CHAPTER 2
Differentiation

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CHAPTER 2
Differentiation

Section 2.1 The Derivative and the Slope of a Graph

Skills Warm Up

1. $P(3, 1), Q(3, 6)$

$m = \dfrac{6 - 1}{3 - 3}$; m is undefined.

$x = 3$

2. $P(2, 2), Q = (-5, 2)$

$m = \dfrac{2 - 2}{-5 - 2} = 0$

$y - 2 = 0(x - 2)$

$y = 2$

3. $P(1, 5), Q(4, -1)$

$m = \dfrac{-1 - 5}{4 - 1} = \dfrac{-6}{3} = -2$

$y - 5 = -2(x - 1)$

$y - 5 = -2x + 2$

$y = -2x + 7$

4. $P(3, 5), Q(-1, -7)$

$m = \dfrac{-7 - 5}{-1 - 3} = \dfrac{-12}{-4} = 3$

$y - 5 = 3(x - 3)$

$y = 3x - 4$

5. $\displaystyle\lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} 2x + \Delta x$

$= 2x$

6. $\displaystyle\lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} \frac{\Delta x\left[3x^2 + 3x\Delta x + (\Delta x)^2\right]}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + (\Delta x)^2$

$= 3x^2$

7. $\displaystyle\lim_{\Delta x \to 0} \frac{1}{x(x + \Delta x)} = \frac{1}{x^2}$

8. $\displaystyle\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$

$\displaystyle= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$

$= 2x$

9. $f(x) = 3x$

Domain: $(-\infty, \infty)$

10. $f(x) = \dfrac{1}{x - 1}$

Domain: $(-\infty, 1) \cup (1, \infty)$

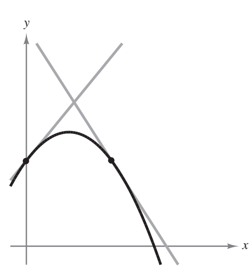
11. $f(x) = \frac{1}{5}x^3 - 2x^2 + \frac{1}{3}x - 1$

Domain: $(-\infty, \infty)$

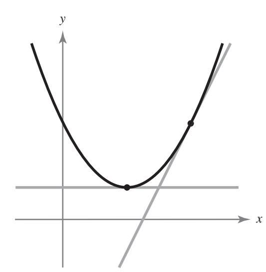
12. $f(x) = \dfrac{6x}{x^3 + x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

1.



2.



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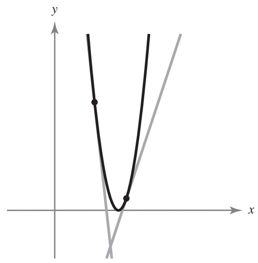

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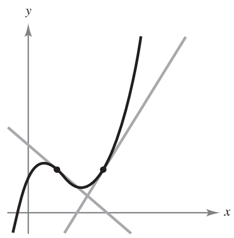

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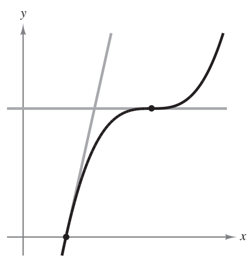
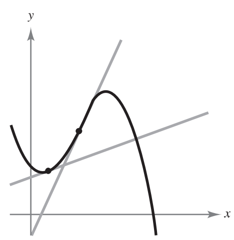


6.



7. The slope is $m = 1$.

8. The slope is $m = \frac{4}{3}$.

9. The slope is $m = 0$.

10. The slope is $m = \frac{1}{4}$.

11. The slope is $m = -\frac{1}{3}$.

12. The slope is $m = -3$.

13. 2009: $m \approx 118$

2011: $m \approx 375$

The slope is the rate of change in millions of dollars per year of revenue for the years 2009 and 2011 for Under Armour.

14. 2010: $m \approx 500$

2012: $m \approx 500$

The slope is the rate of change in millions of dollars per year of sales for the years 2010 and 2012 for Fossil.

15. $t = 3: m \approx 8$

$t = 7: m \approx 1$

$t = 10: m \approx -10$

The slope is the rate of change of the average temperature in degrees Fahrenheit per month in Bland, Virginia, for March, July, and October.

16. (a) At t_1, $f'(t_1) > g'(t_1)$, so the runner given by f is running faster.

(b) At t_2, $g'(t_2) > f'(t_2)$, so the runner given by g is running faster. The runner given by f has traveled farther.

(c) At t_3, the runners are at the same location, but the runner given by g is running faster.

(d) The runner given by g will finish first because that runner finishes the distance at a lesser value of t.

17. $f(x) = -1$ at $(0, -1)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(0 + \Delta x) - f(0)}{\Delta x} \\ &= \frac{-1 - (-1)}{\Delta x} \\ &= \frac{0}{\Delta x} \\ &= 0 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} 0 = 0$$

18. $f(x) = 6$ at $(-2, 6)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-2 + \Delta x) - f(-2)}{\Delta x} \\ &= \frac{6 - 6}{\Delta x} \\ &= \frac{0}{\Delta x} \\ &= 0 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} 0 = 0$$

19. $f(x) = 13 - 4x$ at $(3, 1)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{\left[13 - 4(3 + \Delta x)\right] - 1}{\Delta x} \\ &= \frac{13 - 12 - 4\Delta x - 1}{\Delta x} \\ &= \frac{-4\Delta x}{\Delta x} \\ &= -4 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} (-4) = -4$$

20. $f(x) = 6x + 3$ at $(1, 9)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(1 + \Delta x) - f(1)}{\Delta x} \\ &= \frac{\left[6(1 + \Delta x) + 3\right] - 9}{\Delta x} \\ &= \frac{6 + 6\Delta x + 3 - 9}{\Delta x} \\ &= \frac{6\Delta x}{\Delta x} \\ &= 6 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} 6 = 6$$

21. $f(x) = 2x^2 - 3$ at $(2, 5)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(2 + \Delta x) - f(2)}{\Delta x} \\ &= \frac{\left[2(2 + \Delta x)^2 - 3\right] - 5}{\Delta x} \\ &= \frac{\left[2\left(4 + 4\Delta x + (\Delta x)^2\right) - 3\right] - 5}{\Delta x} \\ &= \frac{\left[8 + 8\Delta x + 2(\Delta x)^2 - 3\right] - 5}{\Delta x} \\ &= \frac{2\Delta x(4 + \Delta x)}{\Delta x} \\ &= 2(4 + \Delta x) \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(2(4 + \Delta x)\right) = 8$$

22. $f(x) = 11 - x^2$ at $(3, 2)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{11 - (3 + \Delta x)^2 - \left[11 - (3)^2\right]}{\Delta x} \\ &= \frac{11 - \left(9 + 6\Delta x + \Delta x^2\right) - 2}{\Delta x} \\ &= \frac{11 - 9 - 6\Delta x + (\Delta x)^2 - 2}{\Delta x} \\ &= \frac{-6\Delta x - (\Delta x)^2}{\Delta x} \\ &= \frac{\Delta x(-6 - \Delta x)}{\Delta x} \\ &= -6 - \Delta x \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} (-6 - \Delta x) = -6$$

23. $f(x) = x^3 - 4x$ at $(-1, 3)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-1 + \Delta x) - f(-1)}{\Delta x} \\ &= \frac{(-1 + \Delta x)^3 - 4(-1 + \Delta x) - \left[(-1)^3 - 4(-1)\right]}{\Delta x} \\ &= \frac{-1 + 3\Delta x - 3(\Delta x)^2 + (\Delta x)^3 + 4 - 4\Delta x - 3}{\Delta x} \\ &= \frac{-\Delta x - 3(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x\left(-1 - 3\Delta x + (\Delta x)^2\right)}{\Delta x} \\ &= -1 - 3\Delta x + (\Delta x)^2 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(-1 - 3\Delta x + (\Delta x)^2\right) = -1$$

24. $f(x) = 7x - x^3$ at $(-3, 6)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-3 + \Delta x) - f(-3)}{\Delta x} \\ &= \frac{7(-3 + \Delta x) - (-3 + \Delta x)^3 - \left[7(-3) - (-3)^3\right]}{\Delta x} \\ &= \frac{-21 + 7\Delta x - \left(-27 + 27\Delta x - 9(\Delta x)^2 + (\Delta x)^3\right) - 6}{\Delta x} \\ &= \frac{-20\Delta x + 9(\Delta x)^2 - (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x\left(-20 + 9\Delta x - (\Delta x)^2\right)}{\Delta x} \\ &= -20 + 9\Delta x - (\Delta x)^2 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(-20 + 9\Delta x - (\Delta x)^2\right) = -20$$

25. $f(x) = 2\sqrt{x}$ at $(4, 4)$

$$\begin{aligned}
m_{\text{sec}} &= \frac{f(4 + \Delta x) - f(4)}{\Delta x} \\
&= \frac{2\sqrt{4 + \Delta x} - 2\sqrt{4}}{\Delta x} \\
&= \frac{2\sqrt{4 + \Delta x} - 4}{\Delta x} \cdot \frac{2\sqrt{4 + \Delta x} + 4}{2\sqrt{4 + \Delta x} + 4} \\
&= \frac{4(4 + \Delta x) - 16}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\
&= \frac{16 + 4\Delta x - 16}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\
&= \frac{4\Delta x}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\
&= \frac{4}{2\sqrt{4 + \Delta x} + 4}
\end{aligned}$$

$$\begin{aligned}
m = \lim_{\Delta x \to 0} m_{\text{sec}} &= \lim_{\Delta x \to 0} \frac{4}{2\sqrt{4 + \Delta x + 4}} \\
&= \frac{4}{2\sqrt{4} + 4} = \frac{1}{2}
\end{aligned}$$

26. $f(x) = \sqrt{x + 1}$ at $(8, 3)$

$$\begin{aligned}
m_{\text{sec}} &= \frac{f(8 + \Delta x) - f(8)}{\Delta x} \\
&= \frac{\sqrt{8 + \Delta x + 1} - \sqrt{8 + 1}}{\Delta x} \\
&= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \\
&= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \cdot \frac{\sqrt{9 + \Delta x} + 3}{\sqrt{9 + \Delta x} + 3} \\
&= \frac{9 + \Delta x - 9}{\Delta x\left(\sqrt{9 + \Delta x} + 3\right)} \\
&= \frac{\Delta x}{\Delta x\left(\sqrt{9 + \Delta x} + 3\right)} \\
&= \frac{1}{\sqrt{9 + \Delta x} + 3}
\end{aligned}$$

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} m_{\text{sec}} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{9 + \Delta x} + 3} \\
&= \frac{1}{\sqrt{9} + 3} \\
&= \frac{1}{6}
\end{aligned}$$

27. $f(x) = 3$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3 - 3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{0}{\Delta x} \\
&= \lim_{\Delta x \to 0} 0 \\
&= 0
\end{aligned}$$

28. $f(x) = -2$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-2 - (-2)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{0}{\Delta x} \\
&= \lim_{\Delta x \to 0} 0 \\
&= 0
\end{aligned}$$

29. $f(x) = -5x$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-5(x + \Delta x) - (-5x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-5x - 5\Delta x + 5x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} -5 \\
&= -5
\end{aligned}$$

30. $f(x) = 4x + 1$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{4(x + \Delta x) + 1 - (4x + 1)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{4x + 4\Delta x + 1 - 4x - 1}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{4\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} 4 \\
&= 4
\end{aligned}$$

31. $g(s) = \frac{1}{3}s + 2$

$$g'(s) = \lim_{\Delta s \to 0} \frac{g(s + \Delta s) - g(s)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{1}{3}(s + \Delta s) + 2 - \left(\frac{1}{3}s + 2\right)}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{1}{3}s + \frac{1}{3}\Delta s + 2 - \frac{1}{3}s - 2}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{\frac{1}{3}\Delta s}{\Delta s}$$
$$= \lim_{\Delta s \to 0} \frac{1}{3}$$
$$= \frac{1}{3}$$

32. $h(t) = 6 - \frac{1}{2}t$

$$h'(t) = \lim_{\Delta t \to 0} \frac{h(t + \Delta t) - h(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{6 - \frac{1}{2}(t + \Delta t) - \left(6 - \frac{1}{2}t\right)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{6 - \frac{1}{2}t - \frac{1}{2}\Delta t - 6 + \frac{1}{2}t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{-\frac{1}{2}\Delta t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} -\frac{1}{2}$$
$$= -\frac{1}{2}$$

33. $f(x) = 4x^2 - 5x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[4(x + \Delta x)^2 - 5(x + \Delta x)\right] - \left(4x^2 - 5x\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 5x - 5\Delta x\right] - 4x^2 + 5x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{4\Delta x\left(2x + \Delta x - \frac{5}{4}\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 4\left(2x + \Delta x - \frac{5}{4}\right) = 8x - 5$$

34. $f(x) = 2x^2 + 7x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[2(x + \Delta x)^2 + 7(x + \Delta x)\right] - \left(2x^2 + 7x\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[2\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + 7x + 7\Delta x\right] - 2x^2 - 7x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 + 7x + 7\Delta x - 2x^2 - 7x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2\Delta x\left(2x + \Delta x + \frac{7}{2}\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 2\left(2x + \Delta x + \frac{7}{2}\right) = 4x + 7$$

35. $h(t) = \sqrt{t - 3}$

$$
\begin{aligned}
h'(t) &= \lim_{\Delta t \to 0} \frac{h(t + \Delta t) - h(t)}{t} \\
&= \lim_{\Delta t \to 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t} \cdot \frac{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}} \\
&= \lim_{\Delta t \to 0} \frac{t + \Delta t - 3 - (t - 3)}{\Delta t\left(\sqrt{t + \Delta t - 3} + \sqrt{t - 3}\right)} \\
&= \lim_{\Delta t \to 0} \frac{\Delta t}{\Delta t\left(\sqrt{t + \Delta t - 3} + \sqrt{t - 3}\right)} \\
&= \lim_{\Delta t \to 0} \frac{1}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}} \\
&= \frac{1}{2\sqrt{t - 13}} \\
&= \frac{\sqrt{t - 3}}{2(t - 3)}
\end{aligned}
$$

36. $f(x) = \sqrt{x + 2}$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}} \\
&= \lim_{\Delta x \to 0} \frac{x + \Delta x + 2 - (x + 2)}{\Delta x\left(\sqrt{x + \Delta x + 2} + \sqrt{x + 2}\right)} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x + 2} + \sqrt{x + 2}\right)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}} \\
&= \frac{1}{2\sqrt{x + 2}}
\end{aligned}
$$

37. $f(t) = t^3 - 12t$

$$\begin{aligned}
f'(t) &= \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^3 - 12(t + \Delta t) - (t^3 - 12t)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12t - 12\Delta t - t^3 + 12t}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12\Delta t}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\Delta t\left(3t^2 + 3t\Delta t + (\Delta t)^2 - 12\right)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 - 12\right) \\
&= 3t^2 - 12
\end{aligned}$$

38. $f(t) = t^3 + t^2$

$$\begin{aligned}
f'(t) &= \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^3 + (t + \Delta t)^2 - (t^3 + t^2)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + t^2 + 2t\Delta t + (\Delta t)^2 - t^3 - t^2}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 2t\Delta t + (\Delta t)^2}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\Delta t\left(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t\right)}{\Delta t} \\
&= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t\right) \\
&= 3t^2 + 2t
\end{aligned}$$

39. $f(x) = \dfrac{1}{x + 2}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x + 2} - \dfrac{1}{x + 2}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x + 2} \cdot \dfrac{x + 2}{x + 2} - \dfrac{1}{x + 2} \cdot \dfrac{x + \Delta x + 2}{x + \Delta x + 2}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{x + 2 - (x + \Delta x + 2)}{(x + \Delta x + 2)(x + 2)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x + 2)(x + 2)} \\
&= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x + 2)(x + 2)} \\
&= -\frac{1}{(x + 2)^2}
\end{aligned}$$

40. $g(s) = \dfrac{1}{s-4}$

$$g'(s) = \lim_{\Delta s \to 0} \frac{g(s+\Delta s) - g(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{1}{s+\Delta s-4} - \dfrac{1}{s-4}}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{s-4-(s+\Delta s-4)}{(s+\Delta s-4)(s-4)}}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{s-4-(s+\Delta s-4)}{(s+\Delta s-4)(s-4)}}{(s+\Delta s-4)(s-4)} \cdot \frac{1}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{-\Delta s}{\Delta s(s+\Delta s-4)(s-4)}$$

$$= \lim_{\Delta s \to 0} \frac{-1}{(s+\Delta s-4)(s-4)}$$

$$= -\frac{1}{(s-4)^2}$$

41. $f(x) = \frac{1}{2}x^2$ at $(2, 2)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x+\Delta x)^2 - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(x+\Delta x)}{\Delta x}$$

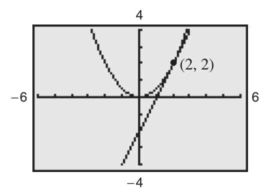
$$= \lim_{\Delta x \to 0} (x+\Delta x)$$

$$= x$$

$m = f'(2) = 2$

$y - 2 = 2(x-2)$

$y = 2x - 2$



42. $f(x) = -\frac{1}{8}x^2$ at $(-4, -2)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}(x+\Delta x)^2 - \left(-\frac{1}{8}x^2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}x^2 - \frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2 + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right)$$

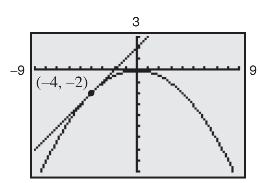
$$= -\frac{1}{4}x$$

$m = f'(-4) = -\frac{1}{4}(-4) = 1$

$y - (-2) = 1[x - (-4)]$

$y + 2 = x + 4$

$y = x + 2$

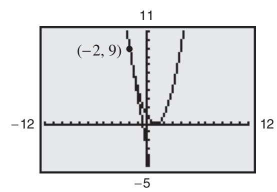


43. $f(x) = (x - 1)^2$ at $(-2, 9)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1)^2 - (x - 1)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x - 2x + (\Delta x)^2 - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x - 2)$$
$$= 2x - 2$$

$$m = f'(-2) = 2(-2) - 2 = -6$$

$$y - 9 = -6[x - (-2)]$$
$$y = -6x - 3$$

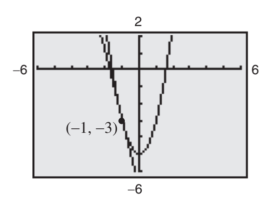


44. $f(x) = 2x^2 - 5$ at $(-1, -3)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2(x + \Delta x)^2 - 5 - (2x^2 - 5)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 5 - 2x^2 + 5}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (4x + 2\Delta x)$$
$$= 4x$$

$$m = f'(-1) = 4(-1) = -4$$

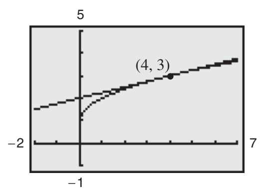
$$y - (-3) = -4(x - (-1))$$
$$y + 3 = -4x - 4$$
$$y = -4x - 7$$



45. $f(x) = \sqrt{x} + 1$ at $(4, 3)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} + 1 - \left(\sqrt{x} + 1\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

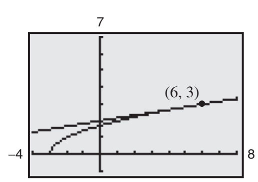
$$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$
$$y - 3 = \frac{1}{4}(x - 4)$$
$$y = \frac{1}{4}x + 2$$



46. $f(x) = \sqrt{x + 3}$ at $(6, 3)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}$$
$$= \frac{1}{2\sqrt{x + 3}}$$
$$= \frac{\sqrt{x + 3}}{2(x + 3)}$$

$$m = f'(6) = \frac{1}{2\sqrt{6 + 3}} = \frac{1}{6}$$
$$y - 3 = \frac{1}{6}(x - 6)$$
$$y - 3 = \frac{1}{6}x - 1$$
$$y = \frac{1}{6}x + 2$$

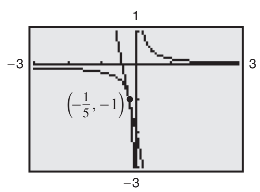


47. $f(x) = \dfrac{1}{5x}$ at $\left(-\dfrac{1}{5}, -1\right)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{5(x + \Delta x)} - \dfrac{1}{5x}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{x - (x + \Delta x)}{5x(x + \Delta x)}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x - (x + \Delta x)}{5x(x + \Delta x)} \cdot \frac{1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{5x \cdot \Delta x \cdot (x + \Delta x)}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{5x(x + \Delta x)}$$
$$= -\frac{1}{5x^2}$$

$$m = f'\left(-\frac{1}{5}\right) = -\frac{1}{5\left(-\frac{1}{5}\right)^2} = -\frac{1}{5\left(\frac{1}{25}\right)} = -5$$

$$y - (-1) = -5\left(x - \left(-\frac{1}{5}\right)\right)$$
$$y + 1 = -5\left(x + \frac{1}{5}\right)$$
$$y + 1 = -5x - 1$$
$$y = -5x - 2$$

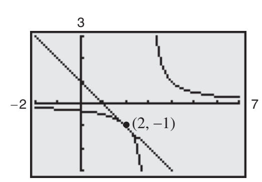


48. $f(x) = \dfrac{1}{x - 3}$ at $(2, -1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 3} - \dfrac{1}{x - 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x - 3} \cdot \dfrac{x - 3}{x - 3} - \dfrac{1}{x - 3} \cdot \dfrac{x + \Delta x - 3}{x + \Delta x - 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{x - 3 - (x + \Delta x - 3)}{(x + \Delta x - 3)(x - 3)}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x + \Delta x - 3)(x - 3)\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 3)(x - 3)} = -\frac{1}{(x - 3)^2}$$

$$m = f'(2) = -\frac{1}{(2 - 3)^2} = -1$$

$$y - (-1) = -1(x - 2)$$
$$y + 1 = -x + 2$$
$$y = -x + 1$$



49. $f(x) = -\frac{1}{4}x^2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}(x + \Delta x)^2 - \left(-\frac{1}{4}x^2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}x^2 - \frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2 + \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x\left(-\frac{1}{2}x - \frac{1}{4}\Delta x\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0}\left(-\frac{1}{2}x - \frac{1}{4}\Delta x\right) \\ &= -\frac{1}{2}x \end{aligned}$$

Since the slope of the given line is -1,

$$\begin{aligned} -\tfrac{1}{2}x &= -1 \\ x &= 2 \text{ and } f(2) = -1. \end{aligned}$$

At the point $(2, -1)$, the tangent line parallel to $x + y = 0$ is $y - (-1) = -1(x - 2)$

$$y = -x + 1.$$

50. $f(x) = x^2 - 7$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 - 7\right] - (x^2 - 7)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7 - x^2 + 7}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x \end{aligned}$$

Since the slope of the given line is -2,

$$\begin{aligned} 2x &= -2 \\ x &= -1 \text{ and } f(-1) = -6. \end{aligned}$$

At the point $(-1, -6)$, the tangent line parallel to $2x + y = 0$ is

$$\begin{aligned} y - (-6) &= -2(x - (-1)) \\ y &= -2x - 8. \end{aligned}$$

51. $f(x) = -\frac{1}{3}x^3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}(x + \Delta x)^3 - \left(-\frac{1}{3}x^3\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}\left(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right) + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}x^3 - x^2\Delta x - x(\Delta x)^2 - \frac{1}{3}(\Delta x)^3 + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x\left(-x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left(-x^2 - x\Delta x - \tfrac{1}{3}(\Delta x)^2\right) = -x^2 \end{aligned}$$

Since the slope of the given line is -9,

$$\begin{aligned} -x^2 &= -9 \\ x^2 &= 9 \\ x &= \pm 3 \text{ and } f(3) = -9 \text{ and } f(-3) = 9. \end{aligned}$$

At the point $(3, -9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$\begin{aligned} y - (-9) &= -9(x - 3) \\ y &= -9x + 18. \end{aligned}$$

At the point $(-3, 9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$\begin{aligned} y - 9 &= -9(x - (-3)) \\ y &= -9x - 18. \end{aligned}$$

52. $f(x) = x^3 + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 + 2 - (x^3 + 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2 - x^3 - 2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x\Delta x + (\Delta x)^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2\right)$$
$$= 3x^2$$

The slope of the given line is

$$3x - y - 4 = 0$$
$$y = 3x - 4$$
$$m = 3.$$

$$3x^2 = 3$$
$$x^2 = 1$$
$$x = \pm 1$$
$$x = 1 \text{ and } f(1) = 3$$
$$x = -1 \text{ and } f(-1) = 1$$

At the point $(1, 3)$, the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 3 = 3(x - 1)$$
$$y - 3 = 3x - 3$$
$$y = 3x.$$

At the point $(-1, 1)$, the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 1 = 3(x - (-1))$$
$$y - 1 = 3(x + 1)$$
$$y - 1 = 3x + 3$$
$$y = 3x + 4.$$

53. y is differentiable for all $x \neq -3$.

