CHAPTER 2: GROUNDWATER PROPERTIES

PROBLEMS

2.1 The porosity and volumetric water content of a sample are 0.40 and 0.18, respectively. Calculate the water saturation of the sample.
Solution: $n = 0.4$ $\theta = 0.18$ Using Equation 2.40: $s = \frac{\theta}{n} = \frac{0.18}{0.4} = 0.45 = 45\%$
2.2 An undisturbed core sample is obtained from an unsaturated aquifer 1.0 m above the water table. The sample is 0.150 m high and 0.050 m in diameter. A dry sample of the aquifer has a specific gravity of 2.65 gr/cm^3. The weight of the sample is 630 grams before drying and 570 grams after drying. Calculate the porosity, volumetric water content, and bulk density of aquifer. Assume that the volume of voids is 450 cm^3.
Solution: $V_P = 450 cm^3 \times 10^{-6} \frac{m^3}{cm^3} = 4.5 \times 10^{-4} m^3$ $V_T = 0.05^2 \times \pi \times 0.15 = 1.17 \times 10^{-3} m^3$ According to the Section 2.8.1, porosity can be calculated as the ratio of volume of voids over the total volume: $n = \frac{V_v}{V_T} = \frac{4.5 \times 10^{-4}}{1.17 \times 10^{-3}} = 0.38$ Water content is calculated using Equation 2.37: $\theta = \frac{W_{(wet)}}{W_{(dry)}} - 1 = \frac{630}{570} - 1 = 0.1$ According to Example 2.3,

$\rho_b = \rho_d(1-n) = 2.65(1-0.38) = 1.643\dfrac{gr}{cm^3}$

2.3 An unconfined aquifer with a specific yield of 0.20 is used as a water supply for irrigation of farm lands. The groundwater is pumped out of this aquifer with the rate of 4.2 m^3/day. The area of the aquifer is 1500 m^2. How long does it take to have one meter drop in water table in this aquifer?

Solution:

Using Equation 2.23:

$$S_y = \frac{V_w}{A\Delta h} = \frac{Q\Delta t}{A\Delta h}$$

$$0.2 = \frac{4.2\dfrac{m^3}{day}\times\Delta t}{1500m^3\times 1.0m}$$

$$\Rightarrow \Delta t = 71.4 days$$

2.4 How much water can be removed from an unconfined aquifer with a specific yield of 0.18 when the water table is lowered 1.0 m? How much water can be removed from a confined aquifer with storage coefficient of 0.0005 when the piezometric surface is lowered 1.0 m? Express your answers in m^3/km^2.

Solution:

As discussed Chapter 2, in an unconfined aquifer, specific yield is the volume of water that is released from storage per unit surface area of the aquifer per unit decline in the water table. Therefore, since there is 1m drop of water table, the volume of water removed from the aquifer is $0.18\dfrac{m^3}{km^2}$.

In a confined aquifer, volume of water released from 1m drop of piezometric surface is represented by storage coefficient. As the piezometric surface is lowered 1m in this problem, the water removed from the aquifer is $0.0005\dfrac{m^3}{km^2}$.

2.5 Determine the total stress acting top on an aquifer, which is underlain a 20 m thick aquitard. The bulk density of the aquitard is 2100 kg/m^3. What is the effective stress in the aquifer if the pressure head is 25 m?

Solution:

The total stress is calculated as the weight of rock and water:

$$\sigma_T = b \times \rho_b \times g = 20 \times 2100 \times 9.806 = 411852 \frac{N}{m^2}$$

And effective stress is calculated using Equation 2.10:

$$\sigma_e = \sigma_T - P = \sigma_T - (\rho_w \times g \times h)$$

$$= 411852 - (1000 \times 9.806 \times 25) = 166707 \frac{N}{m^2}$$

2.6 A sample of silty sand has a volume of 215 cm^3. It has weight of 514.7 gr. After saturating the sample is weighed 594.2 gr. The sample is then drained by gravity until it reaches a constant weight of 483.4 gr. Finally, the sample is dried in oven for ten hours and it reaches the weight 452.1 gr. Assuming the density of water is 1 g/cm^3, compute the following:

(a) Water content of the sample.

(b) Volumetric water content of the sample.

(c) Saturation ratio of the sample.

(d) Porosity

(e) Specific yield

(f) Specific retention

(g) Dry bulk density

Solution:

a) Water content can be calculated based on the mass using Equation 2.37:

$$\theta_w = \frac{W_{wet}}{W_{dry}} - 1 = \frac{514.7}{425.1} - 1 = 0.14 = 14\%$$

b) The volumetric water content is the ratio of the volume of water to the volume of the sample. The volume of water is calculated as:

$$V_w = \frac{W_w}{\rho_w} = \frac{W_{wet} - W_{dry}}{\rho_w} = \frac{514.7 - 452.1}{1} = 62.6 cm^3$$

$$\theta = \frac{V_w}{V_T} = \frac{62.6}{215} = 0.29 = 29\%$$

c) Saturation ratio is the volume of water over the volume of voids. Volume of voids can be determined by dividing the weight of water at saturation by the density of water. Therefore,

$$V_v = \frac{W_{sat} - W_{dry}}{\rho_w} = \frac{594.2 - 452.1}{1} = 142.1 cm^3$$

Using Equation 2.39:

$$S_w = \frac{V_w}{V_v} = \frac{62.6}{142.1} = 0.44 = 44\%$$

d) According to Section 2.8.1:

$$n = \frac{V_v}{V_T} = \frac{142.1}{514.7} = 0.276 = 27.6\%$$

e) Specific yield is the ratio of the volume of water drained by the gravity to the total volume of the sample.

$$V_{drained} = \frac{W_{wet} - W_{drained}}{\rho_w} = \frac{514.7 - 483.4}{1} = 31.3 cm^3$$

$$S_y = \frac{V_{drained}}{V_{sat}} = \frac{31.3}{142.1} = 0.22 = 22\%$$

f) Using Equation 2.25:

$$S_r = n - S_y = 0.276 - 0.22 = 0.056 = 5.6\%$$

g) Dry bulk density is the mass of the soil particles divided by the volume of the sample:

$$\rho_b = \frac{W_{dry}}{V_{wet}} = \frac{452.1 gr}{215 cm^3} = 2.1 \frac{gr}{cm^3}$$

2.7 The porosity of the unsaturated zone in a field is 0.35. The elasticity module of the medium is 9.5×10^6. Determine the storage coefficient for this layer if the thickness of the unsaturated layer is 10 m.

Solution:

Water elasticity module is $2.1 \times 10^9 \frac{N}{m^2}$, therefore:

$$\beta = \frac{1}{E_W} = \frac{1}{2.1 \times 10^9} = 4.76 \times 10^{-10} \frac{m^2}{N}$$

$$\alpha = \frac{1}{E_s} = \frac{1}{9.5 \times 10^6} = 1.05 \times 10^{-7} \frac{m^2}{N}$$

It is assumed that the temperature is $20^0 C$, so $\gamma_w = 9789 \frac{N}{m^2}$. The storage coefficient is now calculated using Equation 2.27:

$$S = \gamma_w.b.(\alpha + n\beta) = 9789 \times 10 \times (1.05 \times 10^{-7} + 0.35 \times 4.76 \times 10^{-10})$$

$$S = 1.03 \times 10^{-2}$$

2.8 What is the specific yield of a soil sample that has the total volume of 250 cm^3, void volume of 150 cm^3 and flow volume of 100 cm^3?

Solution:

$$n = \frac{V_V}{V_T} = \frac{150}{250} = 0.6$$

Effective porosity is calculated using Equation 2.22:

$$\hat{n} = \frac{V_{wet}}{V_T} = \frac{100}{250} = 0.4$$

Using Equation 2.24

$$S_r = n - \hat{n} = 0.6 - 0.4 = 0.2$$

Now Equation 2.25 is used to calculate the specific yield:

$$S_y = n - S_r = 0.6 - 0.2 = 0.4$$

2.9 The average water table elevation has dropped 1.5 m due to the removal of 85 million cubic meters from an unconfined aquifer over an area of 200 km^2. Determine the specific yield for the aquifer. What is the specific retention of the aquifer if its porosity is 0.42?

Solution:

$$S_y = \frac{85 \times 10^6}{200 \times 10^6 \times 1.5} = 0.28$$

Equation 2.25 is used to calculate the specific yield:

$$S_r = n - S_y = 0.42 - 0.28 = 0.14$$

2.10 Resolve Example 2.8 for an unconfined aquifer. Assume that specific yield is 2.5%.

Solution:

In an unconfined aquifer, the storage is calculated by Equation 2.35:

$$S = S_y + b.S_s$$

$$\Rightarrow S = 0.025 + 78 \times 2.64 \times 10^{-4} = 0.046$$

2.11 Records of the average rate of precipitation over an area show 680 mm/yr rainfall, out of

which, 250 mm/yr flows overland. Evaluation of time series of the data shows that 300 mm/yr is evapotranspirated. The area is irrigated with the rate of 150 mm/yr. In this area, 170 mm/yr flows through rivers and the rate of outflow to the other basins is 100 mm/yr. Determine the influence from seepage. Also, calculate the rate of baseflow. Assume groundwater inflow and outflow remain unchanged and the groundwater is not pumped out.
Solution: The inflows to this system are precipitation and irrigation water. The outflows are evapotranspiration, overland flow, and groundwater recharge. Therefore: $P + R_i = E_T + O_f + R_r$ $680 + 150 = 300 + 250 + R_r$ $\Rightarrow R_r = 280 \frac{mm}{gr}$ Now the influent from seepage is calculated as: $I_g + R_r = O_g + R_f$ $I_g + 250 + 170 - 280 = 140 \frac{mm}{gr}$ To calculate baseflow: $I_g + R_i + R_r = O_g + B_f$ $140 + 150 + 280 = 250 + B_f$ $B_f = 320 \frac{mm}{gr}$
2.12 Calculate the density of a soil sample if its porosity and a dry bulk density are 0.28 and 1.65 g/cm^3, respectively.
Solution: According to Example 2.3, $\rho_d = \frac{\rho_b}{(1-n)} = \frac{1.65}{(1-0.28)} = 2.29 \frac{gr}{cm^3}$

2.13 Between bulk density (ρ_b) or particle density (ρ_d), which would be expected to be greater in magnitude? Prove your answer?
Solution: Density of a sample is the ratio of its mass to its volume. In a bulk, the mass of voids can be ignored, therefore: $m_v = 0$ $\rho_b = \frac{m_d + m_v}{V_{Total}} = \frac{\rho_d V_d}{V_{Total}} = \rho_d \frac{V_d}{V_{Total}}$ $\Rightarrow \frac{\rho_b}{\rho d} = \frac{V_d}{V_{Total}}$ (I) Where, m_d, ρ_d and V_d are the particle mass, density, and volume. On the other hand, the volume of only the particles is less than the total volume: $V_d < V_{Total} \Rightarrow \frac{V_d}{V_{Total}} < 1$ (II) By comparing (I) *and* (II) : $\rho_b / \rho_d < 1 \Rightarrow \rho_b < \rho_d$
2.14 The dry and wet bulk densities of a soil sample are measured equal to 1.65 g/cm^3 and 1.98 g/cm^3, respectively. Calculate the porosity of the sample. Assume the sample is completely saturated with water.
Solution: According to the Example 2.4: $n = \frac{\rho_{bw} - \rho_d}{\rho_w}$ The temperature is assumed to be $20^0 C$. Therefore, $\rho_w = 0.998 \frac{gr}{cm^3}$ $n = \frac{1.98 - 1.65}{0.998} = 0.33$
2.15 Determine the specific yield of an aquifer. The water is pumped with the rate of 6.75 m^3/day. There is a 1.2 m drop in water table after 7 years and the area of the aquifer is 8.2 × 10^6 ha (10000 m^2).
Solution:

Using Equation 2.23:

$$S_y = \frac{Q\Delta t}{A\Delta h}$$

$$S_y = \frac{6.75(\frac{m^3}{day}) \times 7\,(yr) \times 365(\frac{days}{year})}{8.2\,(km^2) \times 10^6\,(\frac{m^2}{km^2}) \times 1.2\,(m)} = 1.75 \times 10^{-3}$$