

Chapter 2

$$2.1) \quad T = 2 \text{ sec}, \quad m = 20 \text{ gm} \quad \frac{k}{m} = \omega^2 = \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{2}\right)^2 = \pi^2$$

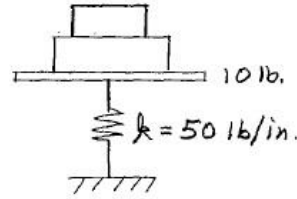
$$\underline{k = \pi^2 m = \pi^2 (20) = 197.4 \text{ dynes/cm}}$$

$$2.2) \quad m = \frac{k}{\omega^2}$$

$$m_{p1} + m_{pkg} = \frac{k}{\omega^2}$$

$$m_{pkg} = \frac{k}{\omega^2} - m_{p1}$$

$$W_{pkg} = m_{pkg} g = \frac{kg}{\omega^2} - m_{p1} g = \frac{kg}{(2\pi\nu)^2} - W_{p1}$$



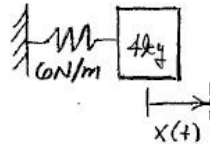
$$W_{pkg} = \frac{50(32.2 \times 12)}{[2\pi(3)]^2} - 10 = 44.4 \text{ lb}$$

$$2.3) \quad \text{Fig. 1.8} \quad \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{12EI}{mL^2}} = 2\sqrt{\frac{3EI}{mL^2}}$$

$$\text{Fig. 1.9} \quad \omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{48EI}{mL^2}} = 4\sqrt{\frac{3EI}{mL^2}}$$

The effect of clamping the roof is to increase the stiffness and, in doing so, double the natural frequency of the structure.

$$2.4) \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{4}} = 1.225 \text{ rad/sec}$$



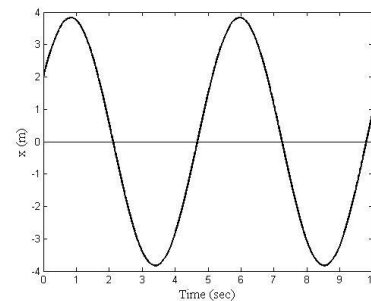
$$x_0 = 2 \text{ m} \\ v_0 = 4 \text{ m/sec}$$

From Eq. (2.16):

$$\underline{A = \sqrt{x_0^2 + (v_0/\omega)^2} = \sqrt{(2)^2 + (4/1.225)^2} = 3.830 \text{ m}}$$

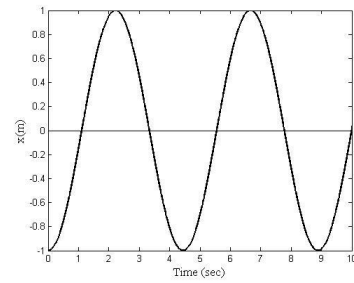
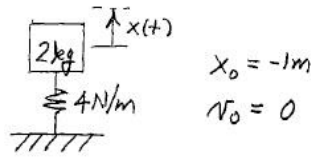
From Eq. (2.17):

$$\underline{\phi = \tan^{-1}\left(\frac{v_0}{\omega x_0}\right) = \tan^{-1}\left(\frac{4}{1.225(2)}\right) = 58.51^\circ = 1.021 \text{ rads}}$$



$$\text{From Eq. (2.15):} \quad \underline{x(t) = 3.830 \cos(1.225t - 1.021) \text{ meters}}$$

$$2.5) \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{2}} = \sqrt{2} \text{ rad/sec}$$



$$A = 1 \text{ m}, \quad \phi = \pi \text{ rads}$$

From Eq. (2.14): $x(t) = \cos(\sqrt{2}t - \pi) = -\cos(\sqrt{2}t) \text{ meters}$

$$2.6) \quad A = \pi R^2 = \pi (1.25)^2 = 4.91 \text{ cm}^2, \quad E_{al} = 6.90 \times 10^6 \text{ N/cm}^2, \quad m = 20 \text{ kg} = 2 \times 10^4 \text{ g}$$

$$k_{eq} = \frac{E_{al} A}{L} = \frac{(6.90 \times 10^6)(4.91)}{30} = 1.13 \times 10^6 \text{ N/cm} = 1.13 \times 10^{11} \text{ dynes/cm}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.13 \times 10^{11}}{2 \times 10^4}} = 2380 \text{ rad/sec} = 378 \text{ cps}$$

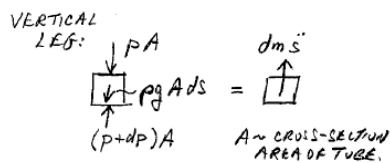
$$2.7) \quad I_p = m r_G^2 = 20(5)^2 = 500 \text{ kg} \cdot \text{cm}^2 = 5 \times 10^5 \text{ g} \cdot \text{cm}^2$$

$$J = \frac{\pi R^4}{2} = \frac{\pi (1.25)^4}{2} = 3.83 \text{ cm}^4$$

From Eq. (1.29): $k_T = \frac{JG}{L} = \frac{(2.76 \times 10^6)(3.83)}{30} = 3.52 \times 10^5 \text{ N} \cdot \text{cm/rad} = 3.52 \times 10^{10} \text{ dyne} \cdot \text{cm/rad}$

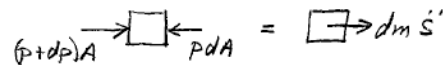
$$\omega = \sqrt{\frac{k_T}{I_p}} = \sqrt{\frac{3.52 \times 10^{10}}{5 \times 10^5}} = 265 \text{ rad/sec}$$