At $(-3, 0)$, the graph has a node.

54. y is differentiable for all $x \neq \pm 3$.

At $(\pm 3, 0)$, the graph has a cusp.

55. y is differentiable for all $x \neq -\frac{1}{2}$.

At $\left(-\frac{1}{2}, 0\right)$, the graph has a vertical tangent line.

56. y is differentiable for all $x > 1$.

The derivative does not exist at endpoints.

57. y is differentiable for all $x \neq \pm 2$.

The function is not defined at $x = \pm 2$.

58. y is differentiable for all $x \neq 0$.

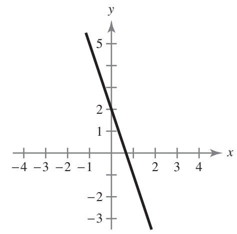
The function is discontinuous at $x = 0$.

59. Since $f'(x) = -3$ for all x, f is a line of the form

$f(x) = -3x + b.$

$f(0) = 2$, so $2 = (-3)(0) + b$, or $b = 2$.

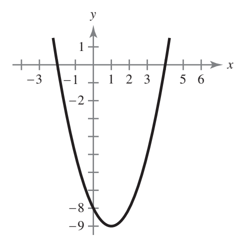
Thus, $f(x) = -3x + 2$.



60. Sample answer: Since $f(-2) = f(4) = 0$, $(x + 2)(x - 4) = 0$.

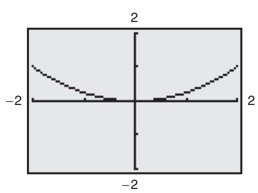
A function with these zeros is $f(x) = x^2 - 2x - 8$.

$$\begin{aligned}
\text{Then } f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 - 2(x + \Delta x) - 8\right] - (x^2 - 2x - 8)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - 8 - x^2 + 2x + 8}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} 2x + \Delta x - 2 \\
&= 2x - 2.
\end{aligned}$$



So $f'(1) = 2(1) - 2 = 0$. Sketching $f(x)$ shows that $f'(x) < 0$ for $x < 1$ and $f'(0) > 0$ for $x > 1$.

61.

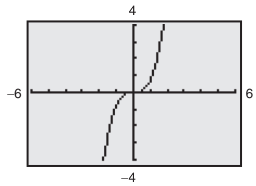


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	1	0.5625	0.25	0.0625	0	0.0625	0.25	0.5625	1
$f'(x)$	–1	–0.75	–0.5	–0.25	0	0.25	0.5	0.75	1

Analytically, the slope of $f(x) = \frac{1}{4}x^2$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}(x + \Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}\left[x^2 + 2x(\Delta x)^2 + (\Delta x)^2\right] - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}x^2 + \frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x\left(\frac{1}{2}x\Delta x + \frac{1}{4}\Delta x\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(\frac{1}{2}x + \frac{1}{4}\Delta x\right) \\
&= \frac{1}{2}x.
\end{aligned}$$

62.

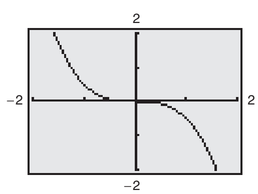


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-2.53	-0.75	-0.1	0	0.1	0.75	2.53	6
$f'(x)$	9	5.0625	2.25	0.5625	0	0.5625	2.25	5.0625	9

Analytically, the slope of $f(x) = \frac{3}{4}x^3$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}(x + \Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}\left(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right) - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}x^3 + \frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}(\Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}x(\Delta x)^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x\left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2\right) \\
&= \frac{9}{4}x^2.
\end{aligned}$$

63.

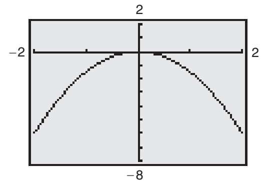


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	4	1.6875	0.5	0.0625	0	-0.0625	-0.5	-1.6875	-4
$f'(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6

Analytically, the slope of $f(x) = -\frac{1}{2}x^3$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}(x + \Delta x)^3 + \frac{1}{2}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}\left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right] + \frac{1}{2}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}\left[3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} -\frac{1}{2}\left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] \\
&= -\frac{3}{2}x^2.
\end{aligned}$$

64.

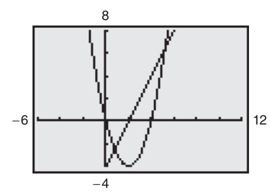


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6
$f'(x)$	6	4.5	3	1.5	0	-1.5	-3	-4.5	-6

Analytically, the slope of $f(x) = -\frac{3}{2}x^2$ is

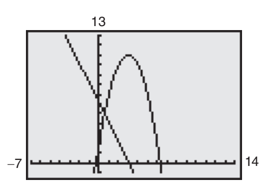
$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}(x + \Delta x)^2 - \left(-\frac{3}{2}x^2\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}\left[x^2 + 2x\Delta x + (\Delta x)^2\right] + \frac{3}{2}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}\left[2x\Delta x + (\Delta x)^2\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} -\frac{3}{2}(2x + \Delta x) \\
&= -3x.
\end{aligned}$$

65.
$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 4(x + \Delta x) - (x^2 - 4x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} (2x + \Delta x - 4) \\
&= 2x - 4
\end{aligned}$$



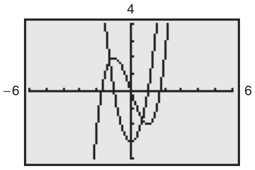
The x-intercept of the derivative indicates a point of horizontal tangency for f.

66.
$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 + 6(x + \Delta x) - (x + \Delta x)^2 - (2 + 6x - x^2)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{6\Delta x - 2x\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (6 - 2x - \Delta x) = 6 - 2x
\end{aligned}$$



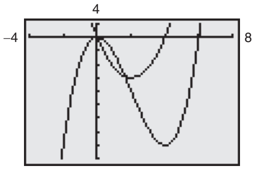
The x-intercept of the derivative indicates a point of horizontal tangency for f.

67. $$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{(x+\Delta x)^3 - 3(x+\Delta x) - (x^3 - 3x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3x - 3\Delta x - x^3 + 3x}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3\Delta x}{\Delta x} \\ &= \lim_{\Delta x\to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 3\right) \\ &= 3x^2 - 3 \end{aligned}$$



The x-intercepts of the derivative indicate points of horizontal tangency for f.

68. $$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{(x+\Delta x)^3 - 6(x+\Delta x)^2 - (x^3 - 6x^2)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{x^3 - 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 6\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - x^3 + 6x^2}{\Delta x} \\ &= \lim_{\Delta x\to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 12x - 6\Delta x\right) \\ &= 3x^2 - 12x \end{aligned}$$



The x-intercepts of the derivative indicate points of horizontal tangency for f.

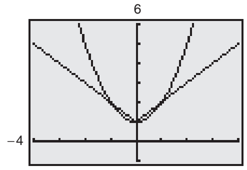
69. True. The slope of the graph is given by $f'(x) = 2x$, which is different for each different x value.

70. False. $f(x) = |x|$ is continuous, but not differentiable at $x = 0$.

71. True. See page 122.

72. True. See page 115.

73. The graph of $f(x) = x^2 + 1$ is smooth at (0, 1), but the graph of $g(x) = |x| + 1$ has a node at (0, 1). The function g is not differentiable at (0, 1).



Section 2.2 Some Rules for Differentiation

Skills Warm Up

1. (a) $2x^2,\ x = 2$

$2(2^2) = 2(4) = 8$

(b) $(5x)^2,\ x = 2$

$[5(2)]^2 = 10^2 = 100$

(c) $6x^{-2},\ x = 2$

$6(2)^{-2} = 6\left(\frac{1}{4}\right) = \frac{3}{2}$

2. (a) $\dfrac{1}{(3x)^2},\ x = 2$

$\dfrac{1}{[3(2)]^2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

(b) $\dfrac{1}{4x^3},\ x = 2$

$\dfrac{1}{4(2^3)} = \dfrac{1}{4(8)} = \dfrac{1}{32}$

(c) $\dfrac{(2x)^{-3}}{4x^{-2}},\ x = 2$

$\dfrac{[2(2)]^{-3}}{4(2)^{-2}} = \dfrac{4^{-3}}{4(2)^{-2}} = \dfrac{2^2}{4(4^3)} = \dfrac{1}{64}$

3. $4(3)x^3 + 2(2)x = 12x^3 + 4x = 4x(3x^2 + 1)$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2} = \frac{3}{2}x^2 - \frac{3}{2}\sqrt{x} = \frac{3}{2}x^{1/2}(x^{3/2} - 1)$

5. $\left(\dfrac{1}{4}\right)x^{-3/4} = \dfrac{1}{4x^{3/4}}$

6. $\dfrac{1}{3}(3)x^2 - 2\left(\dfrac{1}{2}\right)x^{-1/2} + \dfrac{1}{3}x^{-2/3} = x^2 - x^{-1/2} + \dfrac{1}{3}x^{-2/3}$

$= x^2 - \dfrac{1}{\sqrt{x}} + \dfrac{1}{3\sqrt[3]{x^2}}$

7. $3x^2 + 2x = 0$

$x(3x + 2) = 0$

$x = 0$

$3x + 2 = 0 \rightarrow x = -\frac{2}{3}$

8. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0$

$x + 1 = 0 \rightarrow x = -1$

$x - 1 = 0 \rightarrow x = 1$

9. $x^2 + 8x - 20 = 0$

$(x + 10)(x - 2) = 0$

$x + 10 = 0 \rightarrow x = -10$

$x - 2 = 0 \rightarrow x = 2$

10. $3x^2 - 10x + 8 = 0$

$(3x - 4)(x - 2) = 0$

$3x - 4 = 0 \rightarrow x = \frac{4}{3}$

$x - 2 = 0 \rightarrow x = 2$

---

1. $y = 3$

$y' = 0$

2. $f(x) = -8$

$f'(x) = 0$

3. $y = x^5$

$y' = 5x^4$

4. $f(x) = \dfrac{1}{x^6} = x^{-6}$

$f'(x) = -6x^{-7} = -\dfrac{6}{x^7}$

5. $h(x) = 3x^3$

$h'(x) = 9x^2$

6. $h(x) = 6x^5$

$h'(x) = 30x^4$

7. $y = \dfrac{5x^4}{3}$

$y' = \dfrac{20}{6}x^3 = \dfrac{10}{3}x^3$

8. $g(t) = \dfrac{3t^2}{4}$

$g'(t) = \dfrac{3}{2}t$

9. $f(x) = 4x$

$f'(x) = 4$

10. $g(x) = \frac{x}{3} = \frac{1}{3}x$

$g'(x) = \frac{1}{3}$

11. $y = 8 - x^3$

$y = -3x^2$

12. $y = t^2 - 6$

$y' = 2t$

13. $f(x) = 4x^2 - 3x$

$f'(x) = 8x - 3$

14. $g(x) = 3x^2 + 5x^3$

$g'(x) = 6x + 15x^2 = 15x^2 + 6x$

15. $f(t) = -3t^2 + 2t - 4$

$f'(t) = -6t + 2$

16. $y = 7x^3 - 9x^2 + 8$

$y' = 21x^2 - 18x$

17. $s(t) = 4t^4 - 2t + t + 3$

$s'(t) = 16t^2 - 4t + 1$

18. $y = 2x^3 - x^2 + 3x - 1$

$y' = 6x^2 - 2x + 3$

19. $g(x) = x^{2/3}$

$g'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

20. $h(x) = x^{5/2}$

$h'(x) = \frac{5}{2}x^{3/2}$

21. $y = 4t^{4/3}$

$y' = 4\left(\frac{4}{3}\right)t^{1/3} = \frac{16}{3}t^{1/3}$

22. $f(x) = 10x^{1/6}$

$f'(x) = \frac{5}{3}x^{-5/6} = \frac{5}{3x^{5/6}} = \frac{5}{3\sqrt[6]{x^5}}$

23. $y = 4x^{-2} + 2x^2$

$y' = -8x^{-3} + 4x^1 = -\frac{8}{x^3} + 4x$

24. $s(t) = 8t^{-4} + t$

$s'(t) = 8\left(-4t^{-6}\right) + 1 = -\frac{32}{t^6} + 1$

	Function	*Rewrite*	*Differentiate*	*Simplify*
25.	$y = \frac{2}{7x^4}$	$y = \frac{2}{7}x^{-4}$	$y' = \frac{-8}{7}x^{-5}$	$y' = -\frac{8}{7x^5}$
26.	$y = \frac{2}{3x^2}$	$y = \frac{2}{3}x^{-2}$	$y' = -\frac{4}{3}x^{-3}$	$y' = -\frac{4}{3x^3}$
27.	$y = \frac{1}{(4x)^3}$	$y = \frac{1}{64}x^{-3}$	$y' = -\frac{3}{64}x^{-4}$	$y' = -\frac{3}{64x^4}$
28.	$y = \frac{\pi}{(2x)^6}$	$y = \frac{\pi}{64}x^{-6}$	$y' = -\frac{6\pi}{64}x^{-7}$	$y' = -\frac{3\pi}{32x^7}$
29.	$y = \frac{4}{(2x)^{-5}}$	$y = 128x^5$	$y' = 128(5)x^4$	$y' = 640x^4$
30.	$y = \frac{4x}{x^{-3}}$	$y = 4x^4$	$y' = 4(4)x^3$	$y' = 16x^3$
31.	$y = 6\sqrt{x}$	$y = 6x^{1/2}$	$y' = 6\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{\sqrt{x}}$
32.	$y = \frac{3\sqrt{x}}{4}$	$y = \frac{3}{4}x^{1/2}$	$y' = \frac{3}{4}\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{8\sqrt{x}}$

	Function	*Rewrite*	*Differentiate*	*Simplify*
33.	$y = \dfrac{1}{7\sqrt[6]{x}}$	$y = \dfrac{1}{7}x^{-1/6}$	$y' = \dfrac{1}{7}\left(-\dfrac{1}{6}\right)x^{-7/6}$	$y' = -\dfrac{1}{42\sqrt[6]{x^7}}$
34.	$y = \dfrac{3}{2\sqrt[4]{x^3}}$	$y = \dfrac{3}{2}x^{-3/4}$	$y' = \dfrac{3}{2}\left(-\dfrac{3}{4}\right)x^{-7/4}$	$y' = -\dfrac{9}{8\sqrt[4]{x^7}}$
35.	$y = \sqrt[5]{8x}$	$y = (8x)^{1/5} = 8^{1/5}x^{1/5}$	$y' = 8^{1/5}\left(\dfrac{1}{5}x^{-4/5}\right)$	$y' = 8^{1/5}\dfrac{1}{5x^{4/5}} = \dfrac{\sqrt[5]{8}}{5\sqrt[5]{x^4}} = \dfrac{\sqrt[5]{8}}{5x}$
36.	$y = \sqrt[3]{6x^2}$	$y = \sqrt[3]{6}(x)^{2/3}$	$y' = \sqrt[3]{6}\left(\dfrac{2}{3}\right)x^{-1/3}$	$y' = \dfrac{2\sqrt[3]{6}}{3\sqrt[3]{x}}$

37. $y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2}$

At the point $(1, 1), y' = \frac{3}{2}(1)^{1/2} = \frac{3}{2} = m.$

38. $y = x^{-1}$

$y' = x^{-2} = -\dfrac{1}{x^2}$

At the point $\left(\dfrac{3}{4}, \dfrac{4}{3}\right), y' = -\dfrac{1}{\left(\frac{3}{4}\right)^2} = -\dfrac{16}{9} = m.$

39. $f(t) = t^{-4}$

$f'(t) = -4t^{-5} = -\dfrac{4}{t^5}$

At the point

$\left(\dfrac{1}{2}, 16\right), f'\left(\dfrac{1}{2}\right) = -\dfrac{4}{\left(\dfrac{1}{2}\right)^5} = -\dfrac{4}{\dfrac{1}{32}} = -128 = m.$

40. $f(x) = x^{-1/3}$

$f'(x) = -\dfrac{1}{3}x^{-4/3} = -\dfrac{1}{3x^{4/3}}$

At the point $\left(8, \dfrac{1}{2}\right), f'(8) = -\dfrac{1}{3(8)^{4/3}} = -\dfrac{1}{48} = m.$

41. $f(x) = 2x^3 + 8x^2 - x - 4$

$f'(x) = 6x^2 + 16x - 1$

At the point

$(-1, 3), f'(-1) = 6(-1)^2 + 16(-1) - 1 = -11 = m.$

42. $f(x) = x^4 - 2x^3 + 5x^2 - 7x$

$f'(x) = 4x^3 - 6x^2 + 10x - 7$

At the point

$(-1, 15), f'(-1) = 4(-1)^3 - 6(-1)^2 + 10(-1) - 7$

$= -4 - 6 - 10 - 7 = -27 = m.$

43. $f(x) = -\frac{1}{2}x(1 + x^2) = -\frac{1}{2}x - \frac{1}{2}x^3$

$f'(x) = -\frac{1}{2} - \frac{3}{2}x^2$

$f'(1) = -\frac{1}{2} - \frac{3}{2} = -2$

44. $f(x) = 3(5 - x)^2 = 75 - 30x + 3x^2$

$f'(x) = -30 + 6x$

$f'(5) = -30 + (6)(5) = 0$

45. (a) $y = -2x^4 + 5x^2 - 3$

$y' = -8x^3 + 10x$

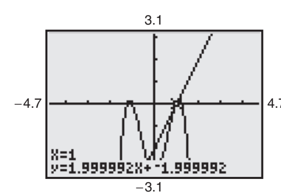
$m = y'(1) = -8 + 10 = 2$

The equation of the tangent line is

$y - 0 = 2(x - 1)$

$y = 2x - 2.$

(b) and (c)



46. (a) $y = x^3 + x + 4$

$y' = 3x^2 + 1$

$m = y'(-2) = 3(-2)^2 + 1 = 13$

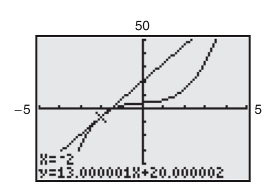
The equation of the tangent line is

$y - (-6) = 13[x - (-2)]$

$y + 6 = 13x + 26$

$y = 13x + 20.$

(b) and (c)



47. (a) $f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$

$$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5} = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$$

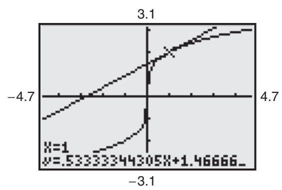
$$m = f'(1) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

The equation of the tangent line is

$$y - 2 = \frac{8}{15}(x - 1)$$

$$y = \frac{8}{15}x + \frac{22}{15}.$$

(b) and (c)



48. (a) $f(x) = \dfrac{1}{\sqrt[3]{x^2}} - x = x^{-2/3} - x$

$$f'(x) = -\frac{2}{3}x^{-5/3} - 1$$

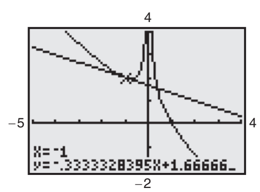
$$m = f'(-1) = \frac{2}{3} - 1 = -\frac{1}{3}$$

The equation of the tangent line is

$$y - 2 = -\tfrac{1}{3}(x + 1)$$

$$y = -\tfrac{1}{3}x + \tfrac{5}{3}.$$

(b) and (c)



49. (a) $y = 3x\left(x^2 - \dfrac{2}{x}\right)$

$$y = 3x^3 - 6$$

$$y' = 9x^2$$

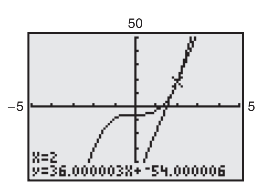
$$m = y' = 9(2)^2 = 36$$

The equation of the tangent line is

$$y - 18 = 36(x - 2)$$

$$y = 36x - 54.$$

(b) and (c)



50. (a) $y = (2x + 1)^2$

$$y = 4x^2 + 4x + 1$$

$$y' = 8x + 4$$

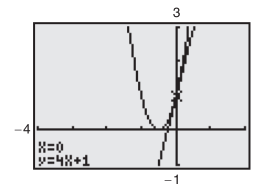
$$m = y' = 8(0) + 4 = 4$$

The equation of the tangent line is

$$y - 1 = 4(x - 0)$$

$$y = 4x + 1.$$

(b) and (c)



51. $f(x) = x^2 - 4x^{-1} - 3x^{-2}$

$$f'(x) = 2x + 4x^{-2} + 6x^{-3} = 2x + \frac{4}{x^2} + \frac{6}{x^3}$$

52. $f(x) = 6x^2 - 5x^{-2} + 7x^{-3}$

$$f'(x) = 12x + 10x^{-3} - 21x^{-4} = 12x + \frac{10}{x^3} - \frac{21}{x^4}$$

53. $f(x) = x^2 - 2x - \dfrac{2}{x^4} = x^2 - 2x - 2x^{-4}$

$$f'(x) = 2x - 2 + 8x^{-5} = 2x - 2 + \frac{8}{x^5}$$

54. $f(x) = x^2 + 4x + \dfrac{1}{x} = x^2 + 4x + x^{-1}$

$$f'(x) = 2x + 4 - x^{-2} = 2x + 4 - \frac{1}{x^2}$$

55. $f(x) = x^{4/5} + x$

$$f'(x) = \frac{4}{5}x^{-1/5} + 1 = \frac{4}{5x^{1/5}} + 1$$

56. $f(x) = x^{1/3} - 1$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

57. $f(x) = x(x^2 + 1) = x^3 + x$

$$f'(x) = 3x^2 + 1$$

58. $f(x) = (x^2 + 2x)(x + 1) = x^3 + 3x^2 + 2x$

$$f'(x) = 3x^2 + 6x + 2$$

59. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2} = 2x - 4 + 3x^{-2}$

$f'(x) = 2 - 6x^{-3} = 2 - \dfrac{6}{x^3} = \dfrac{2x^3 - 6}{x^3} = \dfrac{2(x^3 - 3)}{x^3}$

60. $f(x) = \dfrac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$f'(x) = 2 - x^{-2} = 2 - \dfrac{1}{x^2} = \dfrac{2x^2 - 1}{x^2}$

61. $f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2} = 4x - 3 + 2x^{-1} + 5x^{-2}$

$f'(x) = 4 - 2x^{-2} - 10x^{-3} = 4 - \dfrac{2}{x^2} - \dfrac{10}{x^3} = \dfrac{4x^3 - 2x - 10}{x^3}$

62. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x} = -6x^2 + 3x - 2 + x^{-1}$

$f'(x) = -12x + 3 - x^{-2} = -12x + 3 - \dfrac{1}{x^2}$

63. $y = x^4 - 2x + 3$

$y' = 4x^3 - 4x = 4x(x^2 - 1) = 0$ when $x = 0, \pm 1$

If $x = \pm 1$, then $y = (\pm 1)^4 - 2(\pm 1)^2 + 3 = 2.$

The function has horizontal tangent lines at the points $(0, 3)$, $(1, 2)$, and $(-1, 2)$.

