= 21.9\frac{MJ}{m^2day}$$

$$\text{Oct}, \overline{H}_h = 29.25\left(0.22 + 0.57\frac{70}{100}\right)\frac{MJ}{m^2day}$$

$$= 18.1\frac{MJ}{m^2day}$$

To compare with the ASHRAE clear-sky model, we will need to calculate the radiation over the course of the day and integrate via quadrature. For May 15, τ_b and τ_d are 0.487 and 1.988, respectively (linear interpolation is used to find values for days other than the 21st days of each month). Instantaneous horizontal solar irradiance values are calculated from sunrise to noon, as tabulated below.

Time (hr)	I_h (W/m^2)
5.37	8.4
6	80.3
7	279.9
8	500.0
9	700.3
10	858.3
11	959.0
12	993.5

(The irradiance is nonzero at sunrise due to some diffuse radiation.) We calculate the daily total irradiance as

$$\overline{H}_h = 2\cdot(3862.3Wh) = 27.8\frac{MJ}{m^2day}$$

(The factor of two accounts for the afternoon irradiance.) Similarly, for October (τ_b and τ_d of 0.435 and 2.225, respectively):

$$\overline{H}_h = 2\cdot(2779.3Wh) = 20.0\frac{MJ}{m^2day}$$

The ASHRAE values are larger because they do not take into account weather events that can interfere with the sun's rays. The values are close to those of the Angström-Page method with 100% possible sunshine.

2.21

Eq. (2.55) is given as

$$\frac{\overline{D}_h}{\overline{H}_h} = 1.390 - 4.027\overline{K}_T + 5.531\overline{K}_T^2 - 3.108\,\overline{K}_T^3$$

Where from Eq. (2.50),

$$\overline{K}_T = \frac{\overline{H}_h}{1394W/m^2}$$

Introduce a modified monthly clearness index to be

$$\overline{K}'_T = \frac{\overline{H}_h}{1366.1 W/m^2}$$

Thus,

$$\overline{K}_T = \overline{K}'_T \frac{1366.1}{1394} = 0.980\overline{K}'_T$$

and with the new parameter, Eq. (2.55) becomes

$$\frac{\overline{D}_h}{\overline{H}_h} = 1.390 - 3.946\overline{K}'_T + 5.312\,\overline{K}'^2_T - 2.925\,\overline{K}'^3_T$$

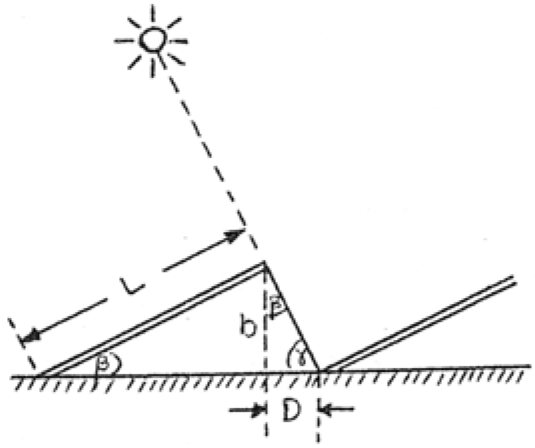
2.22

The Denver ASHRAE clear-sky parameters are available for the 21st day of each month; by interpolation, we have τ_b and τ_d of 0.363 and 2.243, respectively, for September 9. Using the ASHRAE method, the diffuse and beam radiation values are tabulated below for each hour from sunrise to noon.

Time (hr)	I_d (W/m^2)	I_b (W/m^2)
5.37	12.0	0.0
6	21.9	2.6
7	71.9	145.8
8	100.0	357.3
9	118.2	558.5
10	129.8	719.7
11	136.4	823.1
12	138.6	858.7

It can be seen that the diffuse radiation at 9:30AM is approximately 124 W/m^2, while the beam radiation is approximately 639 W/m^2.

2.23



From the law of sines,

$$\frac{D}{\sin\beta} = \frac{b}{\sin\gamma}$$

Further reduce with

$$b = L\sin\beta$$

to obtain the result:

$$D = \frac{L\sin^2\beta}{\sin\gamma}$$

Chapter 3
Solar Thermal Collectors

3.1

The parameter m is determined as

$$m = \sqrt{\frac{U_c}{kt}} = 7.66m^{-1}$$

The conductivity for aluminum is interpolated from Table A3.4 to be 118.1 Btu/hr·ft·°F or 204.4 W/mK. Therefore, the fin efficiency is determined as

$$\eta_f = \frac{\tanh(mw)}{mw} = 0.916$$

Where the length

$$w = \frac{l' - D}{2} = 0.069m$$

From Eq. (3.39), the collector efficiency factor is

$$F' = \frac{1/U_c}{l'[1/(U_c(D + 2w\eta_f) + 1/(h_{c,i}P]} = 0.906$$

where the tube perimeter is simply $P = \pi D$. Then from Eq. (3.45),

$$F_R = \frac{(\dot{m}/A_c)c_p}{U_c}\left[1 - \exp\left(-\frac{U_cF'}{(\dot{m}/A_c)c_p}\right)\right]$$

$$F_R = 0.823$$

where the specific heat for water is 4179 J/kg°C as given in Table A3.2.

3.2

Data for the insolation can be taken from Table A2.6c (40°N). Value for March 1 must be obtained by interpolation. The radiation incident on the collector is obtained directly:

$$I_c = 996W/m^2$$

From Eq. (3.46),

$$\eta_c = F_R\left[\tau\alpha - \frac{U_c(T_{f,i} - T_a)}{I_c}\right]$$

$$\eta_c = 0.823\left[(0.9)(0.9) - \frac{6(330 - 280)}{991}\right] = 0.419$$

The heat removal factor was taken from Problem 3.1.

3.3

Values of the collector heat capacity are found in Example 3.2 to be

$$(mc)_p + (U_c/U_\infty)(mc)_g = 15{,}500\frac{J}{K}$$

Then from Eq. (3.54),

$$T_{p,9\,am} = T_a + \frac{\alpha_s I_s}{U_c} - \left[\frac{\alpha_s I_s}{U_c} - (T_{p,8am} - T_a)\right] \cdot \exp\left[-\frac{U_c A_c t}{15{,}500}\right]$$

For one hour, t = 3600 s. The collector area in example 3.2 is given as 2 m^2. Therefore, at 9 AM,

$$T_{p,9\,am} = 285 + \frac{150}{6} - \left[\frac{150}{6}\right]\exp\left[-\frac{6(2)3600}{15{,}500}\right] = 308.46K$$

Note that during the first hour, 8-9 AM, there was no collector temperature rise as there was no absorbed insolation. Following the same method for the next hour,

$$T_{p,9\,am} = 285 + \frac{270}{6} - \left[\frac{270}{6} - (308.46 - 285)\right] \cdot \exp\left[-\frac{6(2)3600}{15{,}500}\right] = 328.67K$$

3.4

Neglecting edge losses, the total heat losses are from the top and bottom as

$$U_c = U_{bot} + U_{top} = \frac{1}{R_{bot}} + U_{top}$$

where the thermal resistance through the bottom is only according to the insulation conductivity.

$$\frac{1}{R_{bot}} = \frac{k_i}{t_i} = 0.8\frac{W}{m^2K}$$

The top surface convection coefficient and other parameters are found as

$$h_{c,\infty} = 5.7 + 3.8V = 17.1\frac{W}{m^2K}$$

$$C = 250[1 - 0.0044(\beta - 90)] = 299.5$$

$$f = (1 - 0.04h_{c,\infty} + 0.005h_{c,\infty}^2)(1 + 0.091N) = 2.10$$

where N = 2 for two glass covers. From the definition in Eq. (3.13),

$$U_{top} = \frac{q_{top}}{A_c(T_c - T_a)}$$

and so combining with Eq. (3.23),

$$U_{top} = \frac{1}{N(T_c/C)[(T_c - T_a)/(N + f)]^{-0.33} + h_{c,\infty}^{-1}} + \frac{\sigma(T_c^2 + T_a^2)(T_c + T_a)}{1/[\varepsilon_p + 0.05N(1 - \varepsilon_p)] + (2N + f - 1)/\varepsilon_g - N}$$

with the Stefan-Boltzman constant as 5.67x10^{-8} W/m^{2}K^4,