- 2.9) If we treat the fluid as incompressible, we may think of the confined fluid as an incompressible/inextensible rod with vanishing bending stress. We may then recognize that the components of displacement, velocity, and acceleration along the path of the fluid are uniform. That is, they are the same for each fluid particle. Let us isolate a typical fluid element, (1) moving vertically and (2) moving horizontally, and draw the kinetic diagrams for each. Hence,



$$A dp - \rho g A ds = \ddot{s} dm$$

HORIZONTAL LEG:



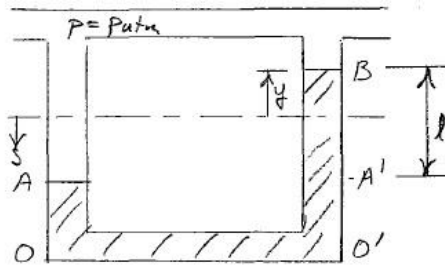
$$A dp = \ddot{s} dm$$

$$\int_A^0 A dp - \int_A^0 \rho g A ds = \int_A^0 \ddot{s} dm \quad (1a)$$

$$\int_{0'}^{A'} A dp - \int_{0'}^{A'} \rho g A ds = \int_{0'}^{A'} \ddot{s} dm \quad (1b)$$

$$\int_{A'}^B A dp - \int_{A'}^B \rho g A ds = \int_{A'}^B \ddot{s} dm \quad (1c)$$

$$\int_0^{0'} A dp = \int_0^{0'} \ddot{s} dm \quad (1d)$$



Adding (1a)-(1d) and recognizing that $\int_A^0 = -\int_{0'}^A$ gives

$$A \int_0^{0'} dp + A \int_{A'}^B dp - \int_{A'}^B \rho g A ds = \ddot{s} \int_A^B dm$$

$$A(p_0 - p_0) + A(p_B - p_A) - \rho g A l = m \ddot{s} \quad (2)$$

where $m = \int_A^B dm = \rho A L$ and L is the total given length of the manometer fluid. Let y measure the height of manometer fluid above its equilibrium position. Then, since acceleration of all particles along the path is the same, $\ddot{s} = \ddot{y}$ and further, $l = 2y$ then substituting into (2):

$$-2\rho g A y = \rho A L \ddot{y}$$

or

$$\ddot{y} + \frac{2g}{L} y = 0$$

Thus,

$$\omega = \sqrt{\frac{2g}{L}}$$

2.10) $k = 400 \text{ lb/in}$, $W = 200 \text{ lb}$

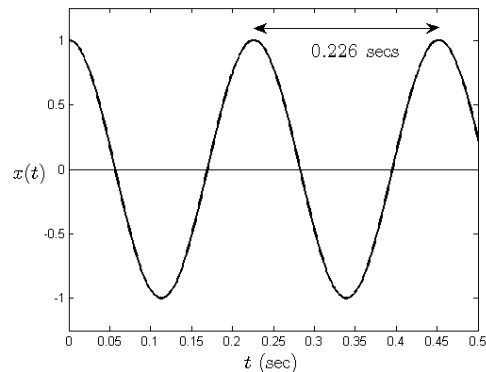
$$\omega = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{400}{200/32.2 \times 12}} = 27.8 \text{ rad/sec}$$

$$T = \frac{2\pi}{27.8} = 0.226 \text{ sec}$$

From Eq. (2.14): $x(t) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$

$$x(t) = (1) \cos 27.8t + \frac{0}{27.8} \sin 27.8t$$

$$\underline{x(t) = \cos 27.8t \text{ inches}}$$

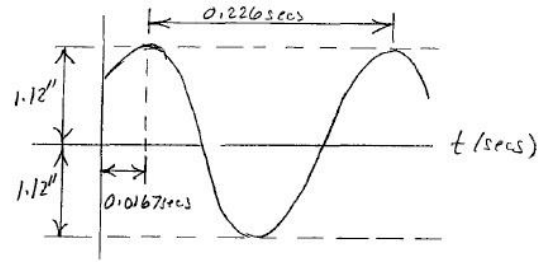


2.11) $x_0 = 1 \text{ in}$, $v_0 = 13.9 \text{ in/sec}$, $\omega = 27.8 \text{ rad/sec}$, $T = \frac{2\pi}{27.8} = 0.226 \text{ sec}$

$$A = \sqrt{(1)^2 + (13.9/27.8)^2} = 1.12 \text{ in}$$

$$\phi = \tan^{-1} \left(\frac{13.9}{27.8(1)} \right) = 0.464 \text{ rads}$$

$$t_\phi = \phi/\omega = 0.0167 \text{ sec}$$



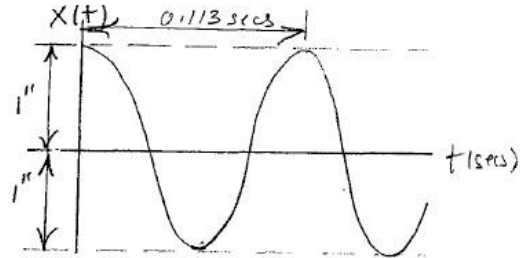
$$\underline{x(t) = 1.12 \cos(27.8t - 0.464) \text{ inches}}$$

2.12) From Prob. 1.6:

$$k = \frac{12EI}{L^3} = 3 \cdot \frac{4EI}{L^3} = 4(400 \text{ lb/in}) = 1600 \text{ lb/in}$$

$$\omega = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{1600}{200/32.2 \times 12}} = 55.6 \text{ rad/sec}$$

$$T = \frac{2\pi}{55.6} = 0.113 \text{ sec}$$



$$\underline{x(t) = \cos 55.6t \text{ inches}}$$

2.13) The spring force is non-impulsive. Conservation of linear momentum:

$$m \cdot 0 + mv_1 = 2mv_2 \Rightarrow v_2 = \frac{1}{2}v_1$$

Thus, the initial velocity for 2-car system is $v_0 = \frac{1}{2}v_1$

$$k_{eq} = \frac{4EA}{L} = \frac{4\pi R^2 E}{L}$$

$$\omega = \sqrt{\frac{k_{eq}}{2m}} = \sqrt{\frac{4\pi R^2 E}{2mL}} = \sqrt{\frac{2\pi R^2 E}{mL}}$$

$$x(t) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$$

$$x_0 = 0, \frac{v_0}{\omega} = \frac{\frac{1}{2}v_1}{\sqrt{2\pi R^2 E/mL}} = \frac{v_1}{2R} \sqrt{\frac{mL}{2\pi E}}$$

$$\underline{x(t) = \frac{v_1}{2R} \sqrt{\frac{mL}{2\pi E}} \sin \sqrt{\frac{2\pi R^2 E}{mL}} t}$$

2.14) From Prob. 1.7: $k = 2.25 \times 10^3$ lb/ft

$$x_0 = 1.01 \text{ in}, \quad v_0 = 0$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.25 \times 10^3}{200/32.2}} = 19.0 \text{ rad/sec}$$

$$\underline{x(t) = 1.01 \cos 19t \text{ inches}}$$

2.15) From Prob. 1.15: $k = 3220$ lb/ft

$$x_0 = 0.447 \text{ in}, \quad v_0 = 0$$

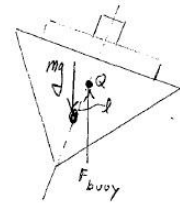
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3220}{200/32.2}} = 22.8 \text{ rad/sec}$$

$$\underline{x(t) = 0.447 \cos 22.8t \text{ inches}}$$

2.16) $\sum M_G = I_G \ddot{\theta}$

$$-lF_{\text{buoy}} \sin \theta = I_G \ddot{\theta}$$

$$I_G \ddot{\theta} + lF_{\text{buoy}} \theta \approx 0$$



$$\underline{\omega^2 = \frac{lF_{\text{buoy}}}{I_G} = \frac{lmg}{I_G}}$$

2.17) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{g/L}}$

$$\underline{L = \frac{gT^2}{4\pi^2} = \frac{32.2(3)^2}{4\pi^2} = 7.34 \text{ ft}}$$

2.18) $\sum \vec{M}_0 = I_0^{(\text{rod})} \ddot{\alpha} + \vec{r}_{\text{bob}} \times m\vec{a}_{\text{bob}}$

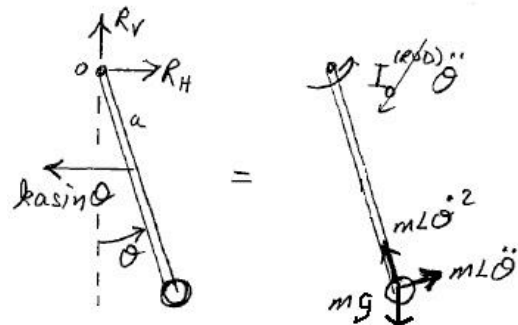
$$-a \cos \theta \cdot ka \sin \theta - L \cos \theta \cdot mg \sin \theta = L \cdot m L \ddot{\theta}$$

$$mL^2 \ddot{\theta} + [ka^2 + mgL] \sin \theta \cos \theta = 0$$

$$\sin \theta \cos \theta = \left[\theta - \frac{\theta^3}{3!} + \dots \right] \cdot \left[1 - \frac{\theta^2}{2!} + \dots \right] \approx \theta \ll 1$$

$$\ddot{\theta} + \left[\frac{ka^2 + mgL}{mL^2} \right] \theta = 0$$

$$\underline{\omega = \sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}}$$



$$2.19) \quad \sum \vec{M}_0 = I_0^{(rod)} \ddot{\alpha} + \vec{r}_{cyl} \times m \vec{a}_{cyl}$$

$$-Lmg \sin \theta + k_T \theta = -L \cdot mL \ddot{\theta}$$

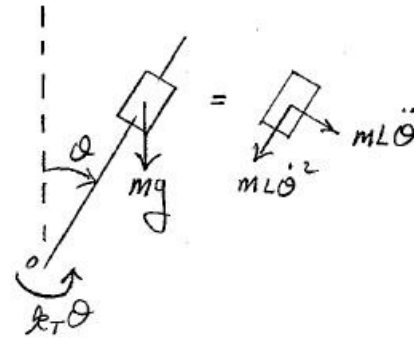
$$mL^2 \ddot{\theta} + k_T \theta - Lmg \sin \theta = 0$$

$$\sin \theta \approx \theta \ll 1$$

$$\ddot{\theta} + \left[\frac{k_T - mgL}{mL^2} \right] \theta = 0$$

$$\omega = \sqrt{\frac{g}{L} \left[\frac{k_T}{mgL} - 1 \right]}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L} \left[\frac{k_T}{mgL} - 1 \right]}}$$



2.20) The solution follows that of Example 2.8, but with the moment of inertia now corresponding to a sphere rather than a disk. Thus, $I = \frac{2}{5} mR^2$