64. $y = x^3 + 3x^2$

$y' = 3x^2 + 6x = 3x(x + 2) = 0$ when $x = 0, -2.$

The function has horizontal tangent lines at the points $(0, 0)$ and $(-2, 4)$.

65. $y = \frac{1}{2}x^2 + 5x$

$y' = x + 5 = 0$ when $x = -5.$

The function has a horizontal tangent line at the point $\left(-5, -\frac{25}{2}\right)$.

66. $y = x^2 + 2x$

$y' = 2x + 2 = 0$ when $x = -1.$

The function has a horizontal tangent line at the point $(-1, -1)$.

67. $y = x^2 + 3$

$y' = 2x$

Set $y' = 4.$

$2x = 4$

$x = 2$

If $x = 2$, $y = (2)^2 + 3 = 7 \rightarrow (2, 7).$

The graph of $y = x^2 + 3$ has a tangent line with slope $m = 4$ at the point $(2, 7)$.

68. $y = x^2 + 2x$

$y' = 2x + 2$

Set $y' = 10.$

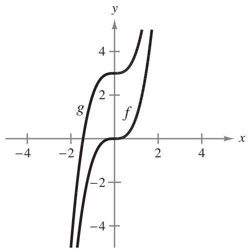
$2x + 2 = 10$

$x = 4$

If $x = 4$, $y = (4)^2 + 2(4) = 24 \rightarrow (4, 24).$

The graph of $y = x^2 + 2x$ has a tangent line with slope $m = 10$ at the point $(4, 24)$.

69. (a)



(b) $f'(x) = g'(x) = 3x^2$

$f'(1) = g'(1) = 3$

(c) Tangent line to f at $x = 1$:

$f(1) = 1$

$y - 1 = 3(x - 1)$

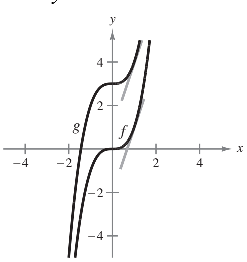
$y = 3x - 2$

Tangent line to g at $x = 1$:

$g(1) = 4$

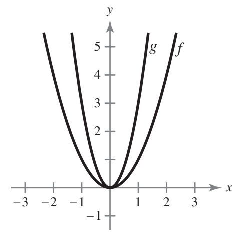
$y - 4 = 3(x - 1)$

$y = 3x + 1$



(d) f' and g' are the same.

70. (a)



(b) $f'(x) = 2x$

$f'(1) = 2$

$g'(x) = 6x$

$g'(1) = 6$

(c) Tangent line to f at $x = 1$:

$f(1) = 1$

$y - 1 = 2(x - 1)$

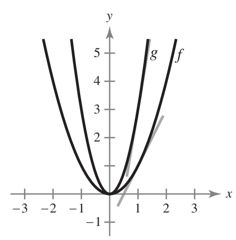
$y = 2x - 1$

Tangent line to g at $x = 1$:

$g(1) = 3$

$y - 3 = 6(x - 1)$

$y = 6x - 3$



(d) g' is 3 times f'.

71. If $g(x) = f(x) + 6$, then $g'(x) = f'(x)$ because the derivative of a constant is 0, $g'(x) = f'(x)$.

72. If $g(x) = 2f(x)$, then $g'(x) = 2f'(x)$ because of the Constant Multiple Rule.

73. If $g(x) = -5f(x)$, then $g'(x) = -5f'(x)$ because of the Constant Multiple Rule.

74. If $g(x) = 3f(x) - 1$, then $g'(x) = 3f'(x)$ because of the Constant Multiple Rule and the derivative of a constant is 0.

75. (a) $R = -4.1685t^3 + 175.037^2 - 1950.88t + 7265.3$

$R' = -12.5055t^2 + 350.074t - 1950.88$

2009: $R'(9) = -12.5055(9)^2 + 350.074(9) - 1950.88 \approx \$186.8$ million per year

2011: $R'(11) = -12.5055(11)^2 + 350.074(1) - 1950.88 \approx \$386.8$ million per year

(b) These results are close to the estimates in Exercise 13 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

76. (a) $R = -2.67538t^4 + 94.0568t^3 - 1155.203t^2 + 6002.42t - 9794.2$

$R' = -10.70152t^3 + 282.1704t^2 - 2310.406t + 6002.42$

2010: $R'(10) = -10.70152(10)^3 + 282.1704(10)^2 - 2310.406(10) + 6002 \approx \$413.88$ million per year

2012: $R'(12) = -10.70152(12)^3 + 282.1704(12)^2 - 2310.406(12) + 6002 \approx \$417.86$ million per year

(b) These results are close to the estimates in Exercise 14 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

77. (a) More men and women seem to suffer from migraines between 30 and 40 years old. More females than males suffer from migraines. Fewer people whose income is greater than or equal to \$30,000 suffer from migraines than people whose income is less than \$10,000.

(b) The derivatives are positive up to approximately 37 years old and negative after about 37 years of age. The percent of adults suffering from migraines increases up to about 37 years old, then decreases. The units of the derivative are percent of adults suffering from migraines per year.

78. (a) The attendance rate for football games, $g'(t)$, is greater at game 1.

(b) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 3.

(c) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 4. In addition, the attendance rate for football games is decreasing at game 4.

(d) At game 5, the attendance rate for football continues to increase, while the attendance rate for basketball continues to decrease.

79. $C = 7.75x + 500$

$C' = 7.75$, which equals the variable cost.

80. $C = 150x + 7000$

$P = R - C$

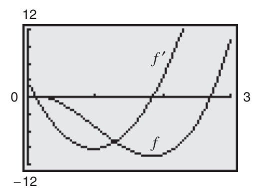
$P = 500x - (150x + 7000)$

$P = 350x - 7000$

$P' = 350$, which equals the profit on each dinner sold.

81. $f(x) = 4.1x^3 - 12x^2 + 2.5x$

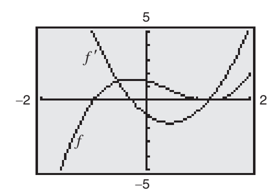
$f'(x) = 12.3x^2 - 24x + 2.5$



f has horizontal tangents at $(0.110, 0.135)$ and $(1.841, -10.486)$.

82. $f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$

$f'(x) = 3x^2 - 2.8x - 0.96$



f has horizontal tangents at $(1.2, 0)$ and $(-0.267, 1.577)$.

83. False. Let $f(x) = x$ and $g(x) = x + 1$.

Then $f'(x) = g'(x) = 1$, but $f(x) \neq g(x)$.

84. True. c is a constant.

Section 2.3 Rates of Change: Velocity and Marginals

Skills Warm Up

1. $\dfrac{-63-(-105)}{21-7} = \dfrac{42}{14}$
$= 3$

2. $\dfrac{-43-35}{6-(-7)} = \dfrac{-78}{13}$
$= -6$

3. $\dfrac{24-33}{9-6} = \dfrac{-9}{3} = -3$

4. $\dfrac{40-16}{18-8} = \dfrac{24}{10} = \dfrac{12}{5}$

5. $y = 4x^2 - 2x + 7$
$y' = 8x - 2$

6. $s = -2t^3 + 8t^2 - 7t$
$s' = -6t^2 + 16t - 7$

7. $s = -16t^2 + 24t + 30$
$s' = -32t + 24$

8. $y = -16x^2 + 54x + 70$
$y' = -32x + 54$

9. $A = \frac{1}{10}\left(-2r^3 + 3r^2 + 5r\right)$
$A' = \frac{1}{10}\left(-6r^2 + 6r + 5\right)$
$A' = -\frac{3}{5}r^2 + \frac{3}{5}r + \frac{1}{2}$

10. $y = \frac{1}{9}\left(6x^3 - 18x^2 + 63x - 15\right)$
$y' = \frac{1}{9}\left(18x^2 - 36x + 63\right)$
$y' = 2x^2 - 4x + 7$

11. $y = 12x - \dfrac{x^2}{5000}$
$y' = 12 - \dfrac{2x}{5000}$
$y' = 12 - \dfrac{x}{2500}$

12. $y = 138 + 74x - \dfrac{x^3}{10{,}000}$
$y' = 74 - \dfrac{3x^2}{10{,}000}$

1. (a) 1980–1986: $\dfrac{120-63}{6-0} = \$9.5$ billion/yr

(b) 1986–1992: $\dfrac{165-120}{12-6} = \$7.5$ billion/yr

(c) 1992–1998: $\dfrac{226-165}{18-12} \approx \$10.2$ billion/yr

(d) 1998–2004: $\dfrac{305-226}{24-18} \approx \$13.2$ billion/yr

(e) 2004–2010: $\dfrac{408-305}{30-24} \approx \$17.2$ billion/yr

(f) 1980–2012: $\dfrac{453-63}{32-0} \approx \$12.2$ billion/yr

(g) 1990–2012: $\dfrac{453-152}{32-10} \approx \$13.7$ billion/yr

(h) 2000–2012: $\dfrac{453-269}{32-20} \approx \$15.3$ billion/yr

2. (a) Imports:

1980–1990: $\dfrac{495 - 245}{10 - 0} = \$25$ billion/yr

(b) Exports:

1980–1990: $\dfrac{394 - 226}{10 - 0} = \$16.8$ billion/yr

(c) Imports:

1990–2000: $\dfrac{1218 - 495}{20 - 10} \approx \$72.3$ billion/yr

(d) Exports:

1990–2000: $\dfrac{782 - 394}{20 - 10} = \$38.8$ billion/yr

(e) Imports:

2000–2010: $\dfrac{1560 - 1218}{29 - 20} = \$38.0$ billion/yr

(f) Exports:

2000–2010: $\dfrac{1056 - 782}{29 - 20} = \$30.4$ billion/yr

(g) Imports:

1980–2013: $\dfrac{2268 - 245}{33 - 0} \approx \$61.3$ billion/yr

(h) Exports:

1980–2013: $\dfrac{1580 - 226}{33 - 0} \approx \$41.0$ billion/yr

3. $f(t) = 3t + 5; [1, 2]$

Average rate of change:

$$\frac{\Delta y}{\Delta t} = \frac{f(2) - f(1)}{2 - 1} = \frac{11 - 8}{1} = 3$$

$f'(t) = 3$

Instantaneous rates of change: $f'(1) = 3,\ f'(2) = 3$

4. $h(x) = 7 - 2x; [1, 3]$

Average rate of change:

$$\frac{\Delta h}{\Delta t} = \frac{h(3) - h(1)}{3 - 1} = \frac{1 - 5}{2} = -2$$

$h'(t) = -2$

Instantaneous rates of change: $h(1) = -2,\ h(3) = -2$

5. $h(x) = x^2 - 4x + 2; [-2, 2]$

Average rate of change:

$$\frac{\Delta h}{\Delta x} = \frac{h(2) - h(-2)}{2 - (-2)} = \frac{-2 - 14}{4} = -4$$

$h'(x) = 2x - 4$

Instantaneous rates of change: $h'(-2) = -8,\ h'(2) = 0$

6. $f(x) = -x^2 - 6x - 5; [-3, 1]$

Average rate of change:

$$\frac{\Delta f}{\Delta x} = \frac{f(1) - f(-3)}{1 - (-3)} = \frac{-12 - 4}{4} = -4$$

$f'(x) = -2x - 6$

Instantaneous rates of change: $f'(-3) = 0,\ f'(1) = -8$

7. $f(x) = 3x^{4/3}; [1, 8]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(1)}{8 - 1} = \frac{48 - 3}{7} = \frac{45}{7}$$

$f'(x) = 4x^{1/3}$

Instantaneous rates of change: $f'(1) = 4,\ f'(8) = 8$

8. $f(x) = x^{3/2}; [1, 4]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{8 - 1}{3} = \frac{7}{3}$$

$f'(x) = \frac{3}{2}x^{1/2}$

Instantaneous rates of change: $f'(1) = \frac{3}{2},\ f'(4) = 3$

9. $f(x) = \frac{1}{x}; [1, 5]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(1)}{5 - 1} = \frac{\frac{1}{5} - 1}{3} = \frac{-\frac{4}{5}}{4} = -\frac{1}{5}$$

$f'(x) = -\frac{1}{x^2}$

Instantaneous rates of change:

$f'(1) = -1,\ f'(5) = -\frac{1}{25}$

10. $f(x) = \frac{1}{\sqrt{x}}; [1, 9]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(9) - f(1)}{9 - 1} = \frac{\frac{1}{3} - 1}{8} = \frac{-\frac{2}{3}}{4} = -\frac{1}{6}$$

$f'(x) = \frac{1}{2x^{3/2}}$

Instantaneous rates of change:

$f'(1) = -\frac{1}{2},\ f'(9) = \frac{1}{54}$

11. $f(t) = t^4 - 2t^2; [-2, -1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(-1) - f(-2)}{-1 - (-2)} = \frac{-1 - 8}{1} = -9$$

$$f'(t) = 4t^3 - 4t$$

Instantaneous rates of change:

$$f'(-2) = -24, f'(-1) = 0$$

12. $g(x) = x^3 - 1; [-1, 1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{0 - (-2)}{2} = 1$$

$$g'(x) = 3x^2$$

Instantaneous rates of change:

$$g'(-1) = 3, g'(1) = 3$$

13. (a) $\approx \dfrac{0 - 1400}{3} \approx -467$

The number of visitors to the park is decreasing at an average rate of 467 people per month from September to December.

(b) Answers will vary. Sample answer: $[4, 11]$

Both the instantaneous rate of change at $t = 8$ and the average rate of change on $[4, 11]$ are about zero.

14. (a) $\dfrac{\Delta M}{\Delta t} = \dfrac{800 - 200}{3 - 1} = \dfrac{600}{2} = 300 \text{ mg/hr}$

(b) Answers will vary. Sample answer: $[2, 5]$

Both the instantaneous rate of change at $t = 4$ and the average rate of change on $[2, 5]$ is about zero.

15. $s = -16t^2 + 30t + 250$

Instantaneous: $v(t) = s'(t) = -32t + 30$

(a) Average: $[0, 1]$:

$$\frac{s(1) - s(0)}{1 - 0} = \frac{264 - 250}{1} = 14 \text{ ft/sec}$$

$$v(0) = s'(0) = 30 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

(b) Average: $[1, 2]$:

$$\frac{s(2) - s(1)}{2 - 1} = \frac{246 - 264}{1} = -18 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

(c) Average: $[2, 3]$:

$$\frac{s(3) - s(2)}{3 - 2} = \frac{196 - 246}{1} = -50 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

(d) Average: $[3, 4]$:

$$\frac{s(4) - s(3)}{4 - 3} = \frac{114 - 196}{1} = -82 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

$$v(4) = s'(4) = -98 \text{ ft/sec}$$

16. (a) $H'(v) = 33\left[10\left(\dfrac{1}{2}v^{-1/2}\right) - 1\right] = 33\left[\dfrac{5}{\sqrt{v}} - 1\right]$

Rate of change of heat loss with respect to wind speed.

(b)
$$\begin{aligned} H'(2) &= 33\left[\frac{5}{\sqrt{2}} - 1\right] \\ &\approx 83.673 \frac{\text{kcal/m}^2\text{/hr}}{\text{m/sec}} \\ &= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}} \\ &= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600} \\ &= 0.023 \text{ kcal/m}^3 \end{aligned}$$

$$\begin{aligned} H'(5) &= 33\left[\frac{5}{\sqrt{5}} - 1\right] \\ &\approx 40.790 \frac{\text{kcal/m}^2\text{/hr}}{\text{m/sec}} \\ &= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}} \\ &= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600} \\ &= 0.11 \text{ kcal/m}^3 \end{aligned}$$

17. $s = -16t^2 + 555$

(a)
$$\begin{aligned} \text{Average velocity} &= \frac{s(3) - s(2)}{3 - 2} \\ &= \frac{411 - 491}{1} \\ &= -80 \text{ ft/sec} \end{aligned}$$

(b) $v = s'(t) = -32t,\ v(2) = -64 \text{ ft/sec},$

$v(3) = -96 \text{ ft/sec}$

(c)
$$\begin{aligned} s &= -16t^2 + 555 = 0 \\ 16t^2 &= 555 \\ t^2 &= \frac{555}{16} \\ t &\approx 5.89 \text{ seconds} \end{aligned}$$

(d) $v(5.89) \approx -188.5 \text{ ft/sec}$

18. (a) $s(t) = -16t^2 - 18t + 210$

$v(t) = s'(t) = -32t - 18$

(b) $[1, 2]$: $\dfrac{s(2) - s(1)}{2 - 1} = \dfrac{110 - 176}{1} = -66$ ft/sec

(c) $v(1) = -50$ ft/sec

$v(2) = -82$ ft/sec

(d) Set $s(t) = 0$.

$-16t^2 - 18t + 210 = 0$

$$t = -\frac{(-18) \pm \sqrt{(-18)^2 - 4(-16)(210)}}{2(-16)} = \frac{18 \pm \sqrt{13{,}764}}{-32} \approx 3.10 \text{ sec}$$

(e) $v(3.10) = -117.2$ ft/sec

19. $C = 205{,}000 + 9800x$

$\dfrac{dC}{dx} = 9800$

20. $C = 150{,}000 + 7x^3$

$\dfrac{dC}{dx} = 21x^2$

21. $C = 55{,}000 + 470x - 0.25x^2,\ 0 \le x \le 940$

$\dfrac{dC}{dx} = 470 - 0.5x$

22. $C = 100\left(9 + 3\sqrt{x}\right)$

$\dfrac{dC}{dx} = 100\left[0 + 3\left(\dfrac{1}{2}x^{-1/2}\right)\right] = \dfrac{150}{\sqrt{x}}$

23. $R = 50x - 0.5x^2$

$\dfrac{dR}{dx} = 50 - x$

24. $R = 30x - x^2$

$\dfrac{dR}{dx} = 30 - 2x$

25. $R = -6x^3 + 8x^2 + 200x$

$\dfrac{dR}{dx} = -18x^2 + 16x + 200$

26. $R = 50\left(20x - x^{3/2}\right)$

$\dfrac{dR}{dx} = 50\left[20 - \dfrac{3}{2}x^{1/2}\right] = 1000 - 75\sqrt{x}$

27. $P = -2x^2 + 72x - 145$

$\dfrac{dP}{dx} = -4x + 72$

28. $P = -0.25x^2 + 2000x - 1{,}250{,}000$

$\dfrac{dP}{dx} = -0.5x + 2000$

29. $P = 0.0013x^3 + 12x$

$\dfrac{dP}{dx} = 0.0039x^2 + 12$

30. $P = -0.5x^3 + 30x^2 - 164.25x - 1000$

$\dfrac{dP}{dx} = -1.5x^2 + 60x - 164.25$

31. $C = 3.6\sqrt{x} + 500$

(a) $C'(x) = 1.8/\sqrt{x}$

$C'(9) = \$0.60$ per unit.

(b) $C(10) - C(9) \approx \$0.584$

(c) The answers are close.

32. $R = 2x\left(900 + 32x - x^2\right)$

(a) $R = 1800x + 64x^2 - 2x^3$

$R'(x) = 1800 + 128x - 6x^2$

$R'(14) = \$2416$

(b) $R(15) - R(14) = 2(15)\left[900 + 32(15) - 15^2\right] - 2(14)\left[900 + 32(14) - 14^2\right]$

$= 34{,}650 - 32{,}256 = \$2394$

(c) The answers are close.

33. $P = -0.04x^2 + 25x - 1500$

(a) $\dfrac{dP}{dx} = -0.08x + 25 = P'(x)$

$P'(150) = \$13$

(b) $\dfrac{\Delta P}{\Delta x} = \dfrac{P(151) - P(150)}{151 - 150} = \dfrac{1362.96 - 1350}{1} = \$12.96$

(c) The results are close.

34. $P = 36{,}000 + 2048\sqrt{x} - \dfrac{1}{8x^2}, 150 \le x \le 275$

$$\frac{dP}{dx} = 2048\left(\frac{1}{2}x^{-1/2}\right) - \frac{1}{8}\left(-2x^{-3}\right)$$
$$= \frac{1024}{\sqrt{x}} + \frac{1}{4x^3}$$

(a) When $x = 150$, $\dfrac{dP}{dx} \approx \$83.61$.

(b) When $x = 175$, $\dfrac{dP}{dx} \approx \$77.41$.

(c) When $x = 200$, $\dfrac{dP}{dx} \approx \$72.41$.

(d) When $x = 225$, $\dfrac{dP}{dx} \approx \$68.27$.

(e) When $x = 250$, $\dfrac{dP}{dx} \approx \$64.76$.

(f) When $x = 275$, $\dfrac{dP}{dx} \approx \$61.75$.

35. $P = 1.73t^2 + 190.6t + 16{,}994$

(a) $P(0) = 16{,}994$ thousand people

$P(3) = 17{,}581.37$ thousand people

$P(6) = 18{,}199.88$ thousand people

$P(9) = 18{,}849.53$ thousand people

$P(12) = 19{,}530.32$ thousand people

$P(15) = 20{,}242.25$ thousand people

$P(18) = 20{,}985.32$ thousand people

$P(21) = 21{,}759.53$ thousand people

The population is increasing from 1990 to 2011.

(b) $\dfrac{dP}{dt} = P'(t) = 3.46t + 190.6$

$\dfrac{dP}{dt}$ represents the population growth rate.

(c) $P'(0) = 190.6$ thousand people per year

$P'(3) = 200.98$ thousand people per year

$P'(6) = 211.36$ thousand people per year

$P'(9) = 221.74$ thousand people per year

$P'(12) = 232.12$ thousand people per year

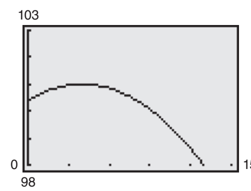
$P'(15) = 242.5$ thousand people per year

$P'(18) = 252.88$ thousand people per year

$P'(21) = 263.26$ thousand people per year

The rate of growth is increasing.

36. (a)



(b) For $t < 4$, the slopes are positive, and the fever is increasing. For $t > 4$, the slopes are negative, and the fever is decreasing.

(c) $T(0) = 100.4°F$

$T'(4) = 101°F$

$T(8) = 100.4°F$

$T(12) = 98.6°F$

(d) $\dfrac{dT}{dt} = -0.075t + 0.3$; the rate of change of temperature with respect to time

(e) $T'(0) = 0.3°F$ per hour

$T'(4) = 0°F$ per hour

$T'(8) = -0.3°F$ per hour

$T(12) = -0.6°F$ per hour

For $0 \le t < 4$, the rate of change of the temperature is positive; therefore, the temperature is increasing. For $4 < t \le 12$, the rate of change of the temperature is decreasing; therefore, the temperature is decreasing back to a normal temperature of $98.6°F$.

