CHAPTER 9 – DIODE CIRCUITS

Exercise 1

A half-wave rectifer as in Figure 9.1 has an input voltage signal $v_s = V_M\, sin(\omega t)$.

Assuming ideal diode ($V_\gamma = 0$*, zero resistance in forward biasing condition), calculate the following:*

- The average output voltage $V_{o,DC}$ across the resistor
- The average current $I_{o,DC}$ flowing throught the resistor
- The RMS value of the output voltage $V_{o,RMS}$ across the resistor
- The RMS value of the current $I_{o,RMS}$ flowing throught the resistor
- The form factor $F_{F,1s}$
- The ripple factor γ

ANSWER

The output voltage function of the half-wave rectifier is represented in Figure 1

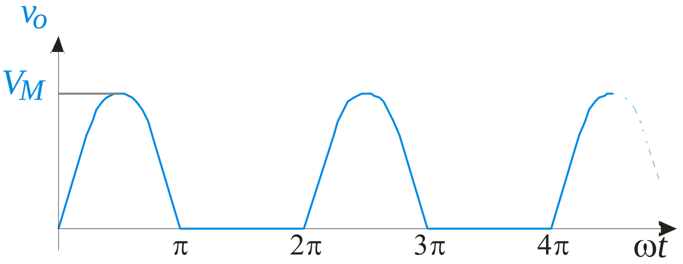

Figure 1: output voltage function of the half-wave rectifier

To obtain the requested values, each time we can start with the respective definitions.
So, the average output (DC) voltage will be:

$$V_{o,DC} = \frac{1}{\pi}\int_0^\pi v_s d(\omega t) = \frac{1}{\pi}\int_0^\pi V_M\, sin(\omega t)\, d(\omega t) = \frac{V_M}{\pi}$$

The average output current:

$$I_{o,DC} = \frac{V_{DC}}{R} = \frac{V_M}{\pi R}$$

The same result can be equivalently obtained considering that this circuit does not produce any phase shift, so that the current can be written as $i(\omega t) = I_M\, sin(\omega t)$). So:

$$I_{o,DC} = \frac{1}{2\pi}\int_0^{2\pi} I_M\, sin(\omega t)\, d(\omega t) = \frac{1}{2\pi}\left[\int_0^{\pi} I_M\, sin(\omega t)\, d(\omega t) + \int_{\pi}^{2\pi} 0 d(\omega t)\right] = \frac{I_M}{2\pi}[-cos(\omega t)]_0^{\pi} + 0$$
$$= \frac{I_M}{2\pi} - 1 - 1 = \frac{I_M}{\pi}$$

To determine the RMS value of the output voltage, we have:

$$V_{o,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} v_S^2 d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_M^2\, sin^2(\omega t)\, d(\omega t)} == \sqrt{\frac{V_M^2}{2\pi}\int_0^{\pi}\left[\frac{1 - cos(2\omega t)}{2}\right] d(\omega t)}$$
$$= V_M\sqrt{\frac{1}{2\pi}\left[\frac{\omega t}{2} - \frac{sin(2\omega t)}{4}\right]_0^{\pi}} = V_M\sqrt{\frac{1}{2\pi}\left[\frac{\pi}{2}\right]} = \frac{V_M}{2}$$

The RMS value of the output current:

$$I_{o,RMS} = \frac{V_{o,RMS}}{R} = \frac{V_M}{2R}$$

The same expression can be equivalently found as:

$$I_{o,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_M^2\, sin^2(\omega t)\, d(\omega t)} = \sqrt{\frac{1}{2\pi} I_M^2 \frac{\pi}{2}} = \frac{I_M}{2}$$

which corresponds to the RMS value of the total current, that is the sum of DC and AC components.

Now, the RMS value of the only AC component, $I_{r,RMS}$, that is the RMS value of the ripple, can be determined starting from the definition applied to the term $(i - I_{DC})$:

$$I_{r,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (i - I_{DC})^2 d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [i^2 - 2iI_{DC} + I_{DC}^2] d(\omega t)}$$

but

$$\sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)} = I_{RMS}^2$$

$$\frac{1}{2\pi}\int_0^{2\pi} i d(\omega t) = I_{DC}$$

so

$$I_{r,RMS} = \sqrt{I_{RMS}^2 - 2I_{o,DC}^2 + I_{o,DC}^2} = \sqrt{I_{RMS}^2 - I_{o,DC}^2}$$

The form factor is defined as the RMS and average ratio, $F_F \stackrel{\text{def}}{=} \frac{V_{o,RMS}}{V_{o,DC}} = \frac{I_{o,RMS}}{I_{o,DC}}$, so:

$$F_{F,1s} = \frac{0{,}5V_M}{0{,}32V_M} \cong 1{,}57$$

Let's see now how we can calculate the ripple. Since the pure AC voltage component is defined as the sinusoidal wave $v_o(= V_M \sin(\omega t)\ which\ is \neq 0\ for\ 0 < \omega t < \pi\ and\ zero\ elsewhere)$ apart from its DC component $V_{o,DC}$, that is $v_o - V_{o,DC}$, we have:

$$V_{r,RMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left[V_M \sin(\omega t) - V_{o,DC}\right]^2 d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left[V_M^2 \sin^2(\omega t) + V_{o,DC}^2 - 2V_{o,DC}V_M \sin(\omega t)\right] d(\omega t)}$$

but we already saw that

$$V_{oRMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_M^2 \sin^2(\omega t)\, d(\omega t)}$$

therefore replacing

$$V_{r,RMS} = \sqrt{\left(V_{o,RMS} - V_{o,DC}\right)^2}$$

as a consequence the ripple of the half-wave rectifier is

$$r = \frac{V_{r,RMS}}{V_{o,DC}} = \frac{\sqrt{\left(V_{o,RMS} - V_{o,DC}\right)^2}}{V_{o,DC}} = \sqrt{\left(\frac{V_{o,RMS}}{V_{o,DC}}\right)^2 - 1} = \sqrt{\left(\frac{V_M/2}{V_M/\pi}\right)^2 - 1} = 1.21$$

The same result can be obtained solving respect to the current rather than to the voltage:

$$r = \frac{I_{r,RMS}}{I_{o,DC}} = \frac{\sqrt{I_{o,RMS}^2 - I_{DC}^2}}{I_{o,DC}} = \sqrt{\left(\frac{I_{o,RMS}}{I_{o,DC}}\right)^2 - 1} = \sqrt{\frac{\frac{I_M}{2}}{\frac{I_M}{\pi}} - 1} = 1.21$$

Summing

$V_{o,DC} = \frac{V_M}{\pi}$; $I_{o,DC} = \frac{V_M}{\pi R}$; $V_{o,RMS} = \frac{V_M}{2}$; $I_{o,RMS} = \frac{V_M}{2R}$; $F_{F,1s} \cong 1{,}57$; $\gamma = 1.21$

Exercise 2

A half-wave rectifier circuit, as in Figure 9.1, has an input sinusoidal voltage source $v_s = V_M \sin(\omega t)$, with $V_M = 10V$, $R = 500\Omega$. Assuming an ideal diode ($V_\gamma = 0$, zero resistance in forward biasing conditions) calculate the following:

- *The maximum current value I_M*
- *DC component of current I_{DC}*
- *RMS value of current I_{RMS}*
- *DC component of voltage in output V_{DC}*
- *DC component of power delivered to the load $P_{o,DC}$*
- *Power value supplied by the source P_s*
- *The ripple value γ*

ANSWER

The maximum current value is

$$I_M = \frac{V_M}{R} = 20mA$$

its DC component

$$I_{DC} = \frac{1}{\pi R}\int_0^{\pi} v_s d(\omega t) = \frac{V_M}{\pi R} = 6.3mA$$

and its RMS value

$$I_{RMS} = \sqrt{\frac{1}{\pi R}\int_0^{\pi} v_s^2 d(\omega t)} = \frac{V_M}{2R} = 10mA$$

The DC component of the output voltage

$$V_{DC} = RI_{DC} = 6.3V$$

The power from the source and the power to the load are respectively

$$P_s = RI_{RMS}^2 = 100mW$$

$$P_{o,DC} = RI_{DC}^2 = 39.7mW$$

Observation

To determine the electric power we consider the absolute values of current and/or voltage when DC but RMS values when AC.
For the current exercise the output voltage is a periodic one so that to calculate the electric power we should utilize the RMS values but, on the contrary, it is commonly adopted the average ones. This is because the final scope of the circuit is to furnish a constant voltage across the load.

Finally, the ripple value

$$\gamma = \sqrt{\left(\frac{I_{RMS}}{I_{DC}}\right)^2 - 1} \cong 1.23$$

Summarizing:
$I_M = 20mA$; $I_{DC} = 6.3mA$; $I_{RMS} = 10mA$; $V_{DC} = 6.3V$; $P_{o,DC} = 39.7\text{m}W$; $P_s = 100mW$; $\gamma \cong 1.23$

Exercise 3

A half-wave rectifier has an input stage with a transformer, turns ratio $n = 20:1$, and a load resistance $R = 50\Omega$. The input voltage source has a RMS value of $V_{s,RMS} = 220V$ (Figure 9.23).

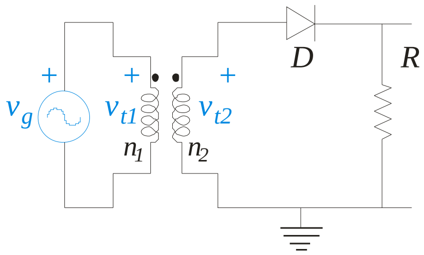

Figure 9.23: half-wave rectifier with a transformer as the first stage

Let's determine the following:

- DC component of voltage in output $V_{o,DC}$
- DC component of current in output $I_{o,DC}$
- RMS value of voltage in output $V_{o,RMS}$
- RMS value of current in output $I_{o,RMS}$
- the ripple value γ

ANSWER

Across the secondary coil there is a RMS voltage equal to $\frac{220}{10} = 22V$, so that the maximum voltage across the half-wave rectifier's input port is

$$V_M = \sqrt{2} * 22 \cong 31.12V$$

According to the previous exercises we know that the DC output voltage can be written as

$$V_{o,DC} = \frac{V_M}{\pi} \cong 9.9V$$

and the output current

$$I_{o,DC} = \frac{V_{o,DC}}{R} \cong 198mA$$

while their RMS values

$$V_{o,RMS} = \frac{V_M}{2} \cong 15.56V$$

$$I_{o,RMS} = \frac{V_{o,RMS}}{R} \cong 0.31A$$

Summarizing

$V_{o,DC} \cong 9.9V$; $I_{o,DC} \cong 198mA$; $V_{o,RMS} \cong 15.56V$; $I_{o,RMS} \cong 0.31A$; $\gamma \cong 1.21$.

Observation

Note that this ripple value is quite high, so the half-wave rectifier is a poor AC to DC converter.

Exercise 4

A half-wave rectifier network is used as a battery charger (Figure 9.24).

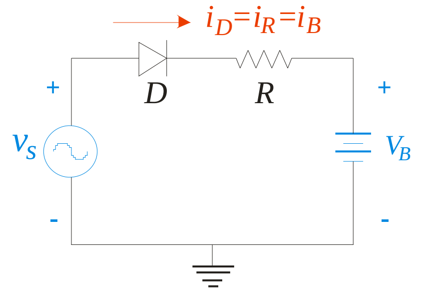

Figure 9.24: a simple battery charger by means of a half-wave rectifier

When the source voltage is higher than that of the battery, the current flows from the source to the battery, which charges. Reverse conditions are not possible because of the presence of the diode. Assuming a voltage source of $v_s = 150\,sin(\omega t)$, an ideal diode ($V_\gamma = 0$ and a short-circuit in forward bias condition, an open-circuit in reverse bias conditions), and a $75V$ battery with a charge current of $750mA$, determine the resistance value of the resistor R.

ANSWER

The charging current can flow only when the diode is in "active" mode (ON), and since $V_B = 75V$ this corresponds to the a-b, a'-b', a"-b",.. intervals of v_s, in green color of Figure 9.24b.

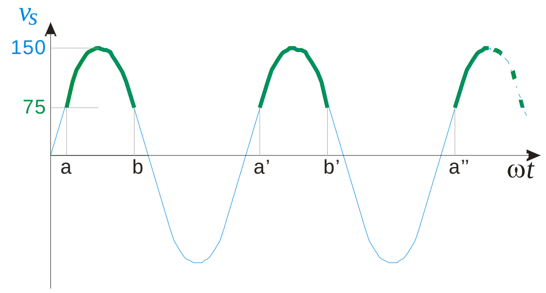

Figure 9.24b: the current flows through the diode during the green part of vs

To determine the value of ωt corresponding to the point a, b, a', b', a'' etc.., we can impose

$$150\,sin(\omega t) = 75$$

so

$$\omega t = \begin{cases} {}^1/_6\,\pi \\ {}^5/_6\,\pi \end{cases}$$

When the current differs from zero, it is equal to

$$i = \frac{v_s - 75}{R} = \frac{150\sin(\omega t) - 75}{R}$$

but we have to consider the DC value of the current which charges the battery

$$I_{DC} = \frac{1}{2\pi}\int_{1/6\pi}^{5/6\pi}\frac{150\sin(\omega t) - 75}{R}d(\omega t) = \frac{1}{2\pi R}[-150\cos(\omega t) - 75\omega t]_{1/6\pi}^{5/6\pi}$$
$$= \frac{1}{2\pi R}\left[-150\left(-\frac{\sqrt{3}}{2}\right) - 75\frac{5}{6}\pi + 150\frac{\sqrt{3}}{2} + \frac{75}{6}\pi\right]$$

Now, according to the request, we impose $I_{DC} = 750mA$, obtaining

$$R \cong 22\Omega$$

Observations

A *battery's capacity* C $[Ah]$ refers to the stored electric charge that can be delivered in an ammount of time at room temperature ($77°F$ or $25°C$). A $500[Ah]$ rated battery can supply $1A$ for $500h$, or $5A$ for $100h$, or $10A$ for $50h$, or $100A$ for $5h$. But the capacity is not the perfect parameter to give a real measure for a battery, because it depends on the discharge conditions: the current's value (not necessary constant), the value of the voltage, the temperature, the discharging rate.

The *C-rate* (or *charge-rate* or *hourly-rate*) of a battery specifies the discharge rate, as a multiple of the *capacity*. So, for example, a battery with a capacity $C = 1.5[Ah]$ and a $C/10$ rate, deliveres $\frac{1.5}{10} = 0.15[A]$; A $1C$ rate means that the battery discharges entirely in $1[h]$. This is similar for the *E-rate* but referred to the power, not the current.

Exercise 5

A battery with $C = 1200mAh$, $V_B = 5V$, and $C - rate = 10$, must be charged by a half-wave rectifier with a transformer at its input port, having a turns ratio $n = 15$. The AC voltage source $v_s = V_M\sin(\omega t)$ has a RMS voltage of $V_{s,RMS} = 220V$ (Figure 9.25).

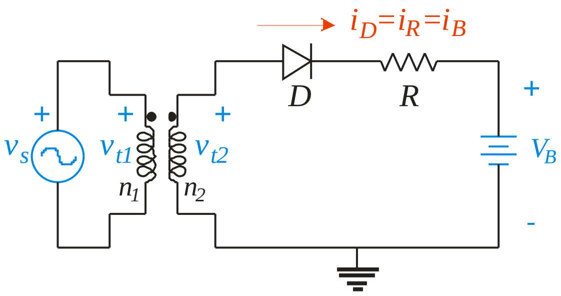

Figure 9.25: Half-wave rectifier with a transformer at its input port

Assuming a diode with $V_\gamma = 0.74V$, calculate the following:

- The average charging current, I_B
- The value of the resistance R necessary to limit the current
- The RMS current flowing throught the battery, I_{RMS}

- The power P_R dissipated by the resistor
- The power P_B delivered to the battery
- The charging time, t_B
- The efficiency of the half-wave rectifier $\eta = \frac{power\ delivered\ to\ the\ battery}{total\ power} = \frac{P_B}{P_{tot}}$

ANSWER

The average charging current is due to the features of the battery, so

$$I_B = \frac{capacity}{C - rate} = \frac{1.2}{10} = 0.12A$$

The voltage across the primary coil is that of the voltage source v_s, while the voltage across the secondary coil is $v_{t2} = \frac{v_{t1}}{n}$, so that the RMS voltage furnished to the circuit is

$$V_{t2,RMS} = \frac{V_{s,RMS}}{n} = \frac{220}{15} = 14.\overline{6}V$$

and its maximum value

$$V_{t2,M} = \sqrt{2}V_{t2,RMS} \cong 20.74V$$

As a consequence, the maximum value of the voltage downstream the diode is

$$V_{M,d} = V_{t2,M} - V_\gamma \cong 20V$$

The charging current will flow only when the diode is "active" (ON), that is when $\left(V_{t2,M} - V_\gamma\right) \geq V_B$, between each α-β segments reported in green in Figure 9.25b.

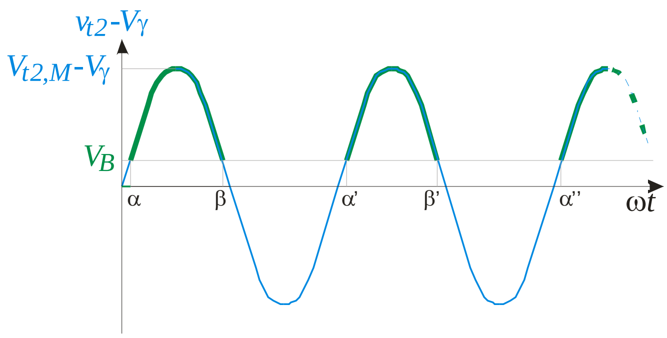

Figure 9.25b: The voltage useful to charge the battery is showed in green

According to the Figure 9.25b, we can determine the α and β values as

$$\alpha = sin^{-1}\left(\frac{V_B}{V_{t2,M} - V_\gamma}\right) \cong 0.25rad \cong 14.48°$$

and

$$\beta = \pi - \alpha \cong 2.89rad \cong 165.52°$$

The average charging current can be determined as

$$I_B = \frac{1}{2\pi}\int_\alpha^\beta \frac{v_{t2} - V_\gamma - V_B}{R} d(\omega t) = \frac{1}{2\pi}\int_\alpha^\beta \frac{V_{t2,M}\, sin(\omega t) - V_\gamma - V_B}{R} d(\omega t)$$
$$= \frac{1}{2\pi R}\left[-V_{t2,M}\, cos(\omega t) - V_\gamma(\omega t) - V_B(\omega t)\right]\Big|_\alpha^\beta$$
$$= \frac{1}{2\pi R}\left[-V_{t2,M}\, cos(\beta) + V_{t2,M}\, cos(\alpha) - (V_\gamma + V_B)\beta + (V_\gamma + V_B)\alpha\right]$$

but $\beta = \pi - \alpha$ and recalling that $\cos(x) = -\cos(\pi - x)$ we can write

$$I_B = \frac{1}{2\pi R}\left[2V_{t2,M}\, cos(\alpha) + 2(V_\gamma + V_B)\alpha - \pi(V_\gamma + V_B)\right]$$

from which, imposing that $I_B \overset{!}{=} 1200mA$, we obtain the requested resistance of the resistor

$$R = \frac{2(V_{t2,M} - V_\gamma)\, cos(\alpha) + 2V_B\alpha - \pi V_B}{2\pi I_B} \cong 34\Omega$$

So, the RMS value of the current flowing through the resistor is

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_\alpha^\beta \frac{(V_{t2,M}\, sin(\omega t) - V_B)^2}{R^2} d(\omega t)}$$
$$= \sqrt{\frac{1}{2\pi}\int_\alpha^\beta \frac{V_{t2,M}^2\, sin^2(\omega t) + V_B^2 - 2(V_{t2,M} - V_\gamma)V_B\, sin(\omega t)}{R^2} d(\omega t)}$$

which can be solved considering that $\sin^2(x) = \frac{1}{2}[1 - \cos(2x)]$, $\sin(-x) = -\sin(x)$, $\cos(\pi - x) = -\cos(x)$, and splitting the integral into three parts (highlighted with three different colors)

$$\int_\alpha^\beta V_{t2,M}^2\, sin^2(\omega t)\, d(\omega t) = \frac{V_{t2,M}^2}{2}\int_\alpha^\beta [1 - cos(2\omega t)] d(\omega t) = \frac{V_{t2,M}^2}{2}\left[\omega t - \frac{1}{2} sin(2\omega t)\right]\Big|_\alpha^\beta$$
$$= \frac{V_{t2,M}^2}{2}(\pi - 2\alpha) + \frac{V_{t2,M}^2}{2} sin(2\alpha)$$

$$\int_\alpha^\beta V_B^2 d(\omega t) = V_B^2[\omega t]|_\alpha^\beta = V_B^2(\pi - 2\alpha)$$

$$\int_\alpha^\beta -2(V_{t2,M} - V_\gamma)V_B\, sin(\omega t)\, d(\omega t) = -2(V_{t2,M} - V_\gamma)V_B[-cos(\omega t)]|_\alpha^\beta = -4(V_{t2,M} - V_\gamma)V_B\, cos(\alpha)$$

therefore, summing up

$$I_{RMS}^2 = \frac{1}{2\pi R^2}\left\{\frac{V_{t2,M}^2}{2}(\pi - 2\alpha) + \frac{V_{t2,M}^2}{2} sin(2\alpha) + V_B^2(\pi - 2\alpha) - 4(V_{t2,M} - V_\gamma)V_B\, cos(\alpha)\right\}$$

numerically $I_{RMS} \cong 0.21A$.

The power P_R dissipated by the resistor is

$$P_R = RI_{RMS}^2 \cong 1.42W$$

The power P_B delivered to the battery is

$$P_B = V_B I_B = 0.6W$$

In order to determine the charging time t_B, being

$$capacity = 1200mAh$$

then

$$t_B = \frac{capacity}{I_B} = \frac{1200mA}{0.12A} = 10h$$

Finally, for the efficiency

$$\eta = \frac{P_B}{P_{tot}} = \frac{P_B}{(P_B + P_R)} \cong 0.30$$

so that it is arond 30%.

Observations

- The charging current must be limited to avoid the battery to be damaged because of the gases which can be released during the charge.
- A *costant voltage (C-V) charging* method refers to the method by which a charger sources current into the battery to force its voltage to a preset value, named *set(-point) voltage*. When this value is reached, the charger sources only the current to maintain the battery at the same constant voltage level.
  It is the opposite for the *constant current (C-I) charging* method.
- Some rechargeable batteries need to be *C-V* while others *C-I* charged. Some chargers are *C-V* others *C-I* type; that's why generally a charger can be used to charge some batteries and not others. Universal chargers are those that have both *C-V* and *C-I* characteristics.

Summarizing
$I_B = 0.12\text{A}$; $R \cong 34\Omega$; $I_{RMS} \cong 0.21A$; $P_R \cong 1.42W$; $P_B = 0.6W$; $t_B = 10h$; $\eta \cong 0.30$

Exercise 6

The bridge full-wave rectifier has better performances than the half-wave rectifier in terms of form factor and ripple. Consider a bridge full-wave rectifier with an input transformer, as in Figure 9.26:

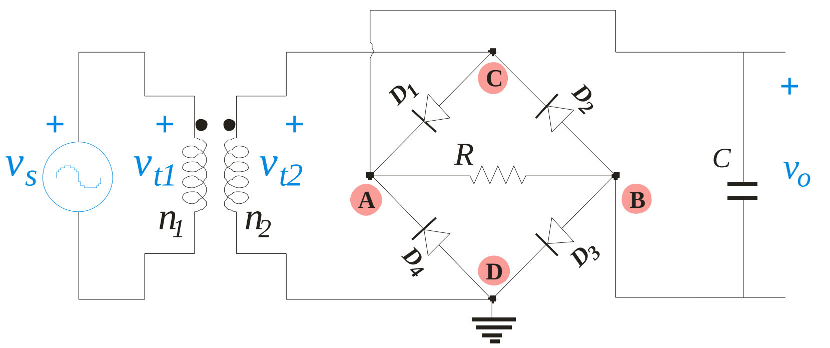

Figure 9.26: Bridge full-wave rectifier with input transformer

Calculate the following:

- DC component of voltage in output $V_{o,DC}$
- DC component of current in output $I_{o,DC}$
- RMS value of voltage in output $V_{o,RMS}$

- RMS value of current in output $I_{o,RMS}$
- The form factor $F_{F,2s}$
- The ripple value γ

ANSWER

For the current rectifier, the voltage period is no more 2π as for the half-wave rectifier, but just π, so that the DC component of voltage in output $V_{o,DC}$ can be written as

$$V_{o,DC} = \frac{1}{\pi}\int_0^{\pi} v_s d(\omega t) = \frac{1}{\pi}\int_0^{\pi} V_M \, sin(\omega t)\, d(\omega t) = \frac{2V_M}{\pi}$$

Accordingly, the DC component of current in output $I_{o,DC}$ is

$$I_{o,DC} = \frac{V_{o,DC}}{R} = \frac{2V_M}{\pi R}$$

The RMS value of voltage in output $V_{o,RMS}$ can be easily obtained as

$$V_{o,RMS} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v_s^2 d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_M^2 \, sin^2(\omega t)\, d(\omega t)} = \frac{V_M}{\sqrt{2}}$$

Such a result should not be surprising, because we could expect it equal to the one of the originating v_s.

The RMS value of current in output $I_{o,RMS}$ can be easily obtained as

$$I_{o,RMS} = \frac{V_{o,RMS}}{R} = \frac{V_M}{R\sqrt{2}}$$

The form factor $F_{F,2s}$ is defined as the $\frac{V_{o,,RMS}}{V_{o,DC}}$ ratio, therefore

$$F_{F,2s} = \frac{\frac{V_M}{\sqrt{2}}}{\frac{2V_M}{\pi}} \cong 1{,}1$$

We start from the definition also to determine the ripple value

$$\gamma = \frac{V_{r,\mathrm{RMS}}}{V_{o,DC}} = \sqrt{\left(\frac{V_{oRMS}}{V_{o,DC}}\right)^2 - 1} = \sqrt{\left(\frac{V_M/\sqrt{2}}{2V_M/\pi}\right)^2 - 1} = 0.482$$

Comparing this result with the one obtained for the half-wave rectifier, we can say that the ripple is halved (0.482 vs. 1.21).

Observations

Please, pay close attention to the fact that for the full-wave rectifier the ground is connected to one terminal of the voltage source, but it cannot be connected to the load as well. So, the bridge configuration cannot be adopted when we must have a *grounded* load.

Unfortunately, the definition of *ripple* is not unambiguous. Sometimes it can be found otherwise.

Summarizing

$V_{o,DC} = \frac{2V_M}{\pi}$; $I_{o,DC} = \frac{2V_M}{\pi R}$; $V_{o,RMS} = \frac{V_M}{\sqrt{2}}$; $I_{o,RMS} = \frac{V_M}{R\sqrt{2}}$; $F_{F,2s} \cong 1{,}1$; $\gamma = 0.482$.

Exercise 7

Given a bridge full-wave rectifier as in Figure 9.26, with diodes that have a threshold voltage $V_\gamma = 0$ and a resistance $R_{on} = 10\Omega$ when "on", sourced by a sinusoidal voltage signal of amplitude $V_{sM} = 30V$, and loaded by a resistance $R_L = 1k\Omega$, calculate the following:

- Maximum, average and RMS values of the output current: I_M, $I_{o,DC}$, $I_{o,RMS}$
- Average and RMS values of the output voltage: $V_{o,DC}$, $V_{o,RMS}$
- Average value of the output power, $P_{o,DC}$
- Average value of the input power, P_i

ANSWER

The maximum output current can be easily calculated applying the KCL to a half path of the bridge

$$I_M = \frac{V_M}{R + 2R_{on}} = \frac{30}{1000 + 20} = 29.4mA$$

therefore its RMS value (see Table 1.3 in Section 1.3.1 of Chapter 1) is

$$I_{o,RMS} = \frac{I_M}{\sqrt{2}} \cong 20.8mA$$

and its average value (as already discussed in Exercise 6) is

$$I_{o,DC} = \frac{2I_M}{\pi} = \frac{2 * 0.0294}{\pi} \cong 18.7mA$$

Average and RMS values of the output voltage are simply due to Ohm's law

$$V_{o,DC} = RI_{o,DC} = 1000 * 0.0187 = 18.7V$$

$$V_{o,RMS} = RI_{o,RMS} = 1000 * 0.0208 = 20.8V$$

Finally, according to the Joule's law (Section 4.4.8 in Chapter 4), we can write for the output and input powers respectively

$$P_{o,DC} = RI_{o,DC}^2 \cong 0.3W$$

$$P_i = (R + 2R_{on})I_{o,eff}^2 \cong 0.41W$$

Summarizing

$I_M = 29.4mA$; $I_{o,RMS} \cong 20.8mA$; $I_{o,DC} \cong 18.7mA$; $V_{o,DC} = 18.7V$; $V_{o,RMS} = 20.8V$; $P_{o,DC} \cong 0.3W$; $P_i \cong 0.41W$

Exercise 8

A resistance load requires a DC voltage $V_{DC} = 12V$ and current $I_{DC} = 0.5A$. To this aim a bridge full-wave rectifier is utilized, sourced by a AC line with a frequency $f = 50Hz$. Calculate the value of the smoothing capacitance for a 10% ripple (Figure 9.27).

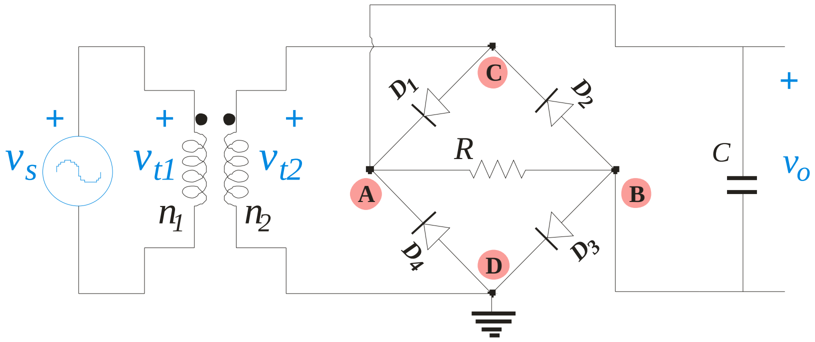

Figure 9.27: Filtered bridge full-wave rectifier

ANSWER

To solve this exercise we need to do some considerations. Let's call *cutin* the instant of time when diodes start to conduct current and *cutoff* the instant of time when the diodes end.

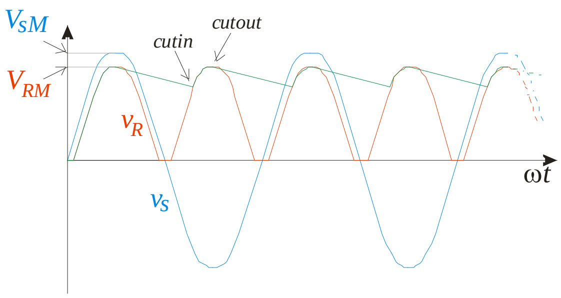

Figure 9.27b:input voltage function (in blue), output voltage function (in red), rectified voltage function (in green)

We can usefully define as

- T: period of the input function
- $\frac{T}{2}$: period of the half-wave
- T_1: time during which the diode is "ON"
- T_2: time during which the diode is "OFF" (discarge of the capacitor)

and we can admit an approximation of the green part of the sinusoidal function as a straight line (this is supposing the time of the capacitor discharge much greater of the period of the input function) as showed in Figure 9.27c.

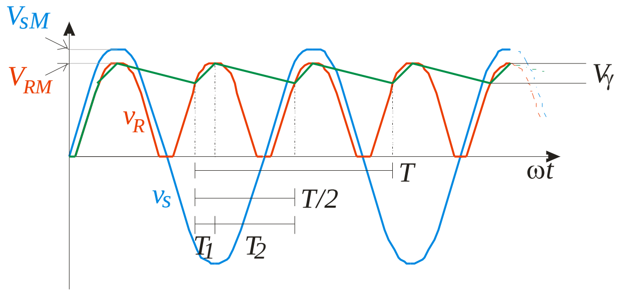

Figure 9.27c: the charge-discharge functions are approximated with straight lines

Defining as V_γ the peak-to-peak value of the residual ripple content, it follows that the RMS value of such a ripple is

$$V_{RMS} = \frac{V_\gamma}{2\sqrt{3}}$$

(please, refer to Table 1.3, Section 1.3.1 in Chapter 1).
During T_2 the capacitor gets discharged through the resistor R, losing an amount of charge Q equal to

$$Q = CV_\gamma$$

but since

$$i(t) = \frac{dQ}{dt}$$

it results

$$Q = \int_0^{T_2} i(t)dt = I_{DC}T_2$$

with I_{DC} the DC component of the current.
But being $T_2 \gg T_1$:

$$T_1 + T_2 = \frac{T}{2} \cong T_2$$

so that

$$V_\gamma = \frac{I_{DC}}{C}\frac{T}{2} = \frac{I_{DC}}{2fC}$$

and since

$$I_{DC} = \frac{V_{DC}}{R_L}$$

then

$$V_\gamma = \frac{V_{DC}}{2fRC}$$

Recalling the ripple's definition $\gamma = \frac{V_{RMS}}{V_{DC}}$, we can put

$$\gamma = \frac{V_{RMS}}{V_{DC}} = \frac{\frac{V_\gamma}{2\sqrt{3}}}{V_{DC}} = \frac{\frac{\frac{V_{DC}}{2fRC}}{2\sqrt{3}}}{V_{DC}} = \frac{1}{4\sqrt{3}fRC}$$

The value of the load resistance is

$$R = \frac{V_{DC}}{I_{DC}} = 24\Omega$$

so that, imposing a 10% ripple as the request of the exercise, and inverting the previous equation we have

$$C = \frac{1}{0.1 * 4\sqrt{3} * 50 * 24} \cong 1.2mF$$

Observation

The choice of the capacitance of the capacitor must satisfy two opposite requests. In fact, to guarantee a low ripple value we need a capacitance as high as possible, but to assure a low current peak we need a capacitance as low as possible. The trade-off usually is guaranteed by capacitance of the order of hundreds of thousands of μF.

Exercise 9

A battery provides a voltage $V_B = 12V$ across a positive clipper circuit configuration, as in Figure 9.28, with $R_1 = 47\Omega$ and $R_2 = 22\Omega$. Assuming the diode to offer a direct resistance $R_{D,on} = 15\Omega$ when "ON" but with no threshold voltage $V_\gamma = 0$, calculate the current flowing through the diode.

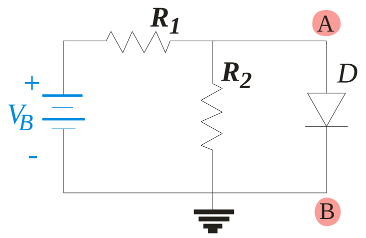

Figure 9.28: A clipper circuit configuration sourced by a constant voltage

ANSWER

It is useful to replace the current circuit with an equivalent one at the A - B terminals, according to the Thevenin's theorem (Figure 9.28b). So, the Thevenin voltage V_{TH} is the open circuited one across the same terminals, the diode being removed

$$V_{TH} = \frac{R_2}{R_1 + R_2} V_B = 3.826V$$

and the Thevenin resistance R_{TH} is the one the battery being short circuited and the diode removed

$$R_{TH} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} \cong 14.9\Omega$$

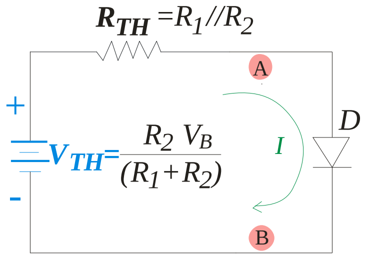

Figure 9.28b: Thevenin's equivalent of the clipper circuit configuration

The current flowing through the diode is then:

$$I = \frac{V_{TH}}{R_{TH} + R_{D,on}} \cong \frac{3.826}{14.9} \cong 257mA$$