

## CHAPTER THREE

# Forces

### MULTIPLE CHOICE QUESTIONS

#### Multiple Choice 3.1

Correct Answer (e).

$$M_p = \frac{1}{2} M_E \text{ and } R_p = 2R_E$$

If the acceleration on earth and the planet

$$\text{are } g_E = \frac{GM_E}{R_E^2} \text{ and } g_p = \frac{GM_p}{R_p^2},$$

respectively, the ratio  $\frac{g_p}{g_E}$  is:

$$\frac{g_p}{g_E} = \frac{\frac{GM_p}{R_p^2}}{\frac{GM_E}{R_E^2}} = \frac{M_p R_E^2}{M_E R_p^2} = \frac{\left(\frac{1}{2} M_E R_E^2\right)}{M_E (4R_E^2)} = \frac{1}{8}$$

$$\frac{g_p}{g_E} = \frac{\frac{GM_p}{R_p^2}}{\frac{GM_E}{R_E^2}} = \frac{M_p R_E^2}{M_E R_p^2} = \frac{\left(\frac{1}{2} M_E R_E^2\right)}{M_E (4R_E^2)} = \frac{1}{8}$$

$$\therefore \frac{g_p}{g_E} = \frac{1}{8}$$

$$g_p = \frac{1}{8} g_E$$

#### Multiple Choice 3.2

Correct Answer (a). If the acceleration on earth and the planet are

$$g_1 = \frac{GM_1}{r_1^2} \text{ and } g_2 = \frac{GM_2}{R_2^2},$$

respectively, the ratio  $\frac{g_2}{g_1}$  is:

$$\frac{g_2}{g_1} = \frac{\frac{GM_2}{r_2^2}}{\frac{GM_1}{r_1^2}} = \frac{M_2 r_1^2}{M_1 r_2^2}$$

The masses of the two planets can be written as:

$$M_1 = \rho V_1 = \frac{4}{3} \pi r_1^3 \text{ and}$$

$$M_2 = \rho V_2 = \frac{4}{3} \pi r_2^3$$

$$\text{so } \frac{g_2}{g_1} = \frac{M_2 r_1^2}{M_1 r_2^2} = \frac{\rho \frac{4}{3} \pi r_2^3 r_1^2}{\rho \frac{4}{3} \pi r_1^3 r_2^2} = \frac{r_2}{r_1}$$

$$\frac{g_2}{g_1} = \frac{r_2}{r_1}$$

#### Multiple Choice 3.3

Correct Answer (c).

#### Multiple Choice 3.4

Correct Answer (d).

#### Multiple Choice 3.5

Correct Answer (d). The force between two charges  $q_1$  and  $q_2$  is

$$F = \frac{kq_1 q_2}{r^2}$$

The force is proportional to  $1/r^2$ . That means doubling the distance quarters the force. In this problem we decrease the distance by 5 times so the force increases by 25 times.

So

$$F_1 = F_2 = \frac{kq_1 q_2}{r^2}$$

$$= \frac{9 \times 10^9 \frac{Nm^2}{C^2} \cdot 1.60 \times 10^{-19} C \cdot 1.60 \times 10^{-19} C}{(2.82 \times 10^{-10} m)^2}$$

$$= 2.90 \times 10^{-9} N$$

The multiplicative factor is 25 not 24, and the answer is (d)

#### Multiple Choice 3.6

Correct Answer (c). The electric force is proportional to  $1/r^2$  so if we increase the distance by a factor of three the force is reduced by a factor of  $1/3^2$  or  $1/9$ .

### Multiple Choice 3.7

Correct Answer (c). Easy way: Doubling the distance reduces the force by a factor of 4. Since  $F$  is proportional to  $q$ , doubling one of the charges doubles the force.

Combining these factors effects we get a factor of

$$\frac{1}{4} \cdot 2 = \frac{1}{2}$$

so the force is reduced by a factor of 2.