37. (a) $TR = -10Q^2 + 160Q$

(b) $(TR)' = MR = -20Q + 160$

(c)

Q	0	2	4	6	8	10
Model	160	120	80	40	0	−40
Table	–	130	90	50	10	−30

38. (a) $R = xp = x(5 - 0.001x) = 5x - 0.001x^2$

(b) $P = R - C = (5x - 0.001x^2) - (35 + 1.5x)$

$= -0.001x^2 + 3.5x - 35$

(c) $\dfrac{dR}{dx} = 5 - 0.002x$

$\dfrac{dP}{dx} = 3.5 - 0.002x$

x	600	1200	1800	2400	3000
dR/dx	3.8	2.6	1.4	0.2	−1.0
dP/dx	2.3	1.1	−0.1	−1.3	−2.5
P	1705	2725	3025	2605	1465

39. (a) $(400, 1.75), (500, 1.50)$

$$\text{Slope} = \frac{1.50 - 1.75}{500 - 400} = -0.0025$$

$$p - 1.75 = -0.0025(x - 400)$$

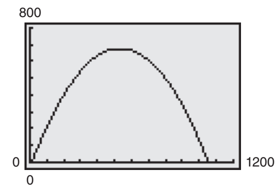
$$p = -0.0025x + 2.75$$

$$P = R - C = xp - c$$

$$= x(-0.0025x + 2.75) - (0.1x + 25)$$

$$= -0.0025x^2 + 2.65x - 25$$

(b)



At $x = 300$, P has a positive slope.

At $x = 530$, P has a 0 slope.

At $x = 700$, P has a negative slope.

(c) $P'(x) = -0.005x + 2.65$

$P'(300) = \$1.15$ per unit

$P'(530) = \$0$ per unit

$P'(700) = -\$.85$ per unit

40. $(36{,}000, 30), (32{,}000, 35)$

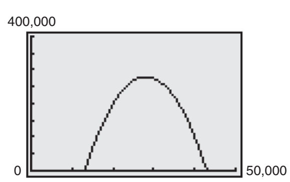
$$\text{Slope} = \frac{35 - 30}{32{,}000 - 36{,}000} = -\frac{5}{4000} = \frac{1}{800}$$

$$p - 30 = -\frac{1}{800}(x - 36{,}000)$$

$$p = -\frac{1}{800}x + 75 \text{ (demand function)}$$

(a) $P = R - C = xp - C = x\left(-\dfrac{1}{800}x + 75\right) - (5x + 700{,}000) = -\dfrac{1}{800}x^2 + 70x - 700{,}000$

(b)



At $x = 18{,}000$, P has a positive slope.

At $x = 28{,}000$, P has a 0 slope.

At $x = 36{,}000$, P has a negative slope.

(c) $P'(x) = -\dfrac{1}{400}x + 70$

$P'(18{,}000) = \$25$ per ticket

$P'(28{,}000) = \$0$ per ticket

$P'(36{,}000) = -\$20$ per ticket

41. (a) $C(x) = \left(\dfrac{15{,}000 \text{ mi}}{\text{yr}}\right)\left(\dfrac{1 \text{ gal}}{x \text{ mi}}\right)\left(\dfrac{2.60 \text{ dollars}}{1 \text{ gal}}\right)$

$C(x) = \dfrac{39{,}000}{x} \dfrac{\text{dollars}}{\text{yr}}$

(b) $\dfrac{dC}{dx} = -\dfrac{39{,}000}{x^2} \dfrac{\frac{\text{dollars}}{\text{yr}}}{\text{mpg}}$

The marginal cost is the change of savings for a 1-mile per gallon increase in fuel efficiency.

(c)

x	10	15	20	25	30	35	40
C	3900	2600	1950	1560	1300	1114.29	975
dC/dx	−390	−173.33	−97.5	−62.4	−43.33	−31.84	−24.38

(d) The driver who gets 15 miles per gallon would benefit more than the driver who gets 35 miles per gallon. The value of dC/dx is a greater savings for $x = 15$ than for $x = 35$.

42. (a) $f'(2.959)$ is the rate of change of the number of gallons of gasoline sold when the price is $\$2.959/\text{gallon}$.

(b) In general, it should be negative. Demand tends to decrease as price increases. Answers will vary.

43. (a) Average rate of change from 2000 to 2013: $\dfrac{\Delta p}{\Delta t} = \dfrac{16{,}576.66 - 10{,}786.85}{13 - 0} \approx \$445.37/\text{yr}$

(b) Average rate of change from 2003 to 2007: $\dfrac{\Delta p}{\Delta t} = \dfrac{13{,}264.82 - 10{,}453.92}{7 - 3} \approx \$702.73/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$702.73/\text{yr}$.

(c) Average rate of change from 2004 to 2006: $\dfrac{\Delta p}{\Delta t} = \dfrac{12{,}463.15 - 10{,}783.01}{6 - 4} \approx \$840.07/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$840.07/\text{yr}$.

(d) The average rate of change from 2004 to 2006 is a better estimate because the data is closer to the years in question.

44. Answers will vary. *Sample answer:*
The rate of growth in the lag phase is relatively slow when compared with the rapid growth in the acceleration phase. The population grows slower in the deceleration phase, and there is no growth at equilibrium. These changes could be explained by food supply or seasonal growth.

Section 2.4 The Product and Quotient Rules

Skills Warm Up

1. $$\begin{aligned}(x^2 + 1)(2) + (2x + 7)(2x) &= 2x^2 + 2 + 4x^2 + 14x \\ &= 6x^2 + 14x + 2 \\ &= 2(3x^2 + 7x + 1)\end{aligned}$$

2. $$\begin{aligned}(2x - x^3)(8x) + (4x^2)(2 - 3x^2) &= 16x^2 - 8x^4 + 8x^2 - 12x^4 \\ &= 24x^2 - 20x^4 \\ &= 4x^2(6 - 5x^2)\end{aligned}$$

3. $x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1) = 8x^2(x^2 + 2)^3(x^2 + 4)$

Skills Warm Up —*continued*—

4. $$x^2(2)(2x+1)(2) + (2x+1)^4(2x) = 4x^2(2x+1) + 2x(2x+1)^4$$
$$= 2x(2x+1)\left[2x + (2x+1)^3\right]$$

5. $$\frac{(2x+7)(5) - (5x+6)(2)}{(2x+7)^2} = \frac{10x + 35 - 10x - 12}{(2x+7)^2}$$
$$= \frac{23}{(2x+7)^2}$$

6. $$\frac{(x^2-4)(2x+1) - (x^2+x)(2x)}{(x^2-4)^2} = \frac{2x^3 + x^2 - 8x - 4 - 2x^3 - 2x^2}{(x^2-4)^2}$$
$$= \frac{-x^2 - 8x - 4}{(x^2-4)^2}$$

7. $$\frac{(x^2+1)(2) - (2x+1)(2x)}{(x^2+1)^2} = \frac{2x^2 + 2 - 4x^2 - 2x}{(x^2+1)^2}$$
$$= \frac{-2x^2 - 2x + 2}{(x^2+1)^2}$$
$$= \frac{-2(x^2 + x - 1)}{(x^2+1)^2}$$

8. $$\frac{(1-x^4)(4) - (4x-1)(-4x^3)}{(1-x^4)^2} = \frac{4 - 4x^4 + 16x^4 - 4x^3}{(1-x^4)^2}$$
$$= \frac{12x^4 - 4x^3 + 4}{(1-x^4)^2}$$
$$= \frac{4(3x^4 - x^3 + 1)}{(1-x^4)^2}$$

9. $$(x^{-1}+x)(2) + (2x-3)(-x^{-2}+1) = 2x^{-1} + 2x + (-2x^{-1} + 2x + 3x^{-2} - 3)$$
$$= 4x + 3x^{-2} - 3$$
$$= 4x + \frac{3}{x^2} - 3$$
$$= \frac{4x^3 - 3x^2 + 3}{x^2}$$

10. $$\frac{(1-x^{-1})(1) - (x-4)(x^{-2})}{(1-x^{-1})^2} = \left(\frac{1 - x^{-1} - x^{-1} + 4x^{-2}}{1 - 2x^{-1} + x^{-2}}\right)\left(\frac{x^2}{x^2}\right)$$
$$= \frac{x^2 - 2x + 4}{x^2 - 2x + 1}$$
$$= \frac{x^2 - 2x + 4}{(x-1)^2}$$

Skills Warm Up —*continued*—

11. $f(x) = 3x^2 - x + 4$

$f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$

$= 12 - 1$

$= 11$

12. $f(x) = -x^3 + x^2 + 8x$

$f'(x) = -3x^2 + 2x + 8$

$f'(2) = -3(2^2) + 2(2) + 8$

$= -3(4) + 4 + 8$

$= 0$

13. $f(x) = \dfrac{2}{7x} = \dfrac{2}{7}x^{-1}$

$f'(x) = -\dfrac{2}{7}x^{-2} = -\dfrac{2}{7x^2}$

$f'(2) = -\dfrac{2}{7(2)^2}$

$= -\dfrac{1}{14}$

14. $f(x) = x^2 - \dfrac{1}{x^2}$

$f'(x) = 2x + \dfrac{2}{x^3}$

$f'(2) = 2(2) + \dfrac{2}{2^3}$

$= 4 + \dfrac{2}{8}$

$= 4 + \dfrac{1}{4}$

$= \dfrac{17}{4}$

1. $f(x) = (2x - 3)(1 - 5x)$

$f'(x) = (2x - 3)(-5) + (1 - 5x)(2)$

$= -10x + 15 + 2 - 10x$

$= -20x + 17$

2. $g(x) = (4x - 7)(3x + 1)$

$g'(x) = (4x - 7)(3) + (3x + 1)(4)$

$= 12x - 21 + 12x + 4$

$= 24x - 17$

3. $f(x) = (6x - x^2)(4 + 3x)$

$f'(x) = (6x - x^2)(3) + (4 + 3x)(6 - 2x)$

$= 18x - 3x^2 + 24 - 8x + 18x - 6x^2$

$= -9x^2 + 28x + 24$

4. $f(x) = (5x - x^3)(2x + 9)$

$f'(x) = (5x - x^3)(2) + (2x + 9)(5 - 3x^2)$

$= 10x - 2x^3 + 10 - 6x^3 + 45 - 27x^2$

$= -8x^3 - 27x^2 + 20x + 45$

5. $f(x) = x(x^2 + 3)$

$f'(x) = x(2x) + (x^2 + 3)(1)$

$= 2x^2 + x^2 + 3$

$= 3x^2 + 3$

6. $f(x) = x^2(3x^3 - 1)$

$f'(x) = x^2(9x^2) + (3x^3 - 1)(2x)$

$= 9x^4 + 6x^4 - 2x$

$= 15x^4 - 2x$

7. $h(x) = \left(\dfrac{2}{x} - 3\right)(x^2 + 7) = (2x^{-1} - 3)(x^2 + 7)$

$h'(x) = (2x^{-1} - 3)(2x) + (x^2 + 7)(-2x^{-2})$

$= 4 - 6x - 2 - 14x^{-2}$

$= -6x + 2 - \dfrac{14}{x^2}$

8. $f(x) = (3 - x)\left(\dfrac{4}{x^2} - 5\right) = (3 - x)(4x^{-2} - 5)$

$f'(x) = (3 - x)(-8x^{-3}) + (4x^{-2} - 5)(-1)$

$= -24x^{-3} + 8x^{-2} - 4x^{-2} + 5$

$= -\dfrac{24}{x^3} + \dfrac{4}{x^2} + 5$

9. $g(x) = (x^2 - 4x + 3)(x - 2)$

$$\begin{aligned} g'(x) &= (x^2 - 4x + 3)(1) + (x - 2)(2x - 4) \\ &= x^2 - 4x + 3 + 2x^2 - 4x - 4x + 8 \\ &= 3x^2 - 12x + 11 \end{aligned}$$

10. $g(x) = (x^2 - 2x + 1)(x^3 - 1)$

$$\begin{aligned} g'(x) &= (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2) \\ &= 3x^4 - 6x^3 + 3x^2 + 2x^4 - 2x^3 - 2x + 2 \\ &= 5x^4 - 8x^3 + 3x^2 - 2x + 2 \end{aligned}$$

11. $h(x) = \dfrac{x}{x - 5}$

$$h'(x) = \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}$$

12. $h(x) = \dfrac{x^2}{x + 3}$

$$\begin{aligned} h'(x) &= \frac{(x + 3)(2x) - x^2(1)}{(x + 3)^2} \\ &= \frac{2x^2 + 6x - x^2}{(x + 3)^2} \\ &= \frac{x^2 + 6x}{(x + 3)^2} \end{aligned}$$

13. $f(t) = \dfrac{2t^2 - 3}{3t + 1}$

$$\begin{aligned} f'(t) &= \frac{(3t + 1)(4t) - (2t^2 - 3)(3)}{(3t + 1)^2} \\ &= \frac{12t^2 + 4t - 6t^2 + 9}{(3t + 1)^2} \\ &= \frac{6t^2 + 4t + 9}{(3t + 1)^2} \end{aligned}$$

14. $f(x) = \dfrac{7x + 3}{4x - 9}$

$$\begin{aligned} f'(x) &= \frac{(4x - 9)(7) - (7x + 3)(4)}{(4x - 9)^2} \\ &= \frac{28x - 63 - 28x - 12}{(x - 1)^2} \\ &= -\frac{75}{(4x - 9)^2} \end{aligned}$$

15. $f(t) = \dfrac{t + 6}{t^2 - 8}$

$$\begin{aligned} f'(t) &= \frac{(t^2 - 8)(1) - (t + 6)(2t)}{(t^2 - 8)^2} \\ &= \frac{t^2 - 8t - 2t^2 - 12t}{(t^2 - 8)^2} \\ &= \frac{-t^2 - 12t - 8}{(t^2 - 8)^2} \end{aligned}$$

16. $g(x) = \dfrac{4x - 5}{x^2 - 1}$

$$\begin{aligned} g'(x) &= \frac{(x^2 - 1)(4) - (4x - 5)(2x)}{(x^2 - 1)^2} \\ &= \frac{4x^2 - 4 - 8x^2 + 10x}{(x^2 - 1)^2} \\ &= \frac{-4x^2 + 10x - 4}{(x^2 - 1)^2} \end{aligned}$$

17. $f(x) = \dfrac{x^2 + 6x + 5}{2x - 1}$

$$\begin{aligned} f'(x) &= \frac{(2x - 1)(2x + 6) - (x^2 + 6x + 5)(2)}{(2x - 1)^2} \\ &= \frac{4x^2 + 12x - 2x - 6 - 2x^2 - 12x - 10}{(2x - 1)^2} \\ &= \frac{2x^2 - 2x - 16}{(2x - 1)^2} \end{aligned}$$

18. $f(x) = \dfrac{4x^2 - x + 2}{3 - 4x}$

$$\begin{aligned} f'(x) &= \frac{(3 - 4x)(8x - 1) - (4x^2 - x + 2)(-4)}{(3 - 4x)^2} \\ &= \frac{24x - 3 - 32x^2 + 4x + 16x^2 - 4x + 8}{(3 - 4x)^2} \\ &= \frac{-16x^2 + 24x + 5}{(3 - 4x)^2} \end{aligned}$$

19. $f(x) = \dfrac{6 + 2x^{-1}}{3x - 1}$

$$\begin{aligned} f'(x) &= \frac{(3x - 1)(-2x^{-2}) - (6 + 2x^{-1})(3)}{(3x - 1)^2} \\ &= \frac{-6x^{-1} + 2x^{-2} - 18 - 6x^{-1}}{(3x - 1)^2} \\ &= \frac{2x^{-2} - 12x^{-1} - 18}{(3x - 1)^2} \\ &= \frac{\frac{2}{x^2} - \frac{12}{x} - 18}{(3x - 1)^2} = \frac{2 - 12x - 18x^2}{x^2(3x - 1)^2} \end{aligned}$$

20. $f(x) = \dfrac{5 - x^{-2}}{x + 2}$

$$\begin{aligned} f'(x) &= \frac{(x + 2)(2x^{-3}) - (5 - x^{-2})(1)}{x + 2} \\ &= \frac{2x^{-2} + 4x^{-3} - 5 + x^{-2}}{(x + 2)^2} \\ &= \frac{4x^{-3} + 3x^{-2} - 5}{(x + 2)^2} \\ &= \frac{\frac{4}{x^3} + \frac{3}{x^2} - 5}{(x + 2)^2} \\ &= \frac{4 + 2x - 5x^3}{x^3(x + 2)^2} \end{aligned}$$

	Function	*Rewrite*	*Differentiate*	*Simplify*
21.	$f(x) = \dfrac{x^3 + 6x}{3}$	$f(x) = \dfrac{1}{3}x^3 + 2x$	$f'(x) = x^2 + 2$	$f'(x) = x^2 + 2$
22.	$f(x) = \dfrac{x^3 + 2x^2}{10}$	$f(x) = \dfrac{1}{10}x^3 + \dfrac{1}{5}x^2$	$f'(x) = \dfrac{3}{10}x^2 + \dfrac{2}{5}x$	$f'(x) = \dfrac{3}{10}x^2 + \dfrac{2}{5}x$
23.	$y = \dfrac{7x^2}{5}$	$y = \dfrac{7}{5}x^2$	$y' = \dfrac{7}{5} \cdot 2x$	$y' = \dfrac{14}{5}x$
24.	$y = \dfrac{2x^4}{9}$	$y = \dfrac{2}{9}x^4$	$y' = \dfrac{2}{9} \cdot 4x^3$	$y' = \dfrac{8}{9}x^3$
25.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
26.	$y = \dfrac{4}{5x^2}$	$y = \dfrac{4}{5}x^{-2}$	$y' = -\dfrac{8}{5}x^{-3}$	$y' = -\dfrac{8}{5x^3}$
27.	$y = \dfrac{4x^2 - 3x}{8\sqrt{x}}$	$y = \dfrac{1}{2}x^{3/2} - \dfrac{3}{8}x^{1/2},\ x \neq 0$	$y' = \dfrac{3}{4}x^{1/2} - \dfrac{3}{16}x^{-1/2}$	$y' = \dfrac{3}{4}\sqrt{x} - \dfrac{3}{16\sqrt{x}}$
28.	$y = \dfrac{5(3x^2 + 2x)}{6\sqrt[3]{x}}$	$y = \dfrac{5}{2}x^{5/3} + \dfrac{5}{3}x^{5/3},\ x \neq 0$	$y' = \dfrac{25}{6}x^{2/3} + \dfrac{10}{9}x^{-1/3},\ x \neq 0$	$y' = \dfrac{25}{6}\sqrt[3]{x^2} + \dfrac{10}{9\sqrt[3]{x}}$
29.	$y = \dfrac{x^2 - 4x + 3}{2(x - 1)}$	$y = \dfrac{1}{2}(x - 3),\ x \neq 1$	$y' = \dfrac{1}{2}(1),\ x \neq 1$	$y' = \dfrac{1}{2},\ x \neq 1$
30.	$y = \dfrac{x^2 - 4}{4(x + 2)}$	$y = \dfrac{1}{4}(x - 2),\ x \neq -2$	$y' = \dfrac{1}{4}(1),\ x \neq -2$	$y' = \dfrac{1}{4},\ x \neq -2$

31.
$$\begin{aligned} f'(x) &= (x^3 - 3x)(4x + 3) + (3x^2 - 3)(2x^2 + 3x + 5) \\ &= 4x^4 + 3x^3 - 12x^2 - 9x + 6x^4 + 9x^3 + 9x^2 - 9x - 15 \\ &= 10x^4 + 12x^3 - 3x^2 - 18x - 15 \end{aligned}$$

Product Rule and Simple Power Rule

32. $h'(t) = (t^5 - 1)(8t - 7) + (5t^4)(4t^2 - 7t - 3)$

$= 8t^6 - 7t^5 - 8t + 7 + 20t^6 - 35t^5 - 15t^4$

$= 28t^6 - 42t^5 - 15t^4 - 8t + 7$

Product Rule and Simple Power Rule

33. $h(t) = \frac{1}{3}(6t - 4)$

$h'(t) = \frac{1}{3}(6) = 2$

Constant Multiple and Simple Power Rules

34. $f(x) = \frac{1}{2}(3x - 8)$

$f'(x) = \frac{1}{2}(3) = \frac{3}{2}$

Constant Multiple and Simple Power Rules

35. $f'(x) = \dfrac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$

$= \dfrac{3x^4 - 3 - 2x^4 - 6x^2 - 4x}{(x^2 - 1)^2}$

$= \dfrac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$

Quotient Rule and Simple Power Rule

36. $f(x) = \dfrac{2x^3 - 4x^2 - 9}{x^3 - 5}$

$f'(x) = \dfrac{(x^3 - 5)(3x^2 - 8x) - (2x^3 - 4x^2 - 9)(3x^2)}{(x^3 - 5)^2}$

$= \dfrac{3x^5 - 8x^4 - 15x^2 + 40x - 6x^5 + 12x^4 - 27x^2}{(x^3 - 5)^2}$

$= \dfrac{-3x^5 + 4x^4 - 42x^2 + 40x}{(x^3 - 5)^2}$

Quotient Rule and Simple Power Rule

37. $f(x) = \dfrac{x^2 - x - 20}{x + 4} = \dfrac{(x - 5)(x + 4)}{(x + 4)} = x - 5, x \neq -4$

$f'(x) = 1$

Simple Power Rule

38. $h(t) = \dfrac{3t^2 + 22t + 7}{t + 7} = \dfrac{(3t + 1)(1 + 7)}{t + 7} = 3t + 1, t \neq -7$

$h'(t) = 3, t \neq -7$

Simple Power Rule

39. $g(t) = (2t^3 - 1)^2 = (2t^3 - 1)(2t^3 - 1)$

$g'(t) = (2t^3 - 1)(6t^2) + (2t^3 - 1)(6t^2)$

$= 12t^2(2t^3 - 1)$

Product Rule and Simple Power Rule

40. $f(x) = (4x^3 - 2x - 3)^2 = (4x^3 - 2x - 3)(4x^3 - 2x - 3)$

$f'(x) = (4x^3 - 2x - 3)(12x^2 - 2) + (4x^3 - 2x - 3)(12x^2 - 2)$

$= 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6 + 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6$

$= 96x^5 - 48x^3 - 72x^2 - 16x^3 + 8x + 12$

Product Rule and Simple Power Rule

41. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}} = \dfrac{s^2 - 2s + 5}{s^{1/2}}$

$$g'(s) = \frac{s^{1/2}(2s - 2) - (s^2 - 2s + 5)\left(\frac{1}{2}s^{-1/2}\right)}{s}$$
$$= \frac{2s^{3/2} - 2s^{1/2} - \frac{1}{2}s^{3/2} + s^{1/2} - \frac{5}{2}s^{-1/2}}{s}$$
$$= \frac{3}{2}s^{1/2} - s^{-1/2} - \frac{5}{2}s^{-3/2} = \frac{3s^2 - 2s - 5}{2s^{3/2}}$$

Quotient Rule and Simple Power Rule

42. $f(x) = \dfrac{x^3 - 5x^2 - 6x}{\sqrt{x}} = \dfrac{x^3 - 5x^2 - 6x}{x^{1/2}}$

$$= x^{5/2} - 5x^{3/2} - 6x^{1/2}$$
$$f'(x) = \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} - 3x^{-1/2}$$
$$= \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} - \frac{3}{x^{1/2}}$$
$$= \frac{5x^2 - 15x - 6}{2x^{1/2}}$$

Constant Multiple and Simple Power Rules

43. $f(x) = \dfrac{(x - 2)(3x + 1)}{4x + 2} = \dfrac{3x^2 - 5x - 2}{4x + 2}$

$$f'(x) = \frac{(4x + 2)(6x - 5) - (3x^2 - 5x - 2)(4)}{(4x + 2)^2}$$
$$= \frac{24x^2 - 8x - 10 - 12x^2 + 20x + 8}{(4x + 2)^2}$$
$$= \frac{12x^2 + 12x - 2}{4(2x + 1)^2}$$
$$= \frac{2(6x^2 + 6x - 1)}{2(2x + 1)^2}$$
$$= \frac{6x^2 + 6x - 1}{2(2x + 1)^2}$$