From Eqs. (h), (j), and (k) of Ex. 2.8:

$$I_p \left(\frac{R-r}{r} \right) \ddot{\theta} + mgr \sin \theta = 0$$

where $I_p = I_G + mr^2 = \frac{2}{5} mr^2 + mr^2 = \frac{7}{5} mr^2$ and $\sin \theta \approx \theta \ll 1$

$$\frac{7}{5} mr^2 \left(\frac{R-r}{r} \right) \ddot{\theta} + mgr \theta = 0 \text{ or } \ddot{\theta} + \frac{g}{L_{eff}} \theta = 0$$

where $L_{eff} = \frac{7}{5} (R-r) = \frac{7}{5} (100-1.5) = 137.9 \text{ cm}$

$$\omega = \sqrt{\frac{g}{L_{eff}}} = \sqrt{\frac{981}{137.9}} = 2.67 \text{ rad/sec}$$

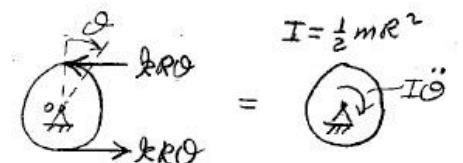
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2.67} = 2.36 \text{ sec}$$

$$t_{rise} = \frac{T}{2} = 1.18 \text{ sec}$$

$$2.21) \quad \sum M_0 = I \alpha: \quad 2kR^2 \theta = -I \ddot{\theta}$$

$$\ddot{\theta} + \frac{2kR^2}{I} \theta = 0$$

$$\omega = 2\sqrt{\frac{k}{m}}$$



2.22) $\theta(t) = \theta_0 \cos \omega t + \frac{\dot{\theta}_0}{\omega} \sin \omega t$

$$\theta(t) = \theta_0 \cos 2\sqrt{\frac{k}{m}}t$$

2.23) Wheel: $\sum M_0 = I\alpha: -PR_0 \cos \beta = -I\ddot{\theta}$

$$I\ddot{\theta} + PR_0 \cos \beta = 0$$

$$\beta \ll 1: I\ddot{\theta} + PR_0 \approx 0$$

Block: $\sum F_x = ma_x: P \cos \psi - kx = m\ddot{x}$

$$\psi \ll 1: m\ddot{x} + kx \approx P$$

(2) into (1)

$$I\ddot{\theta} + mR_0\ddot{x} + kR_0x = 0$$

Need $\theta - x$ relation \rightarrow

$$x = R_0 \sin \theta + L(\cos \psi - \cos \psi_0) \approx R_0 \theta$$

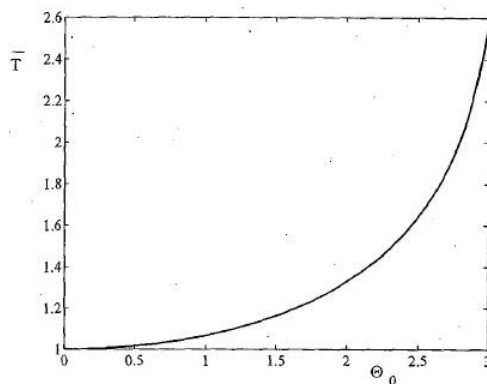
(4) into (3)

$$\left(\frac{I}{R_0} + mR_0 \right) \ddot{\theta} + kR_0\theta = 0$$

$$\omega = \sqrt{\frac{kR_0^2}{I + mR_0^2}}$$

2.24) (a) Execute the following Matlab code:

```
theta=0.01:0.01:3;
q=sin(theta/2);
m=q.*q;
Tbar=(2/pi)*ellipke(m);
plot(theta,Tbar)
```



$$(b) \quad \bar{T} = \frac{1}{1-E} = \frac{2}{\pi} F(m)$$

$$(i) \quad \bar{T} = \frac{1}{1-.05} = 1.05, \quad F(m) = \frac{\pi}{2} \bar{T} = \frac{\pi}{2} (1.05) = 1.65$$

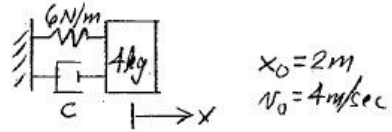
$$\underline{\underline{\theta_0 = 0.873 \text{ rad}}}$$

$$(ii) \quad \bar{T} = \frac{1}{1-.01} = 1.01, \quad F(m) = \frac{\pi}{2} \bar{T} = \frac{\pi}{2} (1.01) = 1.59$$

$$\underline{\underline{\theta_0 = 0.400 \text{ rad}}}$$

2.25) (a) $c = 1 \text{ N} \cdot \text{sec/m}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{4}} = 1.225 \text{ rad/sec}$$



$$\zeta = \frac{c}{2\omega m} = \frac{1}{2(1.225)(4)} = 0.1020$$

$$\omega_d = \omega \sqrt{1 - \zeta^2} = 1.225 \sqrt{1 - (0.1020)^2} = 1.219 \text{ rad/sec}$$

$$\zeta \omega = (0.1020)(1.225) = 0.1250 \text{ rad/sec}$$

$$\underline{\underline{A = 2 \sqrt{1 + \frac{[\{4 / (1.225)(2)\} + 0.1020]^2}{1 - (0.1020)^2}} = 4.020 \text{ m}}}$$

$$\underline{\underline{\phi = \tan^{-1} \left\{ \frac{\frac{4}{(1.225)(2)} + 0.1020}{\sqrt{1 - (0.1020)^2}} \right\} = 1.050 \text{ rad}}}$$

$$\underline{\underline{x(t) = 4.020 e^{-0.1250t} \cos(1.219t - 1.050) \text{ meters}}}$$

(b) $c = 5 \text{ N} \cdot \text{sec/m}, \quad \omega = 1.225 \text{ rad/sec}$

$$\zeta = \frac{c}{2\omega m} = \frac{5}{2(1.225)(4)} = 0.5100$$

$$\omega_d = \omega \sqrt{1 - \zeta^2} = 1.225 \sqrt{1 - (0.5100)^2} = 1.054 \text{ rad/sec}$$

$$\zeta \omega = (0.5100)(1.225) = 0.6248 \text{ rad/sec}$$

$$A = 2\sqrt{1 + \frac{[\{4/(1.225)(2)\} + 0.51]^2}{1 - (0.51)^2}} = 5.368 \text{ m}$$

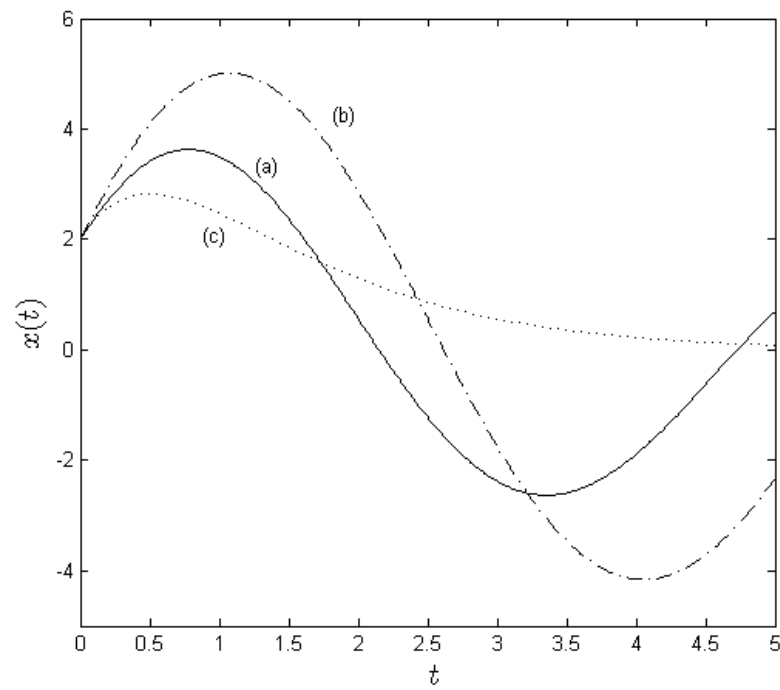
$$\phi = \tan^{-1} \left\{ \frac{\frac{4}{(1.225)(2)} + 0.51}{\sqrt{1 - (0.51)^2}} \right\} = 1.189 \text{ rad}$$

$$\underline{x(t) = 5.368e^{-0.6248t} \cos(1.054t - 1.189) \text{ meters}}$$

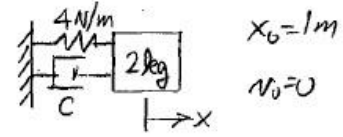
(c) $c = 10 \text{ N} \cdot \text{sec/m}, \quad \omega = 1.225 \text{ rad/sec}$

$$\zeta = \frac{c}{2\omega m} = \frac{10}{2(1.225)(4)} = 1 \text{ (critical damping)}$$

$$\underline{x(t) = [2 + \{4 + (1.225)(2)\}t]e^{-1.225t} = [2 + 6.45t]e^{-1.225t} \text{ meters}}$$



2.26) (a) $c = 2 \text{ N} \cdot \text{sec/m}$, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{2}} = \sqrt{2} = 1.414 \text{ rad/sec}$



$$\zeta = \frac{c}{2\omega m} = \frac{2}{2(\sqrt{2})(2)} = 0.3536$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 1.323 \text{ rad/sec}$$

$$\zeta\omega = 0.500 \text{ rad/sec}$$

$$A = x_0 \sqrt{1 + \frac{\zeta^2}{1-\zeta^2}} = \frac{x_0}{\sqrt{1-\zeta^2}} = \frac{1}{\sqrt{1-(0.3536)^2}} = 1.069 \text{ m}$$

$$\phi = \tan^{-1} \left\{ \frac{\zeta}{\sqrt{1-\zeta^2}} \right\} = \tan^{-1} \left\{ \frac{0.3536}{\sqrt{1-(0.3536)^2}} \right\} = 0.3614 \text{ rad}$$

$$\underline{x(t) = 1.069e^{-0.5t} \cos(1.323t - 0.3614) \text{ meters}}$$

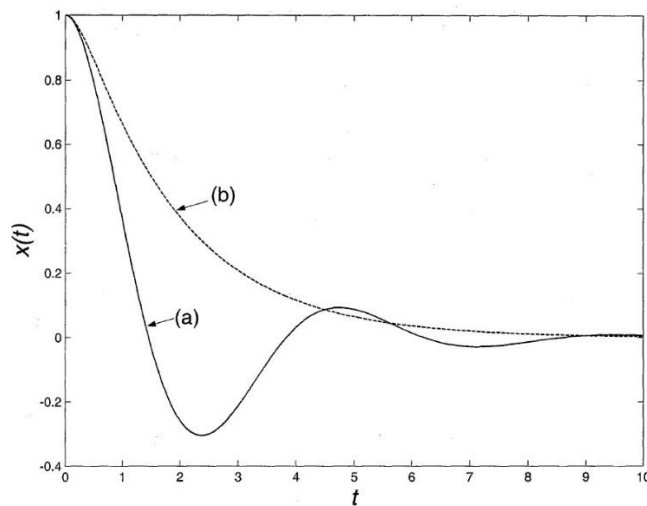
(b) $c = 8 \text{ N} \cdot \text{sec/m}$, $\omega = \sqrt{2} = 1.414 \text{ rad/sec}$

$$\zeta = \frac{c}{2\omega m} = \frac{8}{2(\sqrt{2})(2)} = 1.414 > 1 \text{ (overdamped system)}$$

$$z = \sqrt{\zeta^2 - 1} = \sqrt{(\sqrt{2})^2 - 1} = 1$$

$$\zeta\omega = 2 \text{ rad/sec}$$

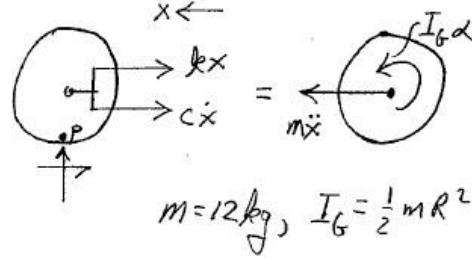
$$\underline{x(t) = e^{-2t} [\cosh \sqrt{2}t + \sqrt{2} \sinh \sqrt{2}t] \text{ meters}}$$



2.27) Kinetics:

$$\sum \vec{M}_P = I_G \ddot{\alpha} + \vec{r}_{PG} \times m \vec{a}_{PG}$$

$$-(kx + c\dot{x})R = I_G \alpha + m\ddot{x}R$$



(1)

Kinematics for no slip:

$$x = R\theta, \dot{x} = R\dot{\theta}, \ddot{x} = R\ddot{\theta} \Rightarrow \alpha = \ddot{\theta} = \frac{1}{R} \ddot{x}$$

(2)

$$-(kx + c\dot{x})R = \frac{1}{R} (I_G + mR^2) \ddot{x}$$

$$\begin{aligned} (2) \text{ into } (1) \quad &= \frac{1}{R} \left(\frac{1}{2} mR^2 + mR^2 \right) \ddot{x} \\ &= \frac{3}{2} mR \ddot{x} \end{aligned}$$

$$\text{Thus, } \ddot{x} + \frac{2}{3} \frac{c}{m} \dot{x} + \frac{2}{3} \frac{k}{m} x = 0 \quad \text{or} \quad \ddot{x} + 2\omega\zeta \dot{x} + \omega^2 x = 0$$

$$\text{where } \omega = \sqrt{\frac{2}{3} \frac{k}{m}} = \sqrt{\frac{2}{3} \left(\frac{8}{12} \right)} = 0.7454 \text{ rad/sec}$$

$$2\omega\zeta = \frac{2}{3} \frac{c}{m} \Rightarrow \zeta = \frac{c}{3\omega m} = \frac{8}{3(0.7454)(12)} = 0.2981$$

$$\omega_d = \omega \sqrt{1 - \zeta^2} = 0.7454 \sqrt{1 - (0.2981)^2} = 0.7115 \text{ rad/sec}$$

2.28) $\sum \vec{M}_0 = I_0^{(rod)} \ddot{\alpha} + \vec{r}_{cyl} \times m \vec{a}_{cyl}$

$$L \cdot cL\dot{\theta} - Lmg \sin \theta + k_T \theta = -L \cdot mL\ddot{\theta}$$

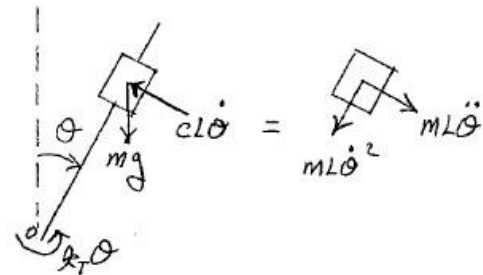
$$mL^2 \ddot{\theta} + cL^2 \dot{\theta} + k_T \theta - Lmg \sin \theta = 0 \quad \text{where } \sin \theta \approx \theta \ll 1$$

$$\text{Thus, } \ddot{\theta} + \frac{c}{m} \dot{\theta} + \left[\frac{k_T}{mL^2} - \frac{g}{L} \right] \theta = 0$$

$$\omega^2 = \frac{k_T}{mL^2} - \frac{g}{L}$$

$$2\omega\zeta = \frac{c}{m} \Rightarrow \zeta = \frac{c}{2\omega m} = \frac{c}{2m \sqrt{\frac{k_T}{mL^2} - \frac{g}{L}}}$$

$$\omega_d = \omega \sqrt{1 - \zeta^2} = \sqrt{\frac{k_T}{mL^2} - \frac{g}{L} - \frac{c^2}{4m^2}}$$



2.29) (a) $T = 2.6 \text{ sec}, T_d = 3 \text{ sec}$

$$\omega_d = \omega \sqrt{1 - \zeta^2} \Rightarrow \zeta^2 = 1 - \left(\frac{\omega_d}{\omega} \right)^2 = 1 - \left(\frac{2\pi/T_d}{2\pi/T} \right)^2 = 1 - \left(\frac{T}{T_d} \right)^2$$

$$\zeta = \sqrt{1 - \left(\frac{T}{T_d} \right)^2} = \sqrt{1 - \left(\frac{2.6}{3} \right)^2} = 0.50$$

(b) $x_1 = 1 \text{ in}, x_4 = 0.1 \text{ in}, n = 3$

From Eq. (2.89), $\bar{\delta} = \frac{1}{n} \ln \frac{x_1}{x_{n+1}} = \frac{1}{3} \ln \frac{1}{0.1} = 0.7675$

$$\zeta = \frac{0.7675}{\sqrt{4\pi^2 + (0.7675)^2}} = 0.1213$$

2.30) $\sum M_0 = I\alpha: k_r \theta + cl^2 \dot{\theta} - \frac{L}{2} W \cos \theta = -I\ddot{\theta}$

$\cos \theta \approx 1$ for $\theta \ll 1$

$$\ddot{\theta} + \frac{cl^2}{I} \dot{\theta} + \frac{k_r}{I} \theta \approx \frac{WL}{2I}$$

or

$$\ddot{\beta} + \frac{cl^2}{I} \dot{\beta} + \frac{k_r}{I} \beta = 0 \text{ where } \beta(t) = \theta(t) - \theta_{\text{static}} = \theta(t) - \frac{WL}{2k_r}$$

$$I = \frac{1}{12} mh^2 + \frac{1}{3} mL^2 = \frac{1}{3} mL^2 \left[\frac{1}{4} \left(\frac{h}{L} \right)^2 + 1 \right] \approx \frac{1}{3} mL^2$$

$$I = \frac{1}{3} \frac{\gamma}{g} AL^3 = \frac{1}{3} \frac{(48)(2)(7)^3}{32.2} = 340.9 \text{ slug} \cdot \text{ft}^2$$

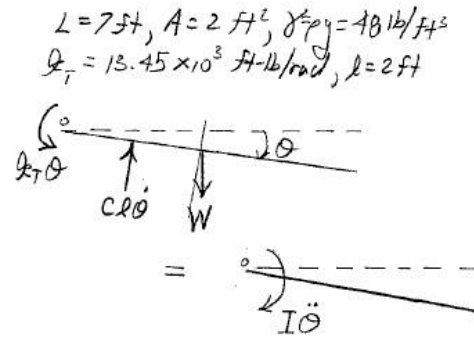
$$W = \gamma AL = (48)(2)(7) = 672 \text{ lb}$$

$$\omega = \sqrt{\frac{k_r}{I}} = \sqrt{\frac{13.45 \times 10^3}{340.9}} = 39.45 \text{ rad/sec}$$

$$\bar{\delta} = \frac{1}{5} \ln \frac{1}{0.02} = 0.7824$$

$$\zeta = \frac{\bar{\delta}}{\sqrt{4\pi^2 + \bar{\delta}^2}} = \frac{0.7824}{\sqrt{4\pi^2 + (0.7824)^2}} = 0.1236$$

$$2\omega\zeta = cl^2 / I \Rightarrow c = \frac{2\omega\zeta I}{l^2} = \frac{2(39.45)(0.1236)(340.9)}{(2)^2} = 831.1 \text{ lb} \cdot \text{sec/ft}$$



$$2.31) \quad \sum \vec{M}_0 = I_0^{(rod)} \ddot{\alpha} + \vec{r}_{bob} \times m \vec{a}_{bob}$$

$$-a \cos \theta \left[c \ddot{\alpha} \sin \theta + ka \sin \theta \right] - L \cdot mg \sin \theta = L \cdot mL \ddot{\theta}$$

$$mL^2 \ddot{\theta} + ca^2 \cos^2 \theta \dot{\theta} + ka^2 \sin \theta \cos \theta + mgL \sin \theta = 0$$

$$\cos^2 \theta = \left[1 - \frac{\theta^2}{2!} + \dots \right]^2 \approx 1$$

$$\sin \theta \cos \theta = \left[\theta - \frac{\theta^3}{3!} + \dots \right] \cdot \left[1 - \frac{\theta^2}{2!} + \dots \right] \approx \theta \ll 1$$

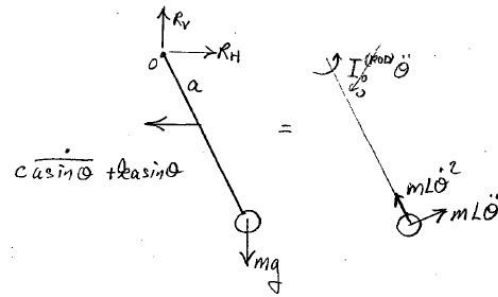
$$mL^2 \ddot{\theta} + ca^2 \dot{\theta} + [ka^2 + mgL] \theta \approx 0$$

$$\ddot{\theta} + \frac{ca^2}{mL^2} \dot{\theta} + \left[\frac{ka^2}{mL^2} + \frac{g}{L} \right] \theta \approx 0$$

$$\omega = \sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}$$

$$2\omega\zeta = \frac{ca^2}{mL^2} \Rightarrow c = 2\zeta m \left(\frac{L}{a} \right)^2 \sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}$$

$$\underline{\underline{c_{cr} = c|_{\zeta=1} = 2m \left(\frac{L}{a} \right)^2 \sqrt{\frac{ka^2}{mL^2} + \frac{g}{L}}}}$$



$$2.32) \quad \zeta = 1 \quad \underline{\underline{c_{cr} = \frac{2\omega\zeta I}{l^2} = \frac{2(39.45)(340.9)}{(2)^2} = 6724 \text{ lb} \cdot \text{sec/ft}}}}$$

$$2.33) \quad \sum M_0 = I\alpha : -ca^2 \dot{\theta} - k_T \theta = I \ddot{\theta}$$

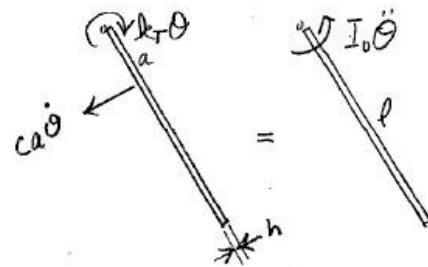
$$\ddot{\theta} + \frac{ca^2}{I} \dot{\theta} + \frac{k_T}{I} \theta = 0$$

$$I = \frac{1}{12} m h^2 + \frac{1}{3} m l^2 = \frac{1}{3} m l^2 \left[\frac{1}{4} \left(\frac{h}{l} \right)^2 + 1 \right] \approx \frac{1}{3} m l^2$$

$$\omega = \sqrt{k_T / I}$$

$$2\omega\zeta = ca^2 / I \Rightarrow c = 2\omega\zeta I / a^2$$

$$\underline{\underline{c_{cr} = c|_{\zeta=1} = \frac{2\omega I}{a^2} = \frac{2l}{a^2} \sqrt{\frac{k_T m}{3}}}}$$



$$2.34) \quad \frac{-4}{1.225(2)} < -1 \quad \checkmark$$

$$\text{Eq. (2.100): } t_a = \frac{1}{\| -4/2 \| - 1.225} = 1.290 \text{ sec}$$

$$\text{Eq. (2.101): } t_{os} = t_a + \frac{1}{\omega} = 1.290 + \frac{1}{1.225} = 2.106 \text{ sec}$$

$$\text{Eq. (2.102):}$$

$$x_{os} = -\frac{x_0}{\omega t_a} e^{-(\omega t_a + 1)} = -\frac{2}{(1.225)(1.290)} e^{-((1.225)(1.290) + 1)} = -0.09588 \text{ m}$$

$$2.35) \quad \frac{-2}{1.414(1)} < -1 \quad \checkmark$$

$$\text{Eq. (2.100): } t_a = \frac{1}{\| -2/1 \| - 1.414} = 1.706 \text{ sec}$$

$$\text{Eq. (2.101): } t_{os} = t_a + \frac{1}{\omega} = 1.706 + \frac{1}{1.414} = 2.413 \text{ sec}$$

$$\text{Eq. (2.102):}$$

$$x_{os} = -\frac{x_0}{\omega t_a} e^{-(\omega t_a + 1)} = -\frac{1}{(1.414)(1.706)} e^{-((1.414)(1.706) + 1)} = -0.01367 \text{ m}$$

$$2.36) \quad m = 4 \text{ kg}, \quad k = 6 \text{ N/m}, \quad \mu_s = \mu_k = 0.1, \quad x_0 = 2 \text{ m}, \quad v_0 = 4 \text{ m/sec}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{4}} = 1.225 \text{ rad/sec}, \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{1.225} = 5.130 \text{ sec}$$

$$f_\mu = \mu_k \frac{mg}{k} = (0.1) \frac{(4)(9.81)}{6} = 0.6540 \text{ m}$$

$$n_{cr} = \frac{1}{2} \left[\frac{2}{0.6540} - 1 \right] = 1.029 \Rightarrow n_{stick} = 2$$

$$t_{stick} = t_2 = \frac{(2)(5.13)}{2} = 5.130 \text{ sec}$$

$$\|x_{stick}\| = x_2 = \|2 - 2(2)(0.6540)\| = 0.616 \text{ m}$$

2.37) $m = 2 \text{ kg}, k = 4 \text{ N/m}, \mu_s = 0.12, \mu_k = 0.1$

$$\bar{\mu} = \frac{\mu_s}{\mu_k} = \frac{0.12}{0.10} = 1.2$$

Eq. (2.116):
$$\Delta = 4f_\mu = 4\mu_k \frac{mg}{k} = 4(0.10) \frac{(2)(9.81)}{4} = 1.962 \text{ m}$$

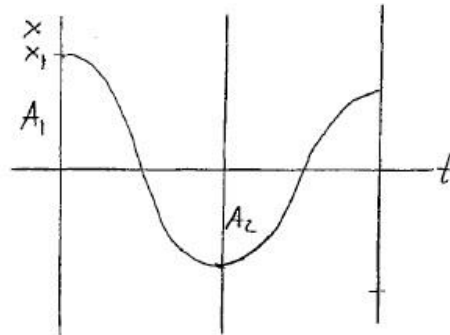
$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{2}} = \sqrt{2} = 1.414 \text{ rad/sec}$$

2.38) Eq. (1.92): $W^{(NC)} = \Delta T + \Delta U$

$$\int_{x_1}^{x_2} F^{(NC)} dx = \left[\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \right] + \left[\frac{1}{2} k x_2^2 - \frac{1}{2} k x_1^2 \right]$$

over $\frac{1}{2}$ cycle: $-\int \mu_k mg dx = [0 - 0] + \frac{1}{2} k [x_2^2 - x_1^2]$

$$-\mu_k mg (x_2 - x_1) = \frac{1}{2} k (x_2 + x_1)(x_2 - x_1)$$



$$\begin{aligned} -\mu_k mg &= \frac{1}{2} k (x_2 + x_1) \\ &= \frac{1}{2} k (-A_2 + A_1) \\ &= -\frac{1}{2} k (A_1 - A_2) \\ &= -\frac{1}{2} k \frac{\Delta}{2} \end{aligned}$$

$$\Delta = 4\mu_k \frac{mg}{k}$$