Longer way:

$$m^2 = \frac{Fr^2}{G}$$

$$\text{so } m = \sqrt{\frac{Fr^2}{G}} = \sqrt{\frac{0.50\text{N} \cdot (2.00\text{m})^2}{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}}} \text{ Let}$$

$$= 1.73 \times 10^5 \text{ kg}$$

$$F_1 = \frac{kq_1q_2}{r_1^2}$$

then

$$F_2 = \frac{k(2q_1)q_2}{(2r_1)^2} = \frac{kq_1q_2}{2r_1^2} = \frac{1}{2} F_1$$

### Multiple Choice 3.8

Correct Answer (d). The strong nuclear force is the strongest of the four fundamental forces and hold the protons together in the nucleus.

### Multiple Choice 3.9

Correct Answer (d). The strong nuclear force is about a 100 times larger than the electric force over the same distance and only acts over very short distances.

### Multiple Choice 3.10

Correct Answer (d). The strong nuclear force does not get weaker with distance. For two quarks it reaches a constant value of about 10,000 N. The weak force diminishes with distance.

### Multiple Choice 3.11

Correct Answer (b).

### Multiple Choice 3.12

Correct Answer (e). Since the kickers foot is no longer in contact with the ball, it no longer exerts a force on the ball. The force exerted by the floor consists of two parts, a normal force  $F_N$  and a frictional force which makes the ball rotate. The force of gravity also acts upon the ball.

### Multiple Choice 3.13

Correct Answer (e). The forces  $T$  and  $F$  are contact forces. If the muscles were suddenly cut the tension would disappear. Similarly, if the dumbbell were released, the force  $F$  would disappear.

### Multiple Choice 3.14

Correct Answer (c). The component of weight acting down the incline is  $Mg\sin\theta$ . Since the block is stationary, static friction must balance this force.

### Multiple Choice 3.15

Correct Answer (a). The magnitude of the force exerted by a spring stretched a distance  $x$  from it's equilibrium position is  $F=kx$ . If  $x$  is doubled then the force must be doubled.

### Multiple Choice 3.16

Correct Answer None of the choices offered in the text are acceptable. The correct solution follows. Both marbles move with constant velocities, which implies that the net force on each marble is zero. This in turn implies that the viscous force on the first marble is equal to its weight and the viscous force on the second marble is equal to the weight. This in other words means that the ratio of the magnitudes of the viscous forces is equal to the ratio of the weights and therefore equal to the ratio of the masses. We conclude that the ratio of the amplitudes of the viscous forces is equal to the cubic power of the ratio of the diameters, 8. This answers can also be obtained using the expression for the viscous force  $F_{\text{vis}}=6\pi\eta rv$ . The ratio of the viscous forces is equal to  $(r_1v_1/r_2v_2)=rv/2r4v=1/8$ .

**Multiple Choice 3.17**

Correct Answer (b).

**Multiple Choice 3.18**

Correct Answer (c). The normal force exerted by the plane must balance the component of the weight perpendicular to the plane. This component is  $Mg\cos\theta$ .

**CONCEPTUAL QUESTIONS****Conceptual Question 3.1**

The diagrams need to be drawn

- (a) Two forces: The weight and the tension
- (b) The weight and the normal to the bowl (along the radius)
- (c) The weight and the normal to the bowl (vertical in this case)
- (d) The weight and the normal force due to the table
- (e) The weight, the normal due to the incline and the tension (no friction).

In the (b) case the object is not in static equilibrium.

**Conceptual Question 3.2**

Both forces are contact forces, that is, contact is required for a force to be exerted. This is in contrast to a force like gravity which can act over a distance, with no contact. The main difference between the normal force and the spring force is that the normal force is a constant force but the spring force varies with distance.

**Conceptual Question 3.3**

Correct Answer (a)

**Conceptual Question 3.4**

The weight of the body part can be neglected if it is perfectly balanced by another force.

**ANALYTICAL PROBLEMS****Problem 3.1**

The distance between the sphere centers is 1.0 m.

The gravitational force between them is:

$$F = \frac{Gmm}{r^2} = \frac{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot 15\text{kg} \cdot 15\text{kg}}{(1.0\text{m})^2}$$

$$= 1.50 \times 10^{-8} \text{ N}$$

If the surface of spheres are separated by 2.0 m, then  $r = 0.5\text{m} + 2.0\text{m} + 0.5\text{m} = 3.0\text{m}$ . The gravitational force between them is now:

$$F = \frac{Gmm}{r^2} = \frac{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot 15\text{kg} \cdot 15\text{kg}}{(3.0\text{m})^2}$$

$$= 1.68 \times 10^{-9} \text{ N}$$

**Problem 3.2**

Take the radius of the Earth

$$R_E = 6.37 \times 10^6 \text{ m}.$$

The acceleration of gravity is:

$$g = \frac{GM_E}{R_E^2} = \frac{6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \cdot 6.00 \times 10^{24}}{(6.37 \times 10^6 \text{ m})^2}$$

$$= 9.86 \text{ m/s}^2$$

at 9800 km above the Earth's surface the gravitational acceleration is:

$$g = \frac{GM_E}{(R_E + 9.8 \times 10^6)^2}$$

$$= 9.86 \times \left( \frac{R_E + 9.8 \times 10^6}{R_E} \right)^2$$

$$= 2.53 \text{ m/s}^2$$

### Problem 3.3

The gravitational force between the spheres is

$$F = \frac{Gmm}{r^2} = 0.50N.$$

Solving for the mass  $m$ , we get:

$$\begin{aligned} m^2 &= \frac{Fr^2}{G} \\ \text{so } m &= \sqrt{\frac{Fr^2}{G}} = \sqrt{\frac{0.50N \cdot (2.00m)^2}{6.67 \times 10^{-11} \frac{Nm^2}{kg^2}}} \\ &= 1.73 \times 10^5 kg \end{aligned}$$

### Problem 3.4

(a) What is the magnitude of force of gravity between the Earth and the Moon, take mass of the Earth  $M_E = 6.00 \times 10^{24} kg$ , mass of the Moon  $M_m = 7.40 \times 10^{22} kg$ , and the distance between centers of them  $R_{EM} = 3.84 \times 10^8 m$ .

(b) At what point between the Earth and the Moon is the net force of gravity on a body by both the Earth and the Moon exactly zero?

$$\begin{aligned} F &= \frac{GM_E M_m}{r^2} \\ &= \frac{6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot 6.00 \times 10^{24} kg \cdot 7.40 \times 10^{22} kg}{(3.84 \times 10^8 m)^2} \\ &= 2.00 \times 10^{20} N \end{aligned}$$

### Problem 3.5

According to Newton's third law the force is equal in magnitude and opposite in direction on each charge. The magnitude of the force is given by:

$$\begin{aligned} F &= \frac{kq_1 q_2}{r^2} \\ &= \frac{9 \times 10^9 \frac{Nm^2}{C^2} \cdot 1.00 \times 10^{-5} C \cdot 1.00 \times 10^{-6} C}{(1.00m)^2} \\ &= 9.00 \times 10^3 N \end{aligned}$$

### Problem 3.6

The electron and proton have the same magnitude of charge, ie.  $e = 1.6 \times 10^{-19} C$ . The magnitude of the electric force between the two is:

$$F = \frac{ke^2}{r^2} = 1.00N$$

Rearranging and plugging in numbers:

$$\begin{aligned} \frac{9 \times 10^9 \frac{Nm^2}{C^2} \cdot (1.60 \times 10^{-19} C)^2}{1.00N} &= r^2 \\ r &= 1.52 \times 10^{-14} m \end{aligned}$$

**Problem 3.7**

Both the electric and gravitational force have the same  $1/r^2$  dependence so the  $r^2$  will cancel out when we take the ratio of the two forces. Also, the electric force between two protons or two electrons is the same, because protons and electrons have the same amount of charge. The ratio of the electric force to the gravitational force for two masses with the same charge can be written:

$$\frac{F_E}{F_s} = \frac{\frac{ke^2}{r^2}}{\frac{Gm_1m_2}{r^2}} = \frac{ke^2}{Gm_1m_2}$$

(a) Two protons  $m_1=m_2=1.67 \times 10^{-27}$  kg so:

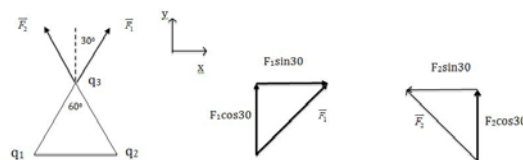
$$\begin{aligned} \frac{F_E}{F_s} &= \frac{ke^2}{Gm_p m_p} \\ &= \frac{9 \times 10^9 \frac{Nm^2}{C^2} \cdot (1.60 \times 10^{-19} C)^2}{6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \cdot (1.67 \times 10^{-27})^2 kg^2} \\ &= 1.24 \times 10^{36} \\ \therefore \frac{F_E}{F_s} &= 1.24 \times 10^{36} \end{aligned}$$

(b) Two electrons  $m_1=m_2=m_e=9.11 \times 10^{-31}$  kg so:

$$\begin{aligned} T &= 4.6 \times 9.81 + 100 \times 9.81 / 2 = 535.6 \text{ N} \\ \therefore \frac{F_E}{F_s} &= 4.17 \times 10^{42} \end{aligned}$$

(c) A proton and an electron  $m_1=m_p=1.67 \times 10^{-27}$  kg,  $m_2=m_e=9.11 \times 10^{-31}$  kg so:

11 kg The ratio is largest for the force between two electrons. The electric force will be the same in all three cases but the gravitational mass will be smallest when the product of the masses is smallest, that is, in the force between two electrons.

**Problem 3.8****Figure 1**

The forces  $\vec{F}_1$  and  $\vec{F}_2$  are of the same magnitude but have different directions. As can be seen in Figure 1 above the x components of the forces cancel and the y components add. The resultant  $F_R$  is simply twice either component ie.,  $F_R = 2F_1 \cos 30 = 2F_2 \cos 30$ .

Now 11 kg so the resultant is

$$5.8 \text{ kg}$$

in the y direction.

**Problem 3.9**

The magnitude of the force will be the same but the direction will be in the opposite direction. The force will be  $7.8 \times 10^{-5}$  N in the negative y direction.

**Problem 3.10**

Singly charged ions each carry one unit of electronic charge ( $e=1.60 \times 10^{-19} C$ ) so the force between these ions is:

$$\begin{aligned} F_1 = F_2 &= \frac{kq_1q_2}{r^2} \\ &= \frac{9 \times 10^9 \frac{Nm^2}{C^2} \cdot 1.60 \times 10^{-19} C \cdot 1.60 \times 10^{-19} C}{(2.82 \times 10^{-10} m)^2} \\ &= 2.90 \times 10^{-9} N \end{aligned}$$

**Problem 3.11**

$$\begin{aligned}
 F_1 &= \frac{kq_1q_2}{r^2} \\
 &= \frac{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \cdot 10^{-4} \text{C} \cdot 45 \times 10^{-6} \text{C}}{(4\text{m})^2} \\
 &= 1.62 \text{N} \\
 F_2 &= \frac{kq_1q_4}{r^2} \\
 &= \frac{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \cdot 10^{-4} \text{C} \cdot 25 \times 10^{-6} \text{C}}{(4\text{m})^2} \\
 &= 1.41 \text{N} \\
 F_3 &= \frac{kq_1q_3}{r^2} \\
 &= \frac{9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \cdot 10^{-4} \text{C} \cdot 125 \times 10^{-6} \text{C}}{(\sqrt{41}\text{m})^2} \\
 &= 2.75 \text{N}
 \end{aligned}$$

These the amplitudes of the three forces acting on the charge  $q_1$ . We now evaluate the components along the vertical and horizontal axes.

$$\begin{aligned}
 F_v &= F_2 - F_3 \times \frac{4}{\sqrt{41}} = 1.41 - 2.75 \times \frac{4}{\sqrt{41}} = -0.3 \text{N} \\
 F_h &= F_3 \times \frac{5}{\sqrt{41}} - F_1 = 2.75 \times \frac{5}{\sqrt{41}} - 1.62 = 0.5 \text{N}
 \end{aligned}$$

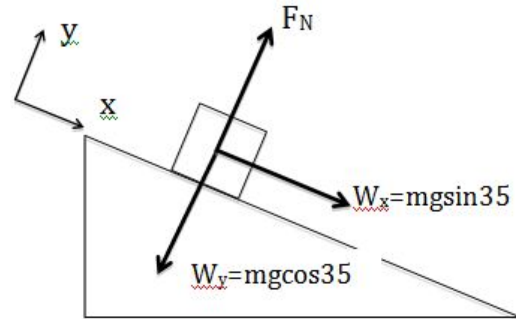
The magnitude of the force and its direction are given by:

$$\begin{aligned}
 F &= \sqrt{F_v^2 + F_h^2} = \sqrt{34} \times 10^{-1} \text{N} \\
 \tan(\theta) &= -\frac{3}{5}
 \end{aligned}$$

**Problem 3.12**

In the y direction:

$$\begin{aligned}
 \Sigma F_y &= 0 \\
 F_N - mg \cos 35 &= 0 \\
 F_N &= mg \cos 35 \\
 &= 5.8 \text{kg} \cdot 9.8 \text{m/s}^2 \cdot \cos 35 \\
 &= 46.6 \text{N}
 \end{aligned}$$



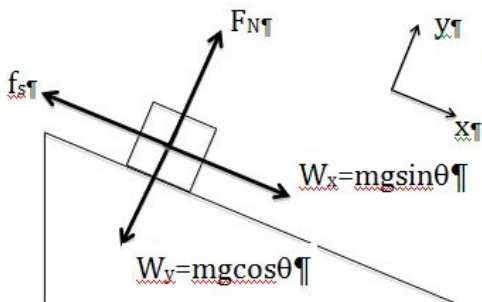
**Figure 2**

**Problem 3.13**

- (a) The weight of the man is  $W=mg$ , that is  $W=70 \text{ (kg)} \times 9.81 \text{ (m/s}^2\text{)}=687 \text{ N}$ .
- (b) The normal force acting on the man is equal and opposite to his weight.
- (c) The man will read 687 N in principle. However, if the scale is not calibrated properly to zero, the weight might be off by the error in calibration. Moreover, the scale has a certain accuracy that may be greater than 1 N, which in turn means that there will be a round off error.

**Problem 3.14**

The climber is stationary so  $a_x=a_y=0$ . In the y direction:

**Figure 3**

$$\Sigma F_y = 0$$

$$F_N - mg \cos 36 = 0$$

$$F_N = mg \cos 36$$

$$= 64 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot \cos 36$$

$$= 5.14 \text{ N}$$

In the x direction:

(c) In the x direction:

$$\Sigma F_x = 0$$

$$f_s - mg \sin 36 = 0$$

$$f_s = mg \sin 36$$

$$= 64 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot \sin 36$$

$$= 369 \text{ N}$$

(c) The maximum static frictional force is:

$$f_{sMax} = \mu_s F_N = 0.86 \cdot 5.14 \text{ N} = 442 \text{ N}$$

The actual frictional force is much less than this.

**Problem 3.15**

A 480 kg sea lion is resting on an inclined wooden surface  $40^\circ$  above the horizontal as illustrated in Figure 4. The coefficient of static friction between the sea lion and the wooden surface is 0.96. Find (a) the normal force on the sea lion by the surface; (b) the magnitude of force of friction; and (c) the maximum force of friction between the sea lion and the wooden surface.

**Figure 4**

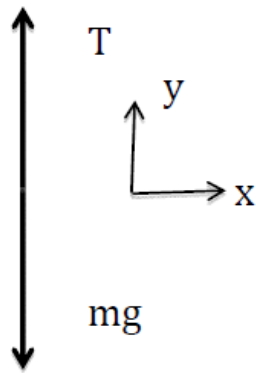
### Problem 3.16

A chandelier, as shown in Figure 5, of mass 11 kg is hanging by a chain from the ceiling. What is the tension force in the chain.



**Figure 5**

The chandelier is in static equilibrium so  $\sum \vec{F} = 0$ . There are no forces to consider in the x direction. In the y direction:



**Figure 6**

$$\begin{aligned}\sum F_y &= 0 \\ T + (-mg) &= 0 \\ T &= mg \\ &= 11\text{kg} \cdot 9.8\text{m/s}^2 \\ &= 109\text{N}\end{aligned}$$

### Problem 3.17

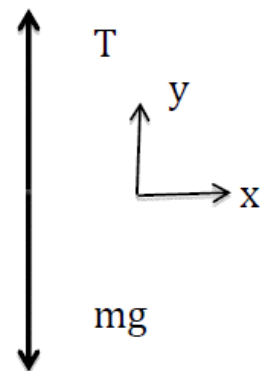
A 85 kg climber is secured by a rope hanging from a rock as shown in Figure 7. Find the tension in the rope.



**Figure 7**

$$\begin{aligned}\sum F_y &= 0 \\ T + (-mg) &= 0 \\ T &= mg \\ &= 85\text{kg} \cdot 9.8\text{m/s}^2 \\ &= 833\text{N}\end{aligned}$$

The climber is in static equilibrium so  $\sum \vec{F} = 0$ . There are no forces to consider in the x direction. In the y direction:

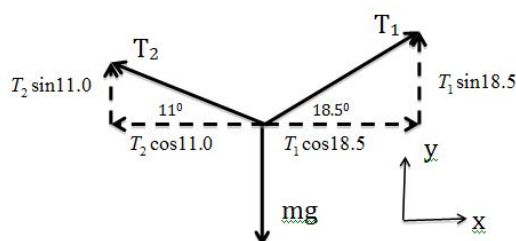


**Figure 8**



**Problem 3.18**

A 76 kg climber is crossing by a rope between two picks of a mountain as shown in Figure 9.

**Figure 9****Figure 10**

The weight  $W$  of the climber is 76.0 kg

The FBD for the climber is

Since the climber is in static equilibrium

$$\sum \vec{F} = 0$$

$$\text{so } \sum F_x = 0 \text{ and } \sum F_y = 0$$

The x component gives

$$T_1 \cos 18.5 - T_2 \cos 11.0 = 0$$

The y component gives

$$T_1 \sin 18.5 + T_2 \sin 11.0 - W = 0$$

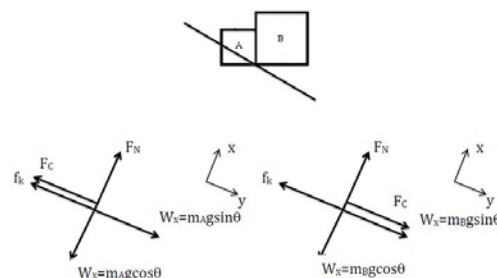
$$W = mg = 76.0 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 745 \text{ N}$$

Solving these two equations in two unknowns gives

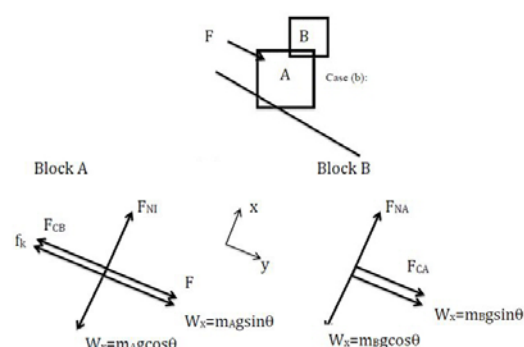
$$T_1 = 1.46 \times 10^3 \text{ N and } T_2 = 1.42 \times 10^3 \text{ N}$$

**Problem 3.19**

Casa (a):

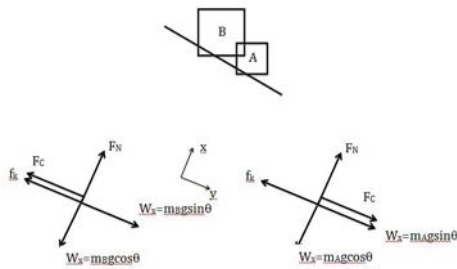
**Figure 11a**

Case (b):

**Figure 11b**

In this case there are two contact forces between block A and B, one parallel to the incline  $F_{CA}$  and one perpendicular to the incline  $F_{NA}$ .  $F_{CA}$  is exerted through friction so if the surfaces are smooth the top block will simply slip on the bottom block. If the surfaces are rough then as the force  $F$  is applied to block A the top block will first follow the bottom block without slipping until the maximum value of the static frictional force is reached. Then the top block B will slip on block A. There is the normal force  $F_{NA}$  exerted by A on B and an equal and opposite force exerted  $F_{NB}$  by B on A.

Case (c):

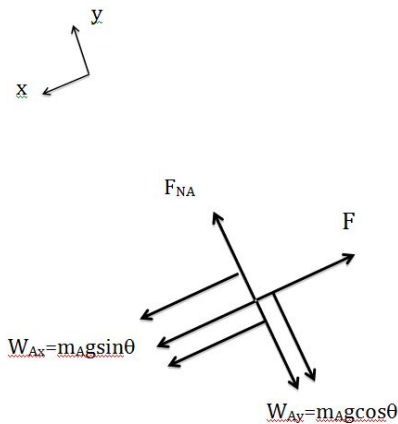


**Figure 11c**

Case (c) is similar to case (a). In fact the acceleration of the two block system will be the same. The main difference between the two cases is that the magnitude of the contact force will be different.

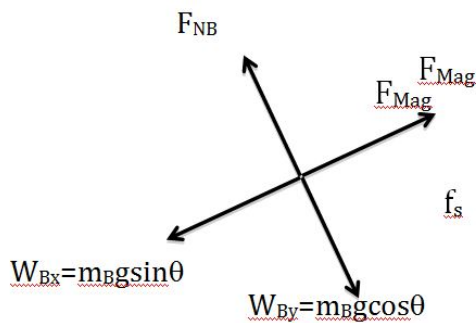
**Problem 3.20**

Block A



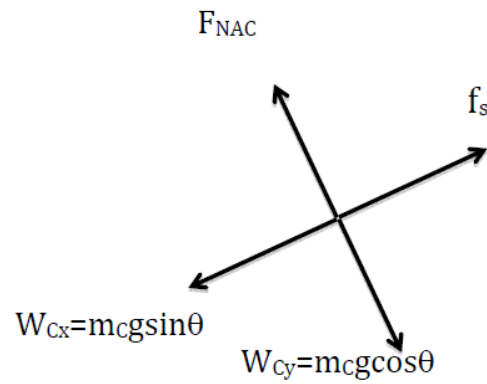
**Figure 12a**

Block B



**Figure 12b**

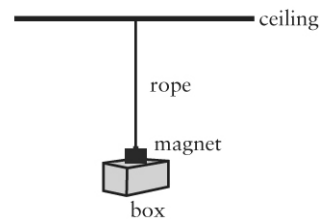
Block C



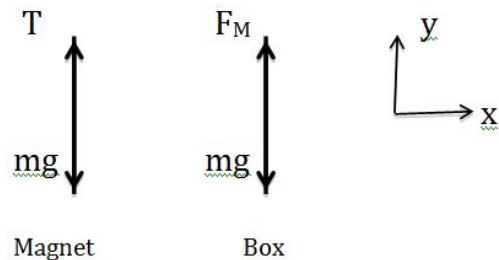
**Figure 12c**

**Problem 3.21**

A box is lifted by a magnet suspended from the ceiling by a rope attached to the magnet as illustrated in Figure 3.46. Draw free body diagram for the box and for the magnet.



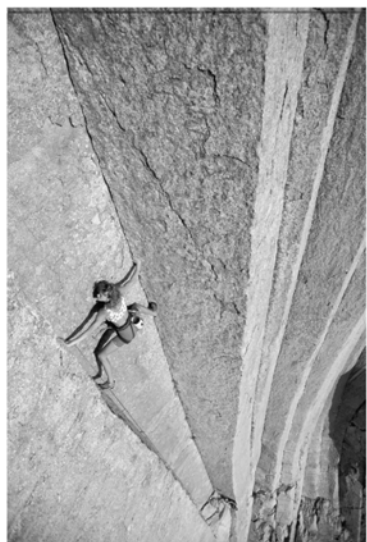
**Figure 3.46**



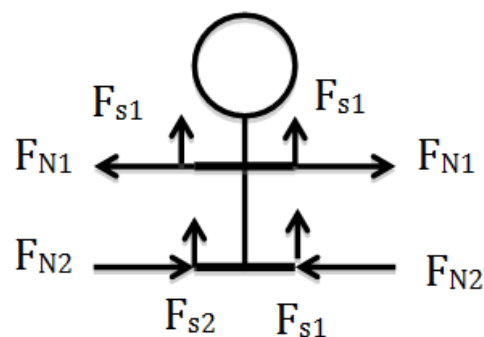
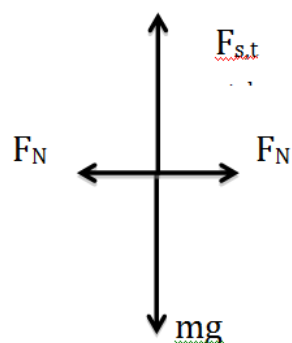
**Figure 13**

**Problem 3.22**

Figure 3.47 shows a rock climber is climbing up Devil's Tower in Wyoming.

**Figure 3.47**

The forces on the climber are his weight, his fingers pulling inward and up against the rock and the force exerted by the climber's legs on the rock. The man cannot be considered as a simple point object in this case. His hands pull inwards producing an outward normal force exerted by the rock. He supports his weight primarily by having his legs at a large angle so that they can push outward on the rock. The outward component of force increases the normal force exerted by the rock wall. This in turn increases the frictional force which is parallel to the wall and upward. This supports most his weight. The frictional force  $F_{s1}$  produced by his hands can also support some of the weight. In the first force diagram below note that the normal forces  $F_{N1}$  and  $F_{N2}$  exerted by the wall on the man are out on the hands and in on the legs. The four upward forces represent static frictional forces which we label  $F_{s1}$  and  $F_{s2}$ . The second diagram shows a simplified FBD. Figure 14b

**Figure 14a****Figure 14b**

**Problem 3.23**

The freebody diagram for each arm is similar to that of figure 4.43 of example 4.25 in the textbook.

The force balance for each arm can be written as

$$T - F - F_{\text{arm}} = 0$$

and the force balance for the bar is

$$2F - W_{\text{bar}} = 0$$

combining the two equations, we find the expression for the tension on the shoulder

$$T = F_{\text{arm}} + W_{\text{bar}} / 2$$

Using the Table 4.1 we can estimate the tension

$$T = 4.6 \times 9.81 + 100 \times 9.81 / 2 = 535.6 \text{ N}$$

**Problem 3.24**

Each hand pulls down the bar with a force  $F_{\text{hand}}$ . The cable attached to the bar provides a tension  $T$ , such that

$$T = 2F_{\text{hand}}$$

On the other hand the system is assumed to be in equilibrium and therefore the tension has to be balanced by the weight of the arms, trunk and head.

$$T = W_{\text{arms+trunk+head}}$$

Combining the two equations, we find:

$$F_{\text{hand}} = W_{\text{arms+trunk+head}} / 2$$

Using table 4.1 of chapter 4, we can determine the weight of arms, trunk and head.

$$\begin{aligned} F_{\text{hand}} &= 9.81 \times (2 \times 70 \times 0.065 + 70 \times 0.48 + 70 \times 0.07) / 2 \\ &= 33.5 \text{ N} \end{aligned}$$