Quotient Rule and Simple Power Rule

44. $f(x) = \dfrac{(x + 1)(2x - 7)}{2x + 1} = \dfrac{2x^2 - 5x - 7}{2x + 1}$

$$f'(x) = \frac{(2x + 1)(4x - 5) - (2x^2 - 5x - 7)(2)}{(2x + 1)^2}$$
$$= \frac{8x^2 - 6x - 5 - 4x^2 + 10x + 14}{(2x + 1)^2}$$
$$= \frac{4x^2 + 4x + 9}{(2x + 1)^2}$$

Quotient Rule and Simple Power Rule

45. $f(x) = (x + 4)(2x + 9)(x - 3)$

$$= (2x^2 + 17x + 36)(x - 3)$$
$$f'(x) = (2x^2 + 17x + 36)(1) + (x - 3)(4x + 17)$$
$$= (2x^2 + 17x + 36) + (4x^2 + 5x - 51)$$
$$= 6x^2 + 22x - 15$$

Product Rule and Simple Power Rule

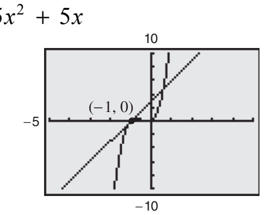
46. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

$$= (3x^3 + 4x)(x^2 - 4x - 5)$$
$$f'(x) = (3x^3 + 4x)(2x - 4) + (x^2 - 4x - 5)(9x^2 + 4)$$
$$= (6x^4 - 12x^3 + 8x^2 - 16x)$$
$$+ (9x^4 - 36x^3 - 41x^2 - 16x - 20)$$
$$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$$

Product Rule and Simple Power Rule

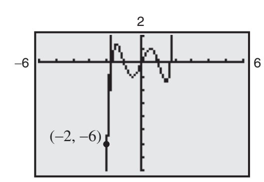
47. $f(x) = (5x + 2)(x^2 + x)$

$$f'(x) = (5x + 2)(2x + 1) + (x^2 + x)(5)$$
$$= 10x^2 + 9x + 2 + 5x^2 + 5x$$
$$= 15x^2 + 14x + 2$$
$$m = f'(-1) = 3$$
$$y - 0 = 3(x - (-1))$$
$$y = 3x + 3$$



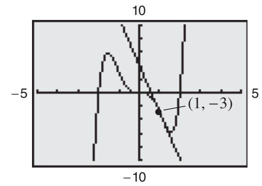
48. $f(x) = (x^2 - 1)(x^3 - 3x)$

$$f'(x) = (x^2 - 1)(3x^2 - 3) + (x^2 - 3x)(2x)$$
$$= 3x^4 - 6x^2 + 3 + 2x^4 - 6x^2$$
$$= 5x^4 - 12x^2 + 3$$
$$m = f'(-2) = 5(-2)^4 - 12(-2)^2 + 3 = 35$$
$$y - (-6) = 35(x - (-2))$$
$$y + 6 = 35x + 70$$
$$y = 35x + 64$$



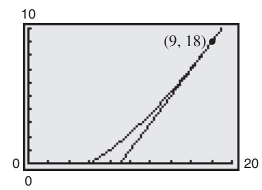
49. $f(x) = x^3(x^2 - 4)$

$$f'(x) = x^2(2x) + (x^2 - 4)(3x^2)$$
$$= 2x^4 + 3x^4 - 12x^2$$
$$= 5x^4 - 12x^2$$
$$m = f'(1) = -7$$
$$y - (-3) = -7(x - 1)$$
$$y = -7x + 4$$



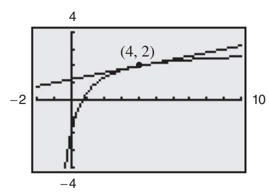
50. $f(x) = \sqrt{x}(x - 3) = x^{1/2}(x - 3)$

$$f'(x) = x^{1/2}(1) + (x - 3)\left(\tfrac{1}{2}x^{-1/2}\right)$$
$$= x^{1/2} + \tfrac{1}{2}x^{1/2} - \tfrac{3}{2}x^{-1/2}$$
$$= \tfrac{3}{2}x^{1/2} - \tfrac{3}{2}x^{-1/2}$$
$$m = f'(9) = \tfrac{3}{2}(9)^{1/2} - \tfrac{3}{2}(9)^{-1/2} = \tfrac{9}{2} - \tfrac{1}{2} = 4$$
$$y - 18 = 4(x - 9)$$
$$y = 4x - 18$$



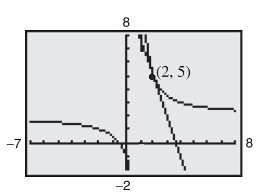
51. $f(x) = \dfrac{3x - 2}{x + 1}$

$$f'(x) = \frac{(x + 1)(3) - (3x - 2)(1)}{(x + 1)^2} = \frac{3x + 3 - 3x + 2}{(x + 1)^2} = \frac{5}{(x + 1)^2}$$
$$f'(4) = \frac{1}{5}$$
$$y - 2 = \frac{1}{5}(x - 4)$$
$$y - 2 = \frac{1}{5}x - \frac{4}{5}$$
$$y = \frac{1}{5}x + \frac{6}{5}$$



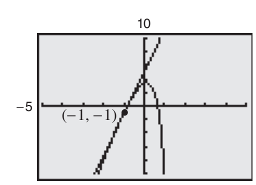
52. $f'(x) = \dfrac{(x - 1)2 - (2x + 1)}{(x - 1)^2} = \dfrac{-3}{(x - 1)^2}$

$$f'(2) = -3$$
$$y - 5 = -3(x - 2)$$
$$y = -3x + 11$$



53. $f(x) = \dfrac{(3x - 2)(6x + 5)}{2x - 3} = \dfrac{18x^2 + 3x - 10}{2x - 3}$

$$f'(x) = \frac{(2x - 3)(36x + 3) - (18x^2 + 3x - 10)(2)}{(2x - 3)^2}$$
$$= \frac{72x^2 - 102x - 9 - 36x^2 - 6x - 20}{(2x - 3)^2}$$
$$= \frac{36x^2 - 108x + 11}{(2x - 3)^2}$$
$$m = f'(-1) = \frac{36(-1)^2 - 108(-1) + 11}{(2(-1) - 3)^2} = \frac{31}{5}$$
$$y - (-1) = \frac{31}{5}(x - (-1))$$
$$y = \frac{31}{5}x + \frac{26}{5}$$



54. $f(x) = \dfrac{(x + 2)(x^2 + x)}{x - 4} = \dfrac{x^3 + 3x^2 + 2x}{x - 4}$

$$f'(x) = \frac{(x - 4)(3x^2 + 6x + 2) - (x^3 + 3x^2 + 2x)(1)}{(x - 4)^2}$$

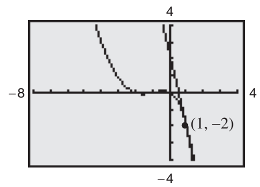
$$= \frac{3x^3 - 12x^2 + 6x^2 - 24x + 2x - 8 - x^3 - 3x^2 - 2x}{(x - 4)^2}$$

$$= \frac{2x^3 - 9x^2 - 24x - 8}{(x - 4)^2}$$

$$m = f'(1) = \frac{2(1)^3 - 9(1)^2 - 24(1) - 8}{(1 - 4)^2} = -\frac{13}{3}$$

$$y - (-2) = -\frac{13}{3}(x - 1)$$

$$y = \frac{13}{3}x + \frac{7}{3}$$



55. $f'(x) = \dfrac{(x - 1)(2x) - x^2(1)}{(x - 1)^2} = \dfrac{x^2 - 2x}{(x - 1)^2}$

$f'(x) = 0$ when $x^2 - 2x = x(x - 2) = 0$, which implies that $x = 0$ or $x = 2$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $(2, 4)$.

56. $f'(x) = \dfrac{(x^2 + 1)(2x) - (x^2)(2x)}{(x^2 + 1)^2} = \dfrac{2x}{(x^2 + 1)^2}$

$f'(x) = 0$ when $2x = 0$, which implies that $x = 0$. Thus, the horizontal tangent line occurs at $(0, 0)$.

57. $f'(x) = \dfrac{(x^3 + 1)(4x^3) - x^4(3x^2)}{(x^3 + 1)^2} = \dfrac{x^6 + 4x^3}{(x^3 + 1)^2}$

$f'(x) = 0$ when $x^6 + 4x^3 = x^3(x^3 + 4) = 0$, which implies that $x = 0$ or $x = \sqrt[3]{-4}$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $(\sqrt[3]{-4}, -2.117)$.

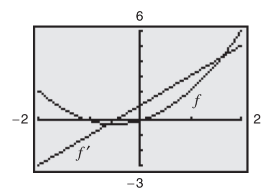
58. $f'(x) = \dfrac{(x^2 + 1)(4x^3) - (x^4 + 3)(2x)}{(x^2 + 1)^2}$

$$= \frac{2x(x^2 + 3)(x^2 - 1)}{(x^2 + 1)^2}$$

$f'(x) = 0$ when $2x(x^2 + 3)(x^2 - 1) = 0$, which implies that $x = 0$ or $x = \pm 1$. Thus, the horizontal tangent lines occur at (0, 3), (1, 2), and (−1, 2).

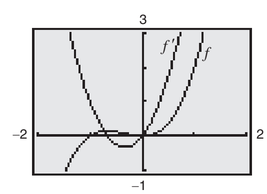
59. $f(x) = x(x + 1) = x^2 + x$

$f'(x) = 2x + 1$



60. $f(x) = x^2(x + 1) = x^3 + x^2$

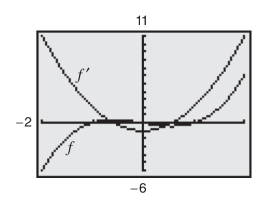
$f'(x) = 3x^2 + 2x = x(3x + 2)$



61. $f(x) = x(x + 1)(x - 1)$

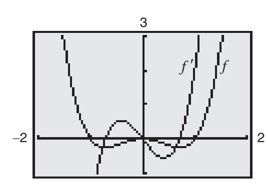
$= x^3 - x$

$f'(x) = 3x^2 - 1$



62. $f(x) = x^2(x + 1)(x - 1) = x^4 - x^2$

$f'(x) = 4x^3 - 2x = 2x(2x^2 - 1)$



63. $x = 275\left(1 - \dfrac{3p}{5p + 1}\right)$

$$\frac{dx}{dp} = -275\left[\frac{(5p + 1)3 - (3p)(5)}{(5p + 1)^2}\right] = -275\left[\frac{3}{(5p + 1)^2}\right]$$

When $p = 4$, $\dfrac{dx}{dp} = -275\left[\dfrac{3}{(21)^2}\right] \approx -1.87$ units per dollar.

64. $\dfrac{dx}{dp} = 0 - 1 - \dfrac{(p + 1)(2) - (2p)(1)}{(p + 1)^2}$

$$= -1 - \frac{2}{(p + 1)^2}$$

$$= \frac{-(p + 1)^2 - 2}{(p + 1)^2}$$

$$= \frac{-p^2 - 2p - 3}{(p + 1)^2}$$

When $p = 3$, $\dfrac{dx}{dp} = \dfrac{-9 - 6 - 3}{16} \approx -1.13$ units per dollar.

65. $P' = 500\left[\dfrac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\dfrac{200 - 4t^2}{(50 + t^2)^2}\right]$

When $t = 2$, $P' = 500\left[\dfrac{184}{(54)^2}\right] \approx 31.55$ bacteria/hour.

66. $\dfrac{dP}{dt} = \dfrac{50(t + 2)(1) - (t + 1750)(50)}{[50(t + 2)]^2}$

$$= \frac{50[(t + 2) - (t + 1750)]}{2500(t + 2)^2}$$

$$= \frac{-1748}{50(t + 2)^2}$$

$$= \frac{-874}{25(t + 2)^2}$$

(a) When $t = 1$, $\dfrac{dP}{dt} = \dfrac{-874}{225} \approx -3.88$ percent/day.

(b) When $t = 10$, $\dfrac{dP}{dt} = \dfrac{-874}{3600} = -\dfrac{437}{1800} \approx -0.24$ percent/day.

67. $P = \dfrac{t^2 - t + 1}{t^2 + 1}$

$$P' = \frac{(t^2 + 1)(2t - 1) - (t^2 - t + 1)(2t)}{(t^2 + 1)^2} = \frac{t^2 - 1}{(t^2 + 1)^2}$$

(a) $P'(0.5) = -0.480$/week

(b) $P'(2) = 0.120$/week

(c) $P'(8) = 0.015$/week

Each rate in parts (a), (b), and (c) is the rate at which the level of oxygen in the pond is changing at that particular time.

68. $T = 10\left(\dfrac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$

Initial temperature: $T(0) = 75°\text{F}$

$$T'(t) = 10\left(\frac{(t^2 + 4t + 10)(8t + 16) - (4t^2 + 16t + 75)(2t + 4)}{(t^2 + 4t + 10)^2}\right) = \frac{-700(t + 2)}{(t^2 + 4t + 10)^2}$$

(a) $T'(1) \approx -9.33°\text{F/hr}$

(b) $T'(3) \approx -3.64°\text{F/hr}$

(c) $T'(5) \approx -1.62°\text{F/hr}$

(d) $T'(10) \approx -0.37°\text{F/hr}$

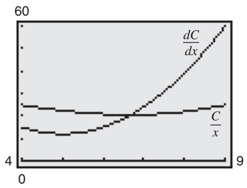
Each rate in parts (a), (b), (c), and (d) is the rate at which the temperature of the food in the refrigerator is changing at that particular time.

69. $C = x^3 - 15x^2 + 87x - 73,\ 4 \le x \le 9$

Marginal cost: $\dfrac{dC}{dx} = 3x^2 - 30x + 87$

Average cost: $\dfrac{C}{x} = x^2 - 15x + 87 - \dfrac{73}{x}$

(a)



(b) Point of intersection:

$$3x^2 - 30x + 87 = x^2 - 15x + 87 - \frac{73}{x}$$

$$2x^2 - 15x + \frac{73}{x} = 0$$

$$2x^3 - 15x^2 + 73 = 0$$

$$x \approx 6.683$$

When $x = 6.683,\ \dfrac{C}{x} = \dfrac{dC}{dx} \approx 20.50.$

Thus, the point of intersection is (6.683, 20.50). At this point average cost is at a minimum.

70. (a) As time passes, the percent of people aware of the product approaches approximately 95%.

(b) As time passes, the rate of change of the percent of people aware of the product approaches zero.

71. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ x \ge 1$

$$C' = 100\left[-2\left(200x^{-3}\right) + \frac{(x+30) - x}{(x+30)^2}\right]$$

$$= 100\left[-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right]$$

(a) $C'(10) = 100\left(-\dfrac{400}{10^3} + \dfrac{30}{40^2}\right) = -38.125$

(b) $C'(15) \approx -10.37$

(c) $C'(20) \approx -3.8$

Increasing the order size reduces the cost per item. An order size of 2000 should be chosen since the cost per item is the smallest at $x = 20$.

72. (a) $P = ax^2 + bx + c$

When $x = 10,\ P = 50$: $50 = 100a + 10b + c$.

When $x = 12,\ P = 60$: $60 = 144a + 12b + c$.

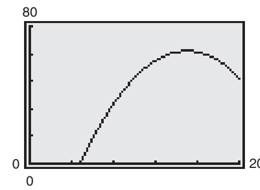
When $x = 14,\ P = 65$: $65 = 196a + 14b + c$.

Solving this system, we have

$a = -\frac{5}{8},\ b = \frac{75}{4}$, and $c = -75$.

Thus, $P = -\frac{5}{8}x^2 + \frac{75}{4}x - 75$.

(b)



(c) Marginal profit: $P' = -\frac{5}{4}x + \frac{75}{4} = 0 \Rightarrow x = 15$

This is the maximum point on the graph of P, so selling 15 units will maximize the profit.

73. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2(-2) + 4 = 0$

74. $f(x) = 3 - g(x)$

$f'(x) = -g'(x)$

$f'(2) = -(-2) = 2$

75. $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$$f'(2) = g(2)h'(2) + h(2)g'(2)$$
$$= (3)(4) + (-1)(-2)$$
$$= 14$$

76. $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$

$$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2} = -10$$

77. Answers will vary.

Chapter 2 Quiz Yourself

1. $f(x) = 5x + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[5(x + \Delta x) + 3\right] - (5x + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x + 3 - 5x - 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 5 = 5$$

At $(-2, -7)$: $m = 5$

2. $f(x) = \sqrt{x + 3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}$$
$$= \frac{1}{2\sqrt{x + 3}}$$

At $(1, 2)$: $m = \dfrac{1}{2\sqrt{1 + 3}} = \dfrac{1}{4}$

3. $f(x) = 3x - x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[3(x + \Delta x) - (x + \Delta x)^2\right] - \left(3x - x^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x - \left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) - \left(3x - x^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x - x^2 - 2x(\Delta x) - (\Delta x)^2 - 3x + x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x - 2x(\Delta x) - (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(3 - 2x - \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (3 - 2x - \Delta) = 3 - 2x$$

At $(4, -4)$: $m = 3 - 2(4) = 3 - 8 = -5$

4. $f(x) = 12$

$f'(x) = 0$

5. $f(x) = 19x + 9$

$f'(x) = 19$

6. $f(x) = x^4 - 3x^3 - 5x^2 + 8$

$f'(x) = 4x^3 - 9x^2 - 10x$

7. $f(x) = 12x^{1/4}$

$f'(x) = 3x^{-3/4} = \dfrac{3}{x^{3/4}}$

8. $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3} = -\dfrac{8}{x^3}$

9. $f(x) = 10x^{-1/5} + x^{-3}$

$f'(x) = -2x^{-6/5} - 3x^{-4} = -\dfrac{2}{x^{-6/5}} - \dfrac{3}{x^4}$

10. $f(x) = \dfrac{2x + 3}{3x + 2}$

$$f'(x) = \frac{(3x + 2)(2) - (2x + 3)(3)}{(3x + 2)^2} = \frac{6x + 4 - 6x - 9}{(3x + 2)^2} = -\frac{5}{(3x + 2)^2}$$

11. $f(x) = (x^2 + 1)(-2x + 4)$

$$f'(x) = (x^2 + 1)(-2) + (-2x + 4)(2x) = -6x^2 + 8x - 2$$

12. $f(x) = (x^2 + 3x + 4)(5x - 2)$

$$f'(x) = (x^2 + 3x + 4)(5) + (5x - 2)(2x + 3) = 5x^2 + 15x + 20 + 10x^2 + 11x - 6 = 15x^2 + 26x + 14$$

13. $f(x) = \dfrac{4x}{x^2 + 3}$

$$f'(x) = \frac{(x^2 + 3)(4) - 4x(2x)}{(x^2 + 3)^2} = \frac{4x^2 + 12 - 8x^2}{(x^2 + 3)^2} = \frac{-4x^2 + 12}{(x^2 + 3)^2} = \frac{-4(x^2 - 3)}{(x^2 + 3)^2}$$

14. $f(x) = x^2 - 3x + 1; [0, 3]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(3) - f(0)}{3 - 0} = \frac{1 - 1}{3} = 0$$

$f'(x) = 2x - 3$

Instantaneous rates of change: $f'(0) = -3,\ f'(3) = 3$

15. $f(x) = 2x^3 + x^3 - x + 4; [-1, 1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{6 - 4}{2} = 1$$

$f'(x) = 6x^2 + 2x - 1$

Instantaneous rates of change: $f'(-1) = 3,\ f'(1) = 7$

16. $f(x) = \dfrac{1}{3x}; [-5, -2]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(-2) - f(5)}{-2 - (-5)} = \frac{-\frac{1}{6} - \left(-\frac{1}{15}\right)}{3} = \frac{-\frac{3}{30}}{3} = -\frac{1}{30}$$

$f'(x) = -\dfrac{1}{3x^2}$

Instantaneous rates of change:

$f'(-2) = -\dfrac{1}{12},\ f'(-5) = -\dfrac{1}{75}$

17. $f(x) = \sqrt[3]{x}; [8, 27]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(27) - f(8)}{27 - 8} = \frac{3 - 2}{19} = \frac{1}{19}$$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

Instantaneous rates of change: $f'(8) = \dfrac{1}{12}$,

$f'(27) = \dfrac{1}{27}$

18. $P = -0.0125x^2 + 16x - 600$

(a) $\dfrac{dP}{dx} = -0.025x + 16$

When $x = 175$, $\dfrac{dP}{dx} = \$11.625$.

(b) $P(176) - P(175) = 1828.8 - 1817.1875 = \$11.6125$

(c) The results are approximately equal.

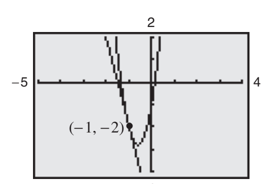
19. $f(x) = 5x^2 + 6x - 1$

$f'(x) = 10x + 6$

At $(-1, -2)$, $m = -4$.

$y + 2 = -4(x + 1)$

$y = -4x - 6$



20. $f(x) = \dfrac{8}{\sqrt{x^3}} = 8x^{-3/2}$

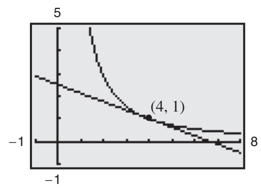
$f'(x) = -12x^{-5/2} = -\dfrac{12}{x^{5/2}} = -\dfrac{12}{x^2\sqrt{x}}$

$m = f'(4) = -\dfrac{12}{(4)^2\sqrt{4}} = -\dfrac{3}{8}$

$y - 1 = -\dfrac{3}{8}(x - 4)$

$y - 1 = -\dfrac{3}{8}x + \dfrac{3}{2}$

$y = -\dfrac{3}{8}x + \dfrac{5}{2}$



21. $f(x) = (x^2 + 1)(4x - 3)$

$f'(x) = (x^2 + 1)(4) + (4x - 3)(2x)$

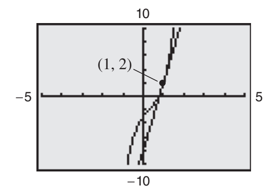
$= 4x^2 + 4 + 8x^2 - 6x$

$= 12x^2 - 6x + 4$

$m = f'(1) = 12(1)^2 - 6(1) + 4 = 10$

$y - 2 = 10(x - 1)$

$y = 10x - 8$



22. $f(x) = \dfrac{5x + 4}{2 - 3x}$

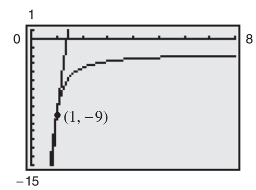
$f'(x) = \dfrac{(2 - 3x)(5) - (5x + 4)(-3)}{(2 - 3x)^2} = \dfrac{10 - 15x + 15x + 12}{(2 - 3x)^2} = \dfrac{22}{(2 - 3x)^2}$

$m = f'(1) = \dfrac{22}{(2 - 3(1))^2} = 22$

$y - (-9) = 22(x - 1)$

$y + 9 = 22x - 22$

$y = 22x - 31$



23. $S = -0.01722t^3 + 0.7333t^2 - 7.657t + 45.47,\ 7 \le t \le 13$

(a) $\dfrac{dS}{dt} = S'(t) = -0.051666t^2 + 1.4666t - 7.657$

(b) 2008: $S'(8) \approx \$0.77/\text{yr}$

2011: $S'(11) \approx \$2.22/\text{yr}$

2012: $S'(12) \approx \$2.50/\text{yr}$

Section 2.5 The Chain Rule

Skills Warm Up

1. $\sqrt[5]{(1 - 5x)^2} = (1 - 5x)^{2/5}$

2. $\sqrt[4]{(2x - 1)^3} = (2x - 1)^{3/4}$

3. $\dfrac{1}{\sqrt{4x^2 + 1}} = (4x^2 + 1)^{-1/2}$

4. $\dfrac{1}{\sqrt[6]{2x^3 + 9}} = (2x^3 + 9)^{-1/6}$

5. $\dfrac{\sqrt{x}}{\sqrt[3]{1 - 2x}} = x^{1/2}(1 - 2x)^{-1/3}$

6. $\dfrac{\sqrt{(3 - 7x)^3}}{2x} = \dfrac{(3 - 7x)^{3/2}}{2x} = (2x)^{-1}(3 - 7x)^{3/2}$

7. $3x^3 - 6x^2 + 5x - 10 = 3x^2(x - 2) + 5(x - 2)$

$= (3x^2 + 5)(x - 2)$

Skills Warm Up —*continued*—

8. $5x\sqrt{x} - x - 5\sqrt{x} + 1 = x(5\sqrt{x} - 1) - 1(5\sqrt{x} - 1)$
$= (x - 1)(5\sqrt{x} - 1)$

9. $4(x^2 + 1)^2 - x(x^2 + 1)^3 = (x^2 + 1)^2[4 - x(x^2 + 1)]$
$= (x^2 + 1)^2(4 - x^3 - x)$

10. $-x^5 + 6x^3 + 7x^2 - 42 = -x^3(x^2 - 6) + 7(x^2 - 6)$
$= (-x^3 + 7)(x^2 - 6)$
$= -(x^3 - 7)(x^2 - 6)$

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2.	$y = (x^2 - 2x + 3)^3$	$u = x^2 - 2x + 3$	$y = u^3$
3.	$y = \sqrt{5x - 2}$	$u = 5x - 2$	$y = \sqrt{u}$
4.	$y = \sqrt[3]{9 - x^2}$	$u = 9 - x^2$	$y = \sqrt[3]{u}$
5.	$y = (3x + 1)^{-1}$	$u = 3x + 1$	$y = u^{-1}$
6.	$y = (x^2 - 3)^{-1/2}$	$u = x^2 - 3$	$y = u^{-1/2}$

7. $y = (4x + 7)^2$
$y' = 2(4x + 7)^1(4)$
$y' = 8(4x + 7)$
$= 32x + 56$

8. $y = (3x^2 - 2)^3$
$y' = 3(3x^2 - 2)^2(6x)$
$y' = 18x(3x^2 - 2)^2$

9. $y = \sqrt{3 - x^2} = (3 - x^2)^{1/2}$
$y' = \frac{1}{2}(3 - x^2)^{-1/2}(-2x)$
$y' = -x(3 - x^2)^{-1/2} = -\frac{x}{(3 - x^2)^{1/2}}$
$y' = -\frac{x}{\sqrt{3 - x^2}}$

10. $y = 4\sqrt[4]{6x + 5} = 4(6x + 5)^{1/4}$
$y' = 4\left(\frac{1}{4}\right)(6x + 5)^{-5/4}(6)$
$y' = 6(6x + 5)^{-5/4} = \frac{6}{(6x + 5)^{5/4}}$

11. $y = (5x^4 - 2x)^{2/3}$
$y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(20x^3 - 2)$
$y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(2)(10x^3 - 1)$
$y' = \left(\frac{4}{3}\right)(5x^4 - 2x)^{-1/3}(10x^3 - 1)$
$y' = \frac{4(10x^3 - 1)}{3(5x^4 - 2x)^{1/3}} = \frac{40x^3 - 4}{3\sqrt[3]{5x^4 - 2x}}$

12. $y = (x^3 + 2x^2)^{-1}$
$y' = (-1)(x^3 + 2x^2)^{-2}(3x^2 + 4x)$
$y' = -\frac{3x^2 + 4x}{(x^3 + 2x^2)^2}$

13. $f(x) = \frac{2}{1 - x^3} = 2(1 - x^3)^{-1}$; (c) General Power Rule

14. $f(x) = \frac{7}{(1 - x)^3} = 7(1 - x)^{-3}$; (c) General Power Rule

15. $f(x) = \sqrt[3]{8^2}$; (b) Constant Rule

16. $f(x) = \sqrt[3]{x^2} = x^{2/3}$; (a) Simple Power Rule

17. $f(x) = \frac{x^2 + 9}{x^3 + 4x^2 - 6}$; (d) Quotient Rule

18. $f(x) = \frac{x^{1/2}}{x^3 + 2x - 5}$; (d) Quotient Rule

19. $y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$

20. $y = (3 - 5x)^4$
$y' = 4(3 - 5x)^3(-5) = -20(3 - 5x)^3$

21. $h'(x) = 2(6x - x^3)(6 - 3x^2) = 6x(6 - x^2)(2 - x^2)$

22. $f(x) = (2x^3 - 6x)^{4/3}$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6x^2 - 6)$$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6)(x^2 - 1)$$

$$f'(x) = 8(2x^3 - 6x)^{1/3}(x^2 - 1)$$

23. $f(t) = \sqrt{t + 1} = (t + 1)^{1/2}$

$$f'(t) = \frac{1}{2}(t + 1)^{-1/2}(1) = \frac{1}{2\sqrt{t + 1}}$$

24. $g(x) = \sqrt{5 - 3x} = (5 - 3x)^{1/2}$

$$g'(x) = \frac{1}{2}(5 - 3x)^{-1/2}(-3) = -\frac{3}{2\sqrt{5 - 3x}}$$

25. $s(t) = \sqrt{2t^2 + 5t + 2} = (2t^2 + 5t + 2)^{1/2}$

$$s'(t) = \frac{1}{2}(2t^2 + 5t + 2)^{-1/2}(4t + 5) = \frac{4t + 5}{2\sqrt{2t^2 + 5t + 2}}$$

26. $y = 9\sqrt[3]{4x^2 + 3} = 9(4x^2 + 3)^{1/3}$

$$y' = 9\left(\frac{1}{3}\right)(4x^2 + 3)^{-2/3}(8x)$$

$$y' = 24x(4x^2 + 3)^{-2/3}$$

$$y' = \frac{24x}{(4x^2 + 3)^{2/3}}$$

27. $f(x) = 2(2 - 9x)^{-3}$

$$f'(x) = 2(-3)(2 - 9x)^{-4}(-9) = \frac{54}{(2 - 9x)^4}$$

28. $g(x) = \dfrac{3}{(7x^2 + 6x)^5} = 3(7x^2 + 6x)^{-5}$

$$g'(x) = 3(-5)(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -15(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -\frac{15(14x + 6)}{(7x^2 + 6x)^6}$$

29. $f(x) = \dfrac{1}{\sqrt{(x^2 + 11)^7}} = (x^2 + 11)^{-7/2}$

$$f'(x) = \left(-\frac{7}{2}\right)(x^2 + 11)^{-9/2}(2x)$$

$$f'(x) = -7x(x^2 + 11)^{-9/2}$$

$$f'(x) = -\frac{7x}{(x^2 + 11)^{9/2}} = -\frac{7x}{\sqrt{(x^2 + 11)^9}}$$

30. $y = (4 - x^3)^{-4/3}$

$$y' = \left(-\frac{4}{3}\right)(4 - x^3)^{-7/3}(-3x^2) = \frac{4x^2}{3(4 - x^2)^{7/3}}$$

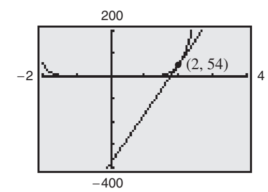
31. $f'(x) = 2(3)(x^2 - 1)^2(2x) = 12x(x^2 - 1)^2$

$$f'(2) = 24(3^2) = 216$$

$$f(2) = 54$$

$$y - 54 = 216(x - 2)$$

$$y = 216x - 378$$



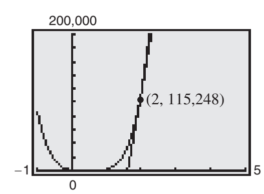
32. $f'(x) = 12(9x - 4)^3(9) = 108(9x - 4)^3$

$$f'(2) = 12(14)^3(9) = 296{,}352$$

$$f(2) = 3(14)^4 = 115{,}248$$

$$y - 115{,}248 = 296{,}352(x - 2)$$

$$y = 296{,}352x - 477{,}456$$



33. $f(x) = \sqrt{4x^2 - 7} = (4x^2 - 7)^{1/2}$

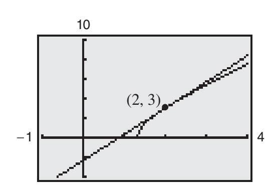
$$f'(x) = \frac{1}{2}(4x^2 - 7)^{-1/2}(8x) = \frac{4x}{\sqrt{4x^2 - 7}}$$

$$f'(2) = \frac{8}{3}$$

$$f(2) = 3$$

$$y - 3 = \frac{8}{3}(x - 2)$$

$$y = \frac{8}{3}x - \frac{7}{3}$$



34. $f(x) = x\sqrt{x^2+5} = x(x^2+5)^{1/2}$

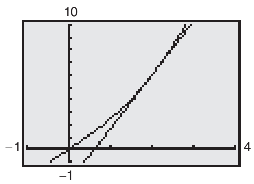
$$f'(x) = x\left[\frac{1}{2}(x^2+5)^{-1/2}(2x)\right] + (x^2+5)^{1/2}(1)$$
$$= x^2(x^2+5)^{-1/2} + (x^2+5)^{1/2}$$
$$= (x^2+5)^{-1/2}\left[x^2 + (x^2+5)\right]$$
$$= \frac{2x^2+5}{\sqrt{x^2+5}}$$

$f'(2) = \dfrac{13}{3}$

$f(2) = 6$

$y - 6 = \dfrac{13}{3}(x-2)$

$y = \dfrac{13}{3}x - \dfrac{8}{3}$



35. $f(x) = \sqrt{x^2-2x+1} = (x^2-2x+1)^{1/2}$

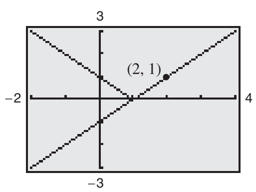
$$f'(x) = \frac{1}{2}(x^2-2x+1)^{-1/2}(2x-2)$$
$$= \frac{x-1}{\sqrt{x^2-2x+1}}$$
$$= \frac{x-1}{|x-1|}$$

$f'(2) = 1$

$f(2) = 1$

$y - 1 = 1(x-2)$

$y = x - 1$



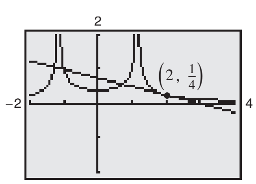
36. $f'(x) = -\dfrac{2}{3}(4-3x^2)^{-5/3}(-6x) = \dfrac{4x}{(4-3x^2)^{5/3}}$

$f'(2) = \dfrac{4(2)}{(-8)^{5/3}} = \dfrac{8}{-32} = -\dfrac{1}{4}$

$f(2) = (-8)^{-2/3} = \dfrac{1}{4}$

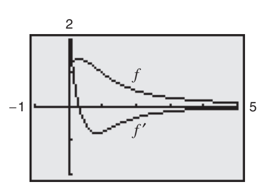
$y - \dfrac{1}{4} = -\dfrac{1}{4}(x-2)$

$y = -\dfrac{1}{4}x + \dfrac{3}{4}$



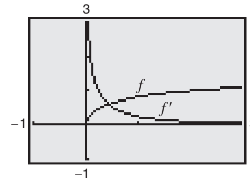
37. $f'(x) = \dfrac{1-3x^2-4x^{3/2}}{2\sqrt{x}(x^2+1)^2}$

f has a horizontal tangent when $f' = 0$.



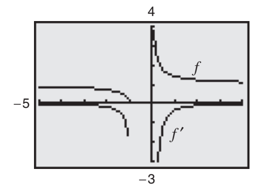
38. $f'(x) = \dfrac{\sqrt{2}}{2\sqrt{x}(x+1)^{3/2}}$

f' is never 0.



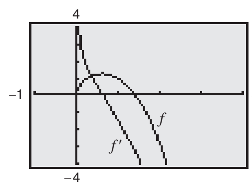
39. $f'(x) = -\dfrac{\sqrt{(x+1)/x}}{2x(x+1)}$

f' is never 0.



40. $f'(x) = \dfrac{2-5x^2}{2\sqrt{x}}$

f has a horizontal tangent when $f' = 0$.



41. $y = (4-x^2)^{-1}$

$$y' = (-1)(4-x^2)^{-2}(-2x)$$
$$= \frac{2x}{(4-x^2)^2}$$

General Power Rule

42. $s(t) = \dfrac{1}{t^2+3t-1} = (t^2+3t-1)^{-1}$

$$s'(t) = -1(t^2+3t-1)^{-2}(2t+3)$$
$$= -\frac{2t+3}{(t^2+3t-1)^2}$$

General Power Rule

43. $y = -\dfrac{5t}{(t+8)^2}$

$$y' = \frac{(t+8)^2(5) - (5t)(2)(t+8)(1)}{\left((t+8)^2\right)^2}$$

$$y' = \frac{5(t+8)\left[(t+8)-2t\right]}{(t^2+8)^4}$$

$$y' = \frac{5(t+8)(-t+8)}{(t+8)^4} = \frac{5(t-8)}{(t+8)^3}$$

Quotient Rule and Chain Rule

44. $f(x) = 3x(x^3 - 4)^{-2}$

$$\begin{aligned} f'(x) &= 3x\left[(-2)(x^3 - 4)^{-3}(3x^2)\right] + (x^3 - 4)^{-2}(3) \\ &= -18x^3(x^3 - 4)^{-3} + 3(x^3 - 4)^{-2} \\ &= -3(x^3 - 4)^{-3}\left[6x^3 - (x^3 - 4)\right] \\ &= \frac{-3(5x^3 + 4)}{(x^3 - 4)^3} \end{aligned}$$

Product Rule and Chain Rule

45. $f(x) = (2x - 1)(9 - 3x^2)$

$$\begin{aligned} f'(x) &= (2x - 1)(-6x) + (9 - 3x^2)(2) \\ &= -12x^2 + 6x + 18 - 6x^2 \\ &= 18 + 6x - 18x^2 \\ &= -6(3x^2 - 2x - 3) \end{aligned}$$

Product Rule and Simple Power Rule

46. $y = (7x + 4)(x^3 - 2x^2)$

$$\begin{aligned} y' &= (7x + 4)(3x^2 - 4x) + (x^3 - 2x^2)(7) \\ &= 21x^3 - 16x^2 - 16x + 7x^3 - 14x^2 \\ &= 28x^3 - 30x^2 - 16x \end{aligned}$$

Product Rule and Simple Power Rule

47. $y = \dfrac{1}{\sqrt{x + 2}} = (x + 2)^{-1/2}$

$$y' = -\frac{1}{2}(x + 2)^{-3/2} = -\frac{1}{2(x + 2)^{3/2}}$$

General Power Rule

48. $g(x) = \dfrac{3}{\sqrt[3]{x^3 - 1}} = 3(x^3 - 1)^{-1/3}$

$$g'(x) = 3\left(-\frac{1}{3}\right)(x^3 - 1)^{-4/3}(3x^2) = -\frac{3x^2}{(x^3 - 1)^{4/3}}$$

General Power Rule

49. $f(x) = x(3x - 9)^3$

$$\begin{aligned} f'(x) &= x(3)(3x - 9)^2(3) + (3x - 9)^3(1) \\ &= (3x - 9)^2\left[9x + (3x - 9)\right] \\ &= 9(x - 3)^2(12x - 9) \\ &= 27(x - 3)^2(4x - 3) \end{aligned}$$

Product and General Power Rule

50. $f(x) = x^3(x - 4)^2$

$$\begin{aligned} f(x) &= x^3(x^2 - 8x + 16) \\ &= x^5 - 8x^4 + 16x^3 \\ f'(x) &= 5x^4 - 32x^3 + 48x^2 \\ &= x^2(5x^2 - 32x + 48) \\ &= x^2(5x - 12)(x - 4) \end{aligned}$$

Simple Power Rule

51. $y = x\sqrt{2x + 3} = x(2x + 3)^{1/2}$

$$\begin{aligned} y' &= x\left[\frac{1}{2}(2x + 3)^{-1/2}(2)\right] + (2x + 3)^{1/2} \\ &= (2x + 3)^{-1/2}\left[x + (2x + 3)\right] \\ &= \frac{3(x + 1)}{\sqrt{2x + 3}} \end{aligned}$$

Product and General Power Rule

52. $y = 2t\sqrt{t + 6} = 2t(t + 6)^{1/2}$

$$\begin{aligned} y' &= 2t\left[\frac{1}{2}(t + 6)^{-1/2}(1)\right] + (t + 6)^{1/2}(2) \\ &= t(t + 6)^{-1/2} + 2(t + 6)^{1/2} \\ &= (t + 6)^{-1/2}\left[t + 2(t + 6)\right] \\ &= (t + 6)^{-1/2}(3t + 12) \\ &= \frac{3t + 12}{\sqrt{t + 6}} = \frac{3(t + 4)}{\sqrt{t + 6}} \end{aligned}$$

Product and General Power Rule

53. $y = t^2\sqrt{t - 2} = t^2(t - 2)^{1/2}$

$$\begin{aligned} y' &= t^2\left[\frac{1}{2}(t - 2)^{-1/2}(1)\right] + 2t(t - 2)^{1/2} \\ &= \frac{1}{2}(t - 2)^{-1/2}\left[t^2 + 4t(t - 2)\right] \\ &= \frac{t^2 + 4t(t - 2)}{2\sqrt{t - 2}} \\ &= \frac{t(5t - 8)}{2\sqrt{t - 2}} \end{aligned}$$

Product and General Power Rule

54. $y = \sqrt{x}(x-2)^2 = x^{1/2}(x-2)^2$

$$y' = x^{1/2}\left[2(x-2)^1(1)\right] + (x-2)^2\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 2\sqrt{x}(x-2) + \frac{(x-2)^2}{2\sqrt{x}}$$

$$= \frac{4x(x-2) + (x-2)^2}{2\sqrt{x}}$$

$$= \frac{(x-2)\left[4x + (x-2)\right]}{2\sqrt{x}}$$

$$= \frac{(x-2)(5x-2)}{2\sqrt{x}}$$

Product and General Power Rule

55. $y = \left(\dfrac{6-5x}{x^2-1}\right)^2$

$$y' = 2\left(\frac{6-5x}{x^2-1}\right)\left[\frac{(x^2-1)(-5) - (6-5x)(2x)}{(x^2-1)^2}\right]$$

$$= \frac{2(6-5x)(5x^2-12x+5)}{(x^2-1)^3}$$

Quotient and General Power Rule

56. $y = \left(\dfrac{4x^2-5}{2-x}\right)^3$

$$y' = 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{(2-x)(8x) - (4x^2-5)(-1)}{(2-x)^2}\right]$$

$$= 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{16x - 8x^2 + 4x^2 - 5}{(2-x)^2}\right]$$

$$= 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{-4x^2 + 16x - 5}{(2-x)^2}\right]$$

$$= \frac{3(4x^2-5)^2(-4x^2+16x-5)}{(2-x)^3}$$

Quotient and General Power Rule

57. $y = (x^3 - 2x^2 - x + 1)^2$

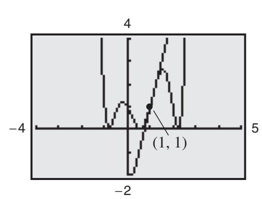
$$y' = 2(x^3 - 2x^2 - x + 1)(3x^2 - 4x - 1)$$

$$m = y'(1) = 2\left((1)^3 - 2(1)^2 - (1) + 1\right)\left(3(1)^2 - 4(1) - 1\right)$$

$$= 2(-1)(-2) = 4$$

$$y - 1 = 4(x-1)$$

$$y = 4x - 3$$



58. $f(x) = (3x^3 + 4x)^{1/5}$

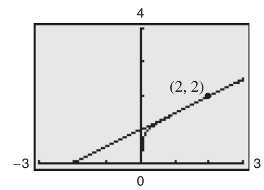
$$f'(x) = \frac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$$

$$= \frac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$$

$$m = f'(2) = \frac{1}{2}$$

$$y - 2 = \frac{1}{2}(x-2)$$

$$y = \frac{1}{2}x + 1$$



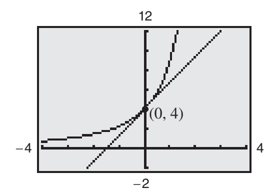
59. $f(t) = \dfrac{36}{(3-t)^2} = 36(3-t)^{-2}$

$$f'(t) = -72(3-t)^{-3}(-1) = \frac{72}{(3-t)^3}$$

$$f'(0) = \frac{72}{27} = \frac{8}{3}$$

$$y - 4 = \frac{8}{3}(t - 0)$$

$$y = \frac{8}{3}t + 4$$



60. $s(x) = \dfrac{1}{\sqrt{x^2 - 3x + 4}} = (x^2 - 3x + 4)^{-1/2}$

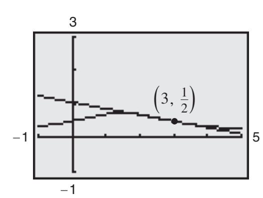
$$s'(x) = -\frac{1}{2}(x^2 - 3x + 4)^{-3/2}(2x - 3)$$

$$= \frac{3 - 2x}{2(x^2 - 3x + 4)^{3/2}}$$

$$s'(3) = \frac{3-6}{2(4)^{3/2}} = -\frac{3}{16}$$

$$y - \frac{1}{2} = -\frac{3}{16}(x - 3)$$

$$y = -\frac{3}{16}x + \frac{17}{16}$$



61. $f(t) = \left(t^2 - 9\right)\sqrt{t + 2} = \left(t^2 - 9\right)(t + 2)^{1/2}$

$$f'(t) = \left(t^2 - 9\right)\left[\frac{1}{2}(t + 2)^{-1/2}\right] + (t + 2)^{1/2}(2t)$$

$$= \frac{1}{2}\left(t^2 - 9\right)(t + 2)^{-1/2} + 2t(t + 2)^{1/2}$$

$$= (t + 2)^{-1/2}\left[\frac{1}{2}\left(t^2 - 9\right) + 2t(t + 2)\right]$$

$$= (t + 2)^{-1/2}\left[\frac{1}{2}t^2 - \frac{9}{2} + 2t^2 + 4t\right]$$

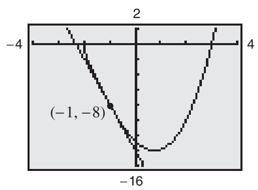
$$= (t + 2)^{-1/2}\left(\frac{5}{2}t^2 + 4t - \frac{9}{2}\right)$$

$$= \frac{\frac{5}{2}t^2 + 4t - \frac{9}{2}}{\sqrt{t + 2}}$$

$$f'(-1) = -6$$

$$y - (-8) = -6\left[t - (-1)\right]$$

$$y = -6t - 14$$



62. $y = -\dfrac{2x}{\sqrt{1 - x}} = -\dfrac{2x}{(1 - x)^{1/2}}$

$$y' = -\left[\frac{(1 - x)^{1/2}(2) - \left(\frac{1}{2}\right)(1 - x)^{-1/2}(-1)(2x)}{\left((1 - x)^{1/2}\right)^2}\right]$$

$$= -\left[\frac{2(1 - x)^{1/2} + x(1 - x)^{-1/2}}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}\left(2(1 - x) + x\right)}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}(2 - 2x + x)}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}(2 - x)}{1 - x}\right]$$

$$= -\left[\frac{(2 - x)}{(1 - x)^{3/2}}\right]$$

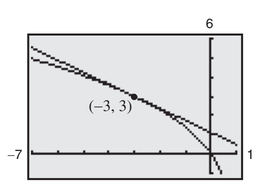
$$= \frac{x - 2}{(1 - x)^{3/2}}$$

$$y'(-3) = \frac{(-3) - 2}{\left(1 - (-3)\right)^{3/2}} = \frac{-5}{4^{3/2}} = -\frac{5}{8}$$

$$y - 3 = -\frac{5}{8}\left(x - (-3)\right)$$

$$y - 3 = -\frac{5}{8}x - \frac{15}{8}$$

$$y = -\frac{5}{8}x + \frac{9}{8}$$



63. $f(x) = \dfrac{x + 1}{\sqrt{2x - 3}} = \dfrac{x + 1}{(2x - 3)^{1/2}}$

$$f'(x) = \frac{(2x - 3)^{1/2}(1) - (x + 1)\left(\frac{1}{2}\right)(2x - 3)^{-1/2}(2)}{(2x - 3)}$$

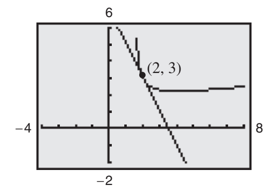
$$= \frac{(2x - 3) - (x + 1)}{(2x - 3)^{3/2}}$$

$$= \frac{x - 4}{(2x - 3)^{3/2}}$$

$$f'(2) = \frac{1 - 3}{1} = -2$$

$$y - 3 = -2(x - 2)$$

$$y = -2x + 7$$



64. $y = \dfrac{x}{\sqrt{25 + x^2}} = x\left(25 + x^2\right)^{-1/2}$

$$y' = x\left[-\tfrac{1}{2}\left(25 + x^2\right)^{-3/2}(2x)\right] + \left(25 + x^2\right)^{-1/2}(1)$$

$$= -x^2\left(25 + x^2\right)^{-3/2} + \left(25 + x^2\right)^{-1/2}$$

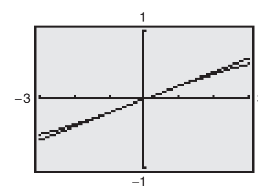
$$= \left(25 + x^2\right)^{-3/2}\left[-x^2 + \left(25 + x^2\right)\right]$$

$$= \frac{25}{\left(25 + x^2\right)^{3/2}}$$

$$y'(0) = \frac{1}{5}$$

$$y - 0 = \frac{1}{5}(x - 0)$$

$$y = \frac{1}{5}x$$



65. $f(x) = \sqrt[3]{x^2 + 4} = \left(x^2 + 4\right)^{1/3}$

$$f'(x) = \frac{1}{3}\left(x^2 + 4\right)^{-2/3}(2x)$$

$$f'(x) = \frac{2x}{3\left(x^2 + 4\right)^{2/3}}$$

Set $f'(x) = \dfrac{2x}{3\left(x^2 + 4\right)^{2/3}} = 0.$

$$2x = 0$$

$$x = 0 \rightarrow y = f(0) = \sqrt[3]{4}$$

Horizontal tangent at: $\left(0, \sqrt[3]{4}\right)$

66. $f(x) = \sqrt{5x^2 + x - 3} = \left(5x^2 + x - 3\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(5x^2 + x - 3\right)^{-1/2}(10x + 1)$

$f'(x) = \frac{10x + 1}{2\left(5x^2 + x - 3\right)^{1/2}}$

Set $f'(x) = \frac{10x + 1}{2\left(5x^2 + x - 3\right)^{1/2}} = 0.$

$10x + 1 = 0$

$x = -\frac{1}{10} \to y = f\left(-\frac{1}{10}\right) = \sqrt{-\frac{61}{20}}$

Because $\sqrt{-\frac{61}{20}}$ is not a real number, there is no point of horizontal tangency.

67. $f(x) = \frac{x}{\sqrt{2x - 1}} = \frac{x}{(2x - 1)^{1/2}}$

$f'(x) = \frac{(2x - 1)^{1/2}(1) - x\left(\frac{1}{2}(2x - 1)^{-1/2}(2)\right)}{\left[(2x + 1)^{1/2}\right]^2}$

$f'(x) = \frac{(2x - 1)^{1/2} - x(2x - 1)^{-1/2}}{(2x + 1)}$

$f'(x) = \frac{(2x - 1)^{-1/2}\left[(2x - 1) - x\right]}{(2x - 1)}$

$f'(x) = \frac{x - 1}{(2x - 1)^{3/2}}$

Set $f'(x) = \frac{x - 1}{(2x - 1)^{3/2}} = 0.$

$x - 1 = 0$

$x = 1 \to y = f(1) = \frac{1}{\sqrt{1}} = 1$

Horizontal tangent at: $(1, 1)$

68. $f(x) = \frac{5x}{\sqrt{3x - 2}} = \frac{5x}{(3x - 2)^{1/2}}$

$f'(x) = \frac{(3x - 2)^{1/2}(5) - 5x\left(\frac{1}{2}(3x - 2)^{-1/2}(3)\right)}{\left[(3x - 2)^{1/2}\right]^2}$

$f'(x) = \frac{5(3x - 2)^{1/2} - \frac{15}{2}x(3x - 2)^{-1/2}}{(2x + 1)}$

$f'(x) = \frac{\frac{5}{2}(3x - 2)^{-1/2}\left[2(3 - x2) - 3x\right]}{(2x - 1)}$

$f'(x) = \frac{5(3x - 4)}{2(3x - 2)^{3/2}}$

Set $f'(x) = \frac{5(3x - 4)}{2(3x - 2)^{3/2}} = 0.$

$5(3x - 4) = 0$

$3x - 4 = 0$

$x = \frac{4}{3} \to y = f\left(\frac{4}{3}\right) = \frac{20}{3\sqrt{2}}$

Horizontal tangent at: $\left(\frac{4}{3}, \frac{10}{3\sqrt{2}}\right)$

69. $A' = 1000(60)\left(1 + \frac{r}{12}\right)^{59}\left(\frac{1}{12}\right) = 5000\left(1 + \frac{r}{12}\right)^{59}$

(a) $A'(0.08) = 50\left(1 + \frac{0.08}{12}\right)^{59}$

$\approx$ \$74.00 per percentage point

(b) $A'(0.10) = 50\left(1 + \frac{0.10}{12}\right)^{59}$

$\approx$ \$81.59 per percentage point

(c) $A'(0.12) = 50\left(1 + \frac{0.12}{12}\right)^{59}$

$\approx$ \$89.94 per percentage point

70. $N = 400\left[1 - 3\left(t^2 + 2\right)^{-2}\right]$

$$\frac{dN}{dt} = N'(t) = 400\left[(-3)(-2)\left(t^2 + 2\right)^{-3}(2t)\right]$$
$$= \frac{4800t}{\left(t^2 + 2\right)^3}$$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) \approx 177.8$ bacteria/day

(c) $N'(2) \approx 44.4$ bacteria/day

(d) $N'(3) \approx 10.8$ bacteria/day

(e) $N'(4) \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as time passes.

71. $V = \dfrac{k}{\sqrt{t + 1}}$

When $t = 0$, $V = 10{,}000$.

$$10{,}000 = \frac{k}{\sqrt{0 + 1}} \Rightarrow k = 10{,}000$$
$$V = \frac{10{,}000}{\sqrt{t + 1}}$$
$$V = 10{,}000(t + 1)^{-1/2}$$
$$\frac{dV}{dt} = -5000(t + 1)^{-3/2}(1) = -\frac{5000}{(t + 1)^{3/2}}$$

When $t = 1$,

$$\frac{dV}{dt} = -\frac{5000}{(2)^{3/2}} = -\frac{2500}{\sqrt{2}} \approx -\$1767.77 \text{ per year.}$$

When $t = 3$, $\dfrac{dV}{dt} = -\dfrac{5000}{(4)^{3/2}} = -\$625.00$ per year.

72. (a) From the graph, the tangent line at $t = 4$ is steeper than the tangent line at $t = 1$. So, the rate of change after 4 hours is greater.

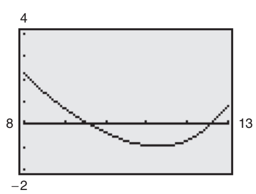
(b) The cost function is a composite function of x units, which is a function of the number of hours, which is not a linear function.

73. (a) $r = \left(0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242\right)^{1/2}$

$$\frac{dr}{dt} = r'(t) = \frac{1}{2}\left(0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242\right)^{-1/2} \cdot \left(1.2068t^3 - 28.971t^2 + 194.7t - 266.8\right)$$
$$= \frac{1.2068t^3 - 28.971t^2 + 194.7t - 266.8}{2\sqrt{0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242}}$$

Chain Rule

(b)



(c) The rate of change appears to be the greatest when $t = 8$ or 2008.

The rate of change appears to be the least when $t \approx 9.60$, or 2009, and when $t \approx 12.57$, or 2012.

Section 2.6 Higher-Order Derivatives

Skills Warm Up

1. $-16t^2 + 292 = 0$

$$-16t^2 = -292$$
$$t^2 = \frac{73}{4}$$
$$t = \pm\frac{\sqrt{73}}{2}$$

2. $-16t^2 + 88t = 0$

$$-8t(2t - 11) = 0$$
$$-8t = 0 \rightarrow t = 0$$
$$2t - 11 = 0 \rightarrow t = \tfrac{11}{2}$$

Skills Warm Up —*continued*—

3. $$\begin{aligned}-16t^2 + 128t + 320 &= 0\\ -16(t^2 - 8t - 20) &= 0\\ -16(t - 10)(t + 2) &= 0\\ t - 10 = 0 &\rightarrow t = 10\\ t + 2 = 0 &\rightarrow t = -2\end{aligned}$$

4. $-16t^2 + 9t + 1440 = 0$

$$\begin{aligned}t &= \frac{-9 \pm \sqrt{9^2 - 4(-16)(1440)}}{2(-16)}\\ &= \frac{-9 \pm \sqrt{92241}}{-32}\\ &= \frac{9 \pm 3\sqrt{10249}}{32}\end{aligned}$$

$t \approx -9.21$ and $t \approx 9.77$

5. $y = x^2(2x + 7)$

$$\begin{aligned}\frac{dy}{dx} &= x^2(2) + 2x(2x + 7)\\ &= 2x^2 + 4x^2 + 14x\\ &= 6x^2 + 14x\end{aligned}$$

6. $y = (x^2 + 3x)(2x^2 - 5)$

$$\begin{aligned}\frac{dy}{dx} &= (x^2 + 3x)(4x) + (2x + 3)(2x^2 - 5)\\ &= 4x^3 + 12x^2 + 4x^3 - 10x + 6x^2 - 15\\ &= 8x^3 + 18x^2 - 10x - 15\end{aligned}$$

7. $y = \dfrac{x^2}{2x + 7}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(2x + 7)(2x) - (x^2)(2)}{(2x + 7)^2}\\ &= \frac{4x^2 + 14x - 2x^2}{(2x + 7)^2}\\ &= \frac{2x^2 + 14x}{(2x + 7)^2}\\ &= \frac{2x(x + 7)}{(2x + 7)^2}\end{aligned}$$

8. $y = \dfrac{x^2 + 3x}{2x^2 - 5}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(2x^2 - 5)(2x + 3) - (x^2 + 3x)(4x)}{(2x^2 - 5)^2}\\ &= \frac{4x^3 + 6x^2 - 10x - 15 - 4x^3 - 12x^2}{(2x^2 - 5)^2}\\ &= \frac{-6x^2 - 10x - 15}{(2x^2 - 5)^2}\end{aligned}$$

9. $f(x) = x^2 - 4$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

10. $f(x) = \sqrt{x - 7}$

Domain: $[7, \infty)$

Range: $[0, \infty)$

1. $$\begin{aligned}f(x) &= 9 - 2x\\ f'(x) &= -2\\ f''(x) &= 0\end{aligned}$$

2. $$\begin{aligned}f(x) &= 4x + 15\\ f'(x) &= 4\\ f''(x) &= 0\end{aligned}$$

3. $$\begin{aligned}f(x) &= x^2 + 7x - 4\\ f'(x) &= 2x + 7\\ f''(x) &= 2\end{aligned}$$

4. $$\begin{aligned}f(x) &= 3x^2 + 4x\\ f'(x) &= 6x + 4\\ f''(x) &= 6\end{aligned}$$

5. $$\begin{aligned}g(t) &= \tfrac{1}{3}t^3 - 4t^2 + 2t\\ g'(t) &= t^2 - 8t + 2\\ g''(t) &= 2t - 8\end{aligned}$$

6. $$\begin{aligned}f(x) &= -\tfrac{5}{4}x^4 + 3x^2 - 6x\\ f'(x) &= -5x^3 + 6x - 6\\ f''(x) &= -15x^2 + 6\end{aligned}$$

7. $f(t) = \dfrac{2}{t^3} = 2t^{-3}$

$f'(t) = -6t^{-4}$

$f''(t) = 24t^{-5} = \dfrac{24}{t^5}$

8. $g(t) = \dfrac{5}{6t^4} = \dfrac{5}{6}t^{-4}$

$g'(t) = -\dfrac{10}{3}t^{-5}$

$g''(t) = \dfrac{50}{3}t^{-6} = \dfrac{50}{3t^6}$

9. $f(x) = 3\left(2 - x^2\right)^3$

$f'(x) = 9\left(2 - x^2\right)^2(-2x) = -18x\left(2 - x^2\right)^2$

$f''(x) = (-18x)2\left(2 - x^2\right)(-2x) + \left(2 - x^2\right)^2(-18)$

$= 18\left(2 - x^2\right)\left[4x^2 - \left(2 - x^2\right)\right]$

$= 18\left(2 - x^2\right)\left(5x^2 - 2\right)$

10. $y = 4\left(x^2 + 5x\right)^3$

$y' = 4(3)\left(x^2 + 5x\right)^2(2x + 5)$

$= (24x + 60)\left(x^4 + 10x^3 + 25x^2\right)$

$= 24x^5 + 300x^4 + 1200x^3 + 1500x^2$

$y'' = 120x^4 + 1200x^3 + 3600x^2 + 3000x$

11. $f(x) = \dfrac{x + 1}{x - 1}$

$f'(x) = \dfrac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$

$= -\dfrac{2}{(x - 1)^2} = -2(x - 1)^{-2}$

$f''(x) = 4(x - 1)^{-3}(1) = \dfrac{4}{(x - 1)^3}$

12. $g(x) = \dfrac{1 - 4x}{x - 3}$

$g'(x) = \dfrac{(x - 3)(-4) - (1 - 4x)(1)}{(x - 3)^2}$

$= \dfrac{-4x + 12 - 1 + 4x}{(x - 3)^2}$

$= \dfrac{11}{(x - 3)^2} = 11(x - 3)^{-2}$

$g''(x) = -22(x - 3)^{-3}(1) = -\dfrac{22}{(x - 3)^3}$

13. $f(x) = x^5 - 3x^4$

$f'(x) = 5x^4 - 12x^3$

$f''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x$

14. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

$f'''(x) = 24x - 12 = 12(2x - 1)$

15. $f(x) = 5x(x + 4)^3$

$= 5x\left(x^3 + 12x^2 + 48x + 64\right)$

$= 5x^4 + 60x^3 + 240x^2 + 320x$

$f'(x) = 20x^3 + 180x^2 + 480x + 320$

$f''(x) = 60x^2 + 360x + 480$

$f'''(x) = 120x + 360$

16. $f(x) = \left(x^3 - 6\right)^4$

$f'(x) = 4\left(x^3 - 6\right)^3\left(3x^2\right)$

$= 12x^{11} - 216x^8 + 1296x^5 - 2592x^2$

$f''(x) = 132x^{10} - 1728x^7 + 6480x^4 - 5184x$

$f'''(x) = 1320x^9 - 12{,}096x^6 + 25{,}920x^3 - 5184$

17. $f(x) = \dfrac{3}{8x^4} = \dfrac{3}{8}x^{-4}$

$f'(x) = -\dfrac{3}{2}x^{-5}$

$f''(x) = \dfrac{15}{2}x^{-6}$

$f'''(x) = -45x^{-7} = -\dfrac{45}{x^7}$

18. $f(x) = -\dfrac{2}{25x^5}$

$f(x) = -\dfrac{2}{25}x^{-5}$

$f'(x) = \dfrac{2}{5}x^{-6}$

$f''(x) = -\dfrac{12}{5}x^{-7}$

$f'''(x) = \dfrac{84}{5}x^{-8} = \dfrac{84}{5x^8}$

19. $g(t) = 5t^4 + 10t^2 + 3$

$g'(t) = 20t^3 + 20t$

$g''(t) = 60t^2 + 20$

$g''(2) = 60(4) + 20 = 260$

20. $f(x) = 9 - x^2$

$f'(x) = -2x$

$f''(x) = -2$

$f''\left(-\sqrt{5}\right) = -2$

21. $f(x) = \sqrt{4 - x} = (4 - x)^{1/2}$

$f'(x) = -\frac{1}{2}(4 - x)^{-1/2}$

$f''(x) = -\frac{1}{4}(4 - x)^{-3/2}$

$f'''(x) = -\frac{3}{8}(4 - x)^{-5/2} = \frac{-3}{8(4 - x)^{5/2}}$

$f'''(-5) = \frac{-3}{8(9)^{5/2}} = -\frac{1}{648}$

22. $f(t) = \sqrt{2t + 3} = (2t + 3)^{1/2}$

$f'(t) = \frac{1}{2}(2t + 3)^{-1/2}(2) = (2t + 3)^{-1/2}$

$f''(t) = -\frac{1}{2}(2t + 3)^{-3/2}(2) = -(2t + 3)^{-3/2}$

$f'''(t) = \frac{3}{2}(2t + 3)^{-5/2}(2) = \frac{3}{(2t + 3)^{5/2}}$

$f'''\left(\frac{1}{2}\right) = \frac{3}{32}$

23. $f(x) = \left(x^3 - 2x\right)^3 = x^9 - 6x^7 + 12x^5 - 8x^3$

$f'(x) = 9x^8 - 42x^6 + 60x^4 - 24x^2$

$f''(x) = 72x^7 - 252x^5 + 240x^3 - 48x$

$f''(1) = 12$

24. $g(x) = \left(x^2 + 3x\right)^4 = x^8 + 12x^7 + 54x^6 + 108x^5 + 81x^4$

$g'(x) = 8x^7 + 84x^6 + 324x^5 + 540x^4 + 324x^3$

$g''(x) = 56x^6 + 504x^5 + 1620x^4 + 2160x^3 + 972x^2$

$g''(-1) = -16$

25. $f'(x) = 2x^2$

$f''(x) = 4x$

26. $f''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x = 12x(5x - 6)$

27. $f'''(x) = 4x^{-4}$

$f^{(4)}(x) = -16x^{-5}$

$f^{(5)}(x) = 80x^{-6} = \frac{80}{x^6}$

28. $f''(x) = 4\sqrt{x - 2} = 4(x - 2)^{1/2}$

$f'''(x) = 4\left(\frac{1}{2}\right)(x - 2)^{-1/2}(1) = 2(x - 2)^{-1/2}$

$f^{(4)}(x) = 2\left(-\frac{1}{2}\right)(x - 2)^{-3/2}(1) = -(x - 2)^{-3/2}$

$f^{(5)}(x) = \frac{3}{2}(x - 2)^{-5/2}(1) = \frac{3}{2(x - 2)^{5/2}}$

29. $f^{(5)}(x) = 2\left(x^2 + 1\right)(2x)$

$= 4x^3 + 4x$

$f^{(6)}(x) = 12x^2 + 4$

30. $f'''(x) = 4x + 7$

$f^{(4)}(x) = 4$

$f^{(5)}(x) = 0$

31. $f'(x) = 3x^2 - 18x + 27$

$f''(x) = 6x - 18$

$f''(x) = 0 \Rightarrow 6x = 18$

$x = 3$

32. $f(x) = (x + 2)(x - 2)(x + 3)(x - 3)$

$= \left(x^2 - 4\right)\left(x^2 - 9\right)$

$= x^4 - 13x^2 + 36$

$f'(x) = 4x^3 - 26x$

$f''(x) = 12x^2 - 26$

$f''(x) = 0 \Rightarrow 12x^2 = 26$

$x = \pm\sqrt{\frac{13}{6}} = \pm\frac{\sqrt{78}}{6}$

33. $f(x) = x\sqrt{x^2 - 1} = x(x^2 - 1)^{1/2}$

$$f'(x) = x\frac{1}{2}(x^2 - 1)^{-1/2}(2x) + (x^2 - 1)^{1/2} = \frac{x^2}{(x^2 - 1)^{1/2}} + (x^2 - 1)^{1/2}$$

$$f''(x) = \frac{(x^2 - 1)^{1/2}(2x) - x^2\left(\frac{1}{2}\right)(x^2 - 1)^{-1/2}(2x)}{x^2 - 1} + \frac{1}{2}(x^2 - 1)^{-1/2}(2x)$$

$$= \frac{(x^2 - 1)(2x) - x^3}{(x^2 - 1)^{3/2}} + \frac{x}{(x^2 - 1)^{1/2}} \cdot \frac{x^2 - 1}{x^2 - 1}$$

$$= \frac{2x^3 - 3x}{(x^2 - 1)^{3/2}}$$

$$f''(x) = 0 \Rightarrow 2x^3 - 3x = x(2x^2 - 3) = 0$$

$$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$$

$x = 0$ is not in the domain of f.

34. $f'(x) = \dfrac{(x^2 + 3)(1) - (x)(2x)}{(x^2 + 3)^2} = \dfrac{3 - x^2}{(x^2 + 3)^2} = (3 - x^2)(x^2 + 3)^{-2}$

$$f''(x) = (3 - x^2)\left[-2(x^2 + 3)^{-3}(2x)\right] + (x^2 + 3)^{-2}(-2x)$$

$$= -2x(x^2 + 3)^{-3}\left[2(3 - x^2) + (x^2 + 3)\right]$$

$$= \frac{-2x(9 - x^2)}{(x^2 + 3)^3}$$

$$= \frac{2x(x^2 - 9)}{(x^2 + 3)^3}$$

$$f''(x) = 0 \Rightarrow 2x(x^2 - 9) = 0$$

$$x = 0, \pm 3$$

35. (a) $s(t) = -16t^2 + 144t$

$v(t) = s'(t) = -32t + 144$

$a(t) = v'(t) = s''(t) = -32$

(b) $s(3) = 288$ ft

$v(3) = 48$ ft/sec

$a(3) = -32$ ft/sec^2

(c) $v(t) = 0$

$-32t + 144 = 0$

$-32t = -144$

$t = 4.5$ sec

$s(4.5) = 324$ ft

(d) $s(t) = 0$

$-16t^2 + 144t = 0$

$-16t(t - 9) = 0$

$t = 0$ sec $\quad t = 9$ sec

$v(9) = -32(9) + 144 = -144$ ft/sec

This is the same speed as the initial velocity.

36. (a) $s(t) = -16t^2 + 1250$

$v(t) = s'(t) = -32t$

$a(t) = v'(t) = -32$

(b) $s(t) = 0$ when $16t^2 = 1250$, or

$t = \sqrt{78.125} \approx 8.8$ sec.

(c) $v(8.8) \approx -282.8$ ft/sec

37. $\dfrac{d^2s}{dt^2} = \dfrac{(t+10)(90) - (90t)(1)}{(t+10)^2} = \dfrac{900}{(t+10)^2}$

t	0	10	20	30	40	50	60
$\dfrac{ds}{dt}$	0	45	60	67.5	72	75	77.14
$\dfrac{d^2s}{dt^2}$	9	2.25	1	0.56	0.36	0.25	0.18

As time increases, the acceleration decreases. After 1 minute, the automobile is traveling at about 77.14 feet per second.

38. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = -16.50t + 66$

$a(t) = s''(t) = -16.50$

t	0	1	2	3	4	5
$s(t)$	0	57.75	99	123.75	132	123.75
$v(t)$	66	49.50	33	16.50	0	–16.50
$a(t)$	–16.50	–16.50	–16.50	–16.50	–16.50	–16.50

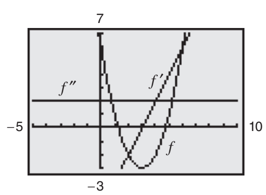
It takes 4 seconds for the car to stop, at which time it has traveled 132 feet.

39. $f(x) = x^2 - 6x + 6$

$f'(x) = 2x - 6$

$f''(x) = 2$

(a)

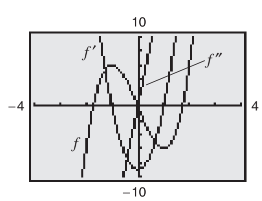


(b) The degree decreased by 1 for each successive derivative.

(c) $f(x) = 3x^2 - 9x$

$f'(x) = 6x - 9$

$f''(x) = 6$



(d) The degree decreases by 1 for each successive derivative.

40. Graph A is the position function. Graph B is the velocity function. Graph C is the acceleration function. Explanations will vary. Sample explanation:
The position function appears to be a third-degree function, while the velocity is a second-degree function, and the acceleration is a linear function.

41. (a) $y(t) = -21.944t^3 + 701.75t^2 - 6969.4t + 27{,}164$

(b) $y'(t) = -65.832t^2 + 1403.5t - 6969.4$

$y''(t) = -131.664t + 1403.5$

(c) Over the interval $8 \le t \le 13$, $y'(t) > 0$; therefore, y is increasing over $8 \le t \le 13$, or from 2008 to 20013.

(d)
$$\begin{aligned} y''(t) &= 0 \\ -131.664t + 1403.5 &= 0 \\ -131.664t &= -1403.5 \\ t &\approx 10.66 \text{ or } 2010 \end{aligned}$$

42. Let $y = xf(x)$.

Then,
$$\begin{aligned} y' &= xf'(x) + f(x) \\ y'' &= xf''(x) + f'(x) + f'(x) \\ &= xf''(x) + 2f'(x) \\ y''' &= xf'''(x) + f''(x) + 2f''(x) \\ &= xf'''(x) + 3f''(x). \end{aligned}$$

In general $y^{(n)} = \left[xf(x)\right]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

43. True. If $y = (x+1)(x+2)(x+3)(x+4)$, then y is a fourth-degree polynomial function and its fifth derivative $\dfrac{d^5y}{dx^5}$ equals o.

44. True. The second derivative represents the rate of change of the first derivative, the same way that the first derivative represents the rate of change of the function.

45. Answers will vary.

Section 2.7 Implicit Differentiation

Skills Warm Up

1.
$$\begin{aligned} x - \frac{y}{x} &= 2 \\ x^2 - y &= 2x \\ -y &= 2x - x^2 \\ y &= x^2 - 2x \end{aligned}$$

2.
$$\begin{aligned} \frac{4}{x-3} &= \frac{1}{y} \\ 4y &= x - 3 \\ y &= \frac{x-3}{4} \end{aligned}$$

3.
$$\begin{aligned} xy - x + 6y &= 6 \\ xy + 6y &= 6 + x \\ y(x+6) &= 6 + x \\ y &= \frac{6+x}{x+6} \\ y &= 1,\ x \neq -6 \end{aligned}$$

4.
$$\begin{aligned} 7 + 4y &= 3x^2 + x^2y \\ 4y - x^2y &= 3x^2 - 7 \\ y(4 - x^2) &= 3x^2 - 7 \\ y &= \frac{3x^2 - 7}{4 - x^2},\ x \neq \pm 2 \end{aligned}$$

5.
$$\begin{aligned} x^2 + y^2 &= 5 \\ y^2 &= 5 - x^2 \\ y &= \pm\sqrt{5 - x^2} \end{aligned}$$

6.
$$\begin{aligned} x &= \pm\sqrt{6 - y^2} \\ x^2 &= 6 - y^2 \\ x^2 - 6 &= -y^2 \\ 6 - x^2 &= y^2 \\ \pm\sqrt{6 - x^2} &= y \end{aligned}$$

7. $\dfrac{3x^2 - 4}{3y^2}$, $(2, 1)$

$$\frac{3(2^2) - 4}{3(1^2)} = \frac{3(4) - 4}{3} = \frac{8}{3}$$

8. $\dfrac{x^2 - 2}{1 - y}$, $(0, -3)$

$$\frac{0^2 - 2}{1 - (-3)} = \frac{-2}{4} = -\frac{1}{2}$$

9. $\dfrac{7x}{4y^2 + 13y + 3}$, $\left(-\dfrac{1}{7}, -2\right)$

$$\frac{7\left(-\frac{1}{7}\right)}{4(-2)^2 + 13(-2) + 3} = \frac{-1}{16 - 26 + 3} = \frac{-1}{-7} = \frac{1}{7}$$

1.
$$\begin{aligned} x^3y &= 6 \\ x^3\frac{dy}{dx} + 3x^2y &= 0 \\ x^3\frac{dy}{dx} &= -3x^2y \\ \frac{dy}{dx} &= -\frac{3x^2}{x^3}y = -\frac{3}{x}y \end{aligned}$$

2.
$$\begin{aligned} 3x^2 - y &= 8x \\ 6x - \frac{dy}{dx} &= 8 \\ -\frac{dy}{dx} &= 8 - 6x \\ \frac{dy}{dx} &= 6x - 8 \end{aligned}$$

3.
$$\begin{aligned} y^2 &= 1 - x^2 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

4.
$$y^3 = 5x^3 + 8x$$
$$3y^2\frac{dy}{dx} = 15x^2 + 8$$
$$\frac{dy}{dx} = \frac{15x^2 + 8}{3y^2}$$

5.
$$y^4 - y^2 + 7y - 6x = 9$$
$$4y^3\frac{dy}{dx} - 2y\frac{dy}{dx} + 7\frac{dy}{dx} - 6 = 0$$
$$\left(4y^3 - 2y + 7\right)\frac{dy}{dx} = 6$$
$$\frac{dy}{dx} = \frac{6}{4y^3 - 2y + 7}$$

6.
$$4y^3 + 5y^2 - y - 3x^3 = 8x$$
$$12y^2\frac{dy}{dx} + 10y\frac{dy}{dx} - \frac{dy}{dx} - 9x^2 = 8$$
$$\left(12y^2 + 10y - 1\right)\frac{dy}{dx} = 8 + 9x^2$$
$$\frac{dy}{dx} = \frac{8 + 9x^2}{12y^2 + 10y - 1}$$

7.
$$xy^2 + 4xy = 10$$
$$y^2 + 2xy\frac{dy}{dx} + 4y + 4x\frac{dy}{dx} = 0$$
$$(2xy + 4x)\frac{dy}{dx} = -y^2 - 4y$$
$$\frac{dy}{dx} = -\frac{y^2 + 4y}{2xy + 4x}$$

8.
$$2xy^3 - x^2y = 2$$
$$2y^3 + 6xy^2\frac{dy}{dx} - 2xy - x^2\frac{dy}{dx} = 0$$
$$\left(6xy^2 - x^2\right)\frac{dy}{dx} = 2xy - 2y^3$$
$$\frac{dy}{dx} = \frac{2xy - 2y^3}{6xy^2 - x^2}$$

9.
$$\frac{2x + y}{x - 5y} = 1$$
$$2x + y = x - 5y$$
$$6y = -x$$
$$y = -\frac{1}{6}x$$
$$\frac{dy}{dx} = -\frac{1}{6}$$

10.
$$\frac{xy - y^2}{y - x} = 1$$
$$xy - y^2 = y - x$$
$$y(x - y) = -(x - y)$$
$$y = -1$$
$$\frac{dy}{dx} = 0$$

11.
$$\frac{2y}{y^2 + 3} = 4x$$
$$2y = 4x\left(y^2 + 3\right)$$
$$2y = 4xy^2 + 12x$$
$$2\frac{dy}{dx} = 8xy\frac{dy}{dx} + 4y^2 + 12$$
$$2\frac{dy}{dx} - 8xy\frac{dy}{dx} = 4y^2 + 12$$
$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

12.
$$\frac{4y^2}{y^2 - 9} = x^2$$
$$\frac{\left(y^2 - 9\right)\left(8y\dfrac{dy}{dx}\right) - 4y^2\left(2y\dfrac{dy}{dx}\right)}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{8y\dfrac{dy}{dx}\left(y^2 - 9 - y^2\right)}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{-72y\dfrac{dy}{dx}}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{dy}{dx} = \frac{2x\left(y^2 - 9\right)^2}{-72y}$$
$$\frac{dy}{dx} = -\frac{x\left(y^2 - 9\right)^2}{36y}$$

13.
$$x^2 + y^2 = 16$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$2y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

At $(0, 4)$, $\dfrac{dy}{dx} = -\dfrac{0}{4} = 0.$

14.
$$x^2 - y^2 = 25$$
$$2x - 2y\frac{dy}{dx} = 0$$
$$-2y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{x}{y}$$

At $(5, 0)$, $\frac{dy}{dx}$ is undefined.

15.
$$y + xy = 4$$
$$\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$
$$\frac{dy}{dx}(1 + x) = -y$$
$$\frac{dy}{dx} = -\frac{y}{x + 1}$$

At $(-5, -1)$, $\frac{dy}{dx} = -\frac{1}{4}$.

16.
$$xy - 3y^2 = 2$$
$$x\frac{dy}{dx} + y - 6y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(x - 6y) = -y$$
$$\frac{dy}{dx} = -\frac{y}{x - 6y}$$

At $(7, 2)$, $\frac{dy}{dx} = -\frac{2}{7 - 6(2)} = \frac{2}{5}$.

17.
$$x^2 - xy + y^2 = 4$$
$$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$
$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(2y - x) = y - 2x$$
$$\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}$$

At $(-2, -1)$, $\frac{dy}{dx} = \frac{(-1) - 3(-2)^2}{2(-1) - (-2)} = \frac{-7}{0}$; $\frac{dy}{dx}$ is undefined.

18.
$$x^2y + y^3x = -6$$
$$x^2\frac{dy}{dx} + 2xy + y^3 + 3y^2\frac{dy}{dx}x = 0$$
$$\frac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$$
$$\frac{dy}{dx} = \frac{-\left(2xy + y^3\right)}{x^2 + 3xy^2}$$
$$\frac{dy}{dx} = \frac{y\left(2x + y^2\right)}{x\left(x + 3y^2\right)}$$

At $(2, -1)$, $\frac{dy}{dx} = -\frac{(-1)\left(2(2) + (-1)^2\right)}{(2)\left((2) + 3(-1)^2\right)} = -\frac{-5}{10} = \frac{1}{2}$.

19.
$$xy - x = y$$
$$x\frac{dy}{dx} + y - 1 = \frac{dy}{dx}$$
$$x\frac{dy}{dx} - \frac{dy}{dx} = 1 - y$$
$$\frac{dy}{dx}(x - 1) = 1 - y$$
$$\frac{dy}{dx} = \frac{1 - y}{x - 1} = -\frac{y - 1}{x - 1}$$

At $\left(\frac{3}{2}, 3\right)$, $\frac{dy}{dx} = -\frac{3 - 1}{\frac{3}{2} - 1} = -\frac{2}{\frac{1}{2}} = -4$.

20.
$$x^3 + y^3 = 6xy$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6\left(x\frac{dy}{dx} + y\right)$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$
$$\left(3y^2 - 6x\right)\frac{dy}{dx} = 6y - 3x^2$$
$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$
$$\frac{dy}{dx} = \frac{3\left(2y - x^2\right)}{3\left(y^2 - 2x\right)}$$
$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

At $\left(\frac{4}{3}, \frac{8}{3}\right)$, $\frac{dy}{dx} = \frac{4}{5}$.

21.
$$x^{1/2} + y^{1/2} = 9$$
$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$
$$x^{-1/2} + y^{-1/2}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = -\sqrt{\frac{y}{x}}$$

At $(16, 25)$, $\frac{dy}{dx} = -\frac{5}{4}$.

22.
$$x^{2/3} + y^{2/3} = 5$$
$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}} = -\sqrt[3]{\frac{y}{x}}$$

At $(8, 1)$, $\dfrac{dy}{dx} = -\dfrac{1}{2}$.

23.
$$\sqrt{xy} = x - 2y$$
$$\sqrt{x}\sqrt{y} = x - 2y$$
$$\sqrt{x}\left(\frac{1}{2}y^{-1/2}\frac{dy}{dx}\right) + \sqrt{y}\left(\frac{1}{2}x^{-1/2}\right) = 1 - 2\frac{dy}{dx}$$
$$\frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} + 2\frac{dy}{dx} = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$
$$\frac{dy}{dx} = \frac{1 - \dfrac{\sqrt{y}}{2\sqrt{x}}}{\dfrac{\sqrt{x}}{2\sqrt{y}} + 2} \cdot \frac{2\sqrt{x}\sqrt{y}}{2\sqrt{x}\sqrt{y}}$$
$$= \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$
$$= \frac{2(x - 2y) - y}{x + 4(x - 2y)}$$
$$= \frac{2x - 5y}{5x - 8y}$$

At $(4, 1)$, $\dfrac{dy}{dx} = \dfrac{1}{4}$.

24.
$$(x + y)^3 = x^3 + y^3$$
$$3(x + y)^2\left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$$
$$3(x + y)^2 + 3(x + y)^2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$$
$$(x + y)^2\frac{dy}{dx} - y^2\frac{dy}{dx} = x^2 - (x + y)^2$$
$$\frac{dy}{dx}\left[(x + y)^2 - y^2\right] = x^2 - (x^2 + 2xy + y^2)$$
$$\frac{dy}{dx} = \frac{-(2xy + y^2)}{x^2 + 2xy} = -\frac{y(2x + y)}{x(x + 2y)}$$

At $(-1, 1)$, $\dfrac{dy}{dx} = -1$.

25.
$$y^2(x^2 + y^2) = 2x^2$$
$$y^2\left(2x + 2y\frac{dy}{dx}\right) + (x^2 + y^2)\left(2y\frac{dy}{dx}\right) = 4x$$
$$2xy^2 + 2y^3\frac{dy}{dx} + 2x^2y\frac{dy}{dx} + 2y^3\frac{dy}{dx} = 4x$$
$$\frac{dy}{dx}(4y^3 + 2x^2y) = 4x - 2xy^2$$
$$\frac{dy}{dx} = \frac{2x(2 - y^2)}{2y(2y^2 + x^2)}$$
$$\frac{dy}{dx} = \frac{x(2 - y^2)}{y(2y^2 + x^2)}$$

At $(1, 1)$, $\dfrac{dy}{dx} = \dfrac{1}{3}$.

26.
$$(x^2 + y^2)^2 = 8x^2y$$
$$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = 8x^2\frac{dy}{dx} + y(16x)$$
$$4x^3 + 4x^2y\frac{dy}{dx} + 4xy^2 + 4y^3\frac{dy}{dx} = 8x^2\frac{dy}{dx} + 16xy$$
$$\frac{dy}{dx}(4x^2y + 4y^3 - 8x^2) = 16xy - 4x^3 - 4xy^2$$
$$\frac{dy}{dx} = \frac{4(4xy - x^3 - xy^2)}{4(x^2y + y^3 - 2x^2)}$$
$$\frac{dy}{dx} = \frac{x(4y - x^2 - y^2)}{x^2y + y^3 - 2x^2}$$

At $(2, 2)$, $\dfrac{dy}{dx} = 0$.

27. $3x^2 - 2y + 5 = 0$
$$6x - 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = 3x$$

At $(1, 4)$, $\dfrac{dy}{dx} = 3$.

28. $4x^2 + 2y - 1 = 0$
$$8x + 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{8x}{2} = -4x$$
$$\frac{dy}{dx}(-1) = -4(-1) = 4$$

29.
$$x^2 + y^2 = 4$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

At $\left(\sqrt{3}, 1\right)$, $\dfrac{dy}{dx} = -\dfrac{\sqrt{3}}{1} = -\sqrt{3}.$

30.
$$4x^2 + 9y^2 = 36$$
$$8x + 18\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{4x}{9y}$$

At $\left(\sqrt{5}, \dfrac{4}{3}\right)$, $\dfrac{dy}{dx} = -\dfrac{4\sqrt{5}}{9(4/3)} = -\dfrac{\sqrt{5}}{3}.$

31.
$$x^2 - y^3 = 0$$
$$2x - 3y^2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

At $(-1, 1)$, $\dfrac{dy}{dx} = -\dfrac{2}{3}.$

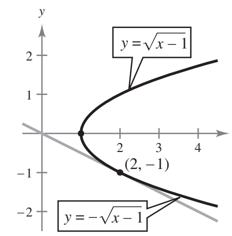
32.
$$(4 - x)y^2 = x^3$$
$$y^2 = \frac{x^3}{4 - x}$$
$$2y\frac{dy}{dx} = \frac{(4 - x)\left(3x^2\right) - x^3(-1)}{(4 - x)^2}$$
$$2y\frac{dy}{dx} = \frac{12x^2 - 3x^3 + x^3}{(4 - x)^2}$$
$$2y\frac{dy}{dx} = \frac{12x^2 - 2x^3}{(4 - x)^2}$$
$$\frac{dy}{dx} = -\frac{2x^2(x - 6)}{2y(4 - x)^2}$$
$$\frac{dy}{dx} = -\frac{x^2(x - 6)}{y(4 - x)^2}$$

At $(2, 7)$, $\dfrac{dy}{dx} = 2.$

33. Implicitly: $1 - 2y\dfrac{dy}{dx} = 0$
$$\frac{dy}{dx} = \frac{1}{2y}$$

Explicitly:
$$y = \pm\sqrt{x - 1}$$
$$= \pm(x - 1)^{1/2}$$
$$\frac{dy}{dx} = \pm\frac{1}{2}(x - 1)^{-1/2}(1)$$
$$= \pm\frac{1}{2\sqrt{x - 1}}$$
$$= \frac{1}{2\left(\pm\sqrt{x - 1}\right)}$$
$$= \frac{1}{2y}$$

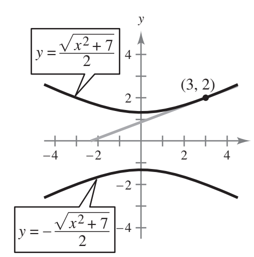
At $(2, -1)$, $\dfrac{dy}{dx} = -\dfrac{1}{2}.$



34. Implicitly: $8y\dfrac{dy}{dx} - 2x = 0$
$$\frac{dy}{dx} = \frac{x}{4y}$$

Explicitly:
$$y = \pm\frac{1}{2}\sqrt{x^2 + 7}$$
$$= \pm\frac{1}{2}\left(x^2 + 7\right)^{1/2}$$
$$\frac{dy}{dx} = \pm\frac{1}{4}\left(x^2 + 7\right)^{-1/2}(2x)$$
$$= \pm\frac{x}{2\sqrt{x^2 + 7}}$$
$$= \frac{x}{4\left(\pm\frac{1}{2}\sqrt{x^2 + 7}\right)}$$
$$= \frac{x}{4y}$$

At (3, 2), $\dfrac{dy}{dx} = \dfrac{3}{8}.$



35. $$x^2 + y^2 = 100$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (8, 6):

$$m = -\frac{4}{3}$$

$$y - 6 = -\frac{4}{3}(x - 8)$$

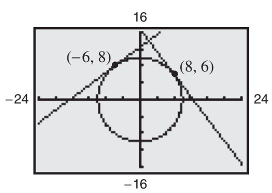
$$y = -\frac{4}{3}x + \frac{50}{3}$$

At $(-6, 8)$:

$$m = \frac{3}{4}$$

$$y - 8 = \frac{3}{4}(x + 6)$$

$$y = \frac{3}{4}x + \frac{25}{2}$$



36. $$x^2 + y^2 = 9$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (0, 3):

$$m = 0$$

$$y - 3 = 0(x - 0)$$

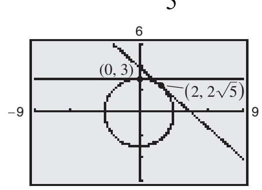
$$y = 3$$

At $\left(2, \sqrt{5}\right)$:

$$m = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$y - \sqrt{5} = -\frac{2\sqrt{5}}{5}(x - 2)$$

$$y = -\frac{2\sqrt{5}}{5}x + \frac{9\sqrt{5}}{5}$$



37. $$y^2 = 5x^3$$

$$2y\frac{dy}{dx} = 15x^2$$

$$\frac{dy}{dx} = \frac{15x^2}{2y}$$

At $\left(1, \sqrt{5}\right)$:

$$m = \frac{15}{2\sqrt{5}} = \frac{3\sqrt{5}}{2}$$

$$y - \sqrt{5} = \frac{3\sqrt{5}}{2}(x - 1)$$

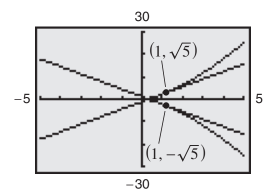
$$y = \frac{3\sqrt{5}}{2}x - \frac{\sqrt{5}}{2}$$

At $\left(1, -\sqrt{5}\right)$:

$$m = \frac{-15}{2\sqrt{5}} = -\frac{3\sqrt{5}}{2}$$

$$y + \sqrt{5} = -\frac{3\sqrt{5}}{2}(x - 1)$$

$$y = -\frac{3\sqrt{5}}{2}x + \frac{\sqrt{5}}{2}$$



38. $$4xy + x^2 = 5$$

$$4x\frac{dy}{dx} + 4y + 2x = 0$$

$$\frac{dy}{dx} = -\frac{4y + 2x}{4x} = -\frac{2y + x}{2x}$$

At (1, 1):

$$m = -\frac{3}{2}$$

$$y - 1 = -\frac{3}{2}(x - 1)$$

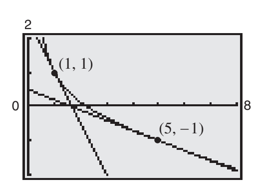
$$y = -\frac{3}{2}x + \frac{5}{2}$$

At $(5, -1)$:

$$m = -\frac{3}{10}$$

$$y + 1 = -\frac{3}{10}(x - 5)$$

$$y = -\frac{3}{10}x + \frac{1}{2}$$



39.

$$\begin{aligned} x^3 + y^3 &= 8 \\ 3x^2 + 3y^2\frac{dy}{dx} &= 0 \\ 3y^2\frac{dy}{dx} &= -3x^2 \\ \frac{dy}{dx} &= -\frac{x^2}{y^2} \end{aligned}$$

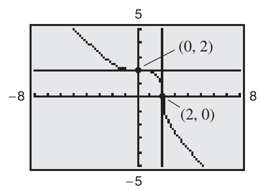
At $(0, 2)$:

$$m = \frac{dy}{dx} = 0$$

$$\begin{aligned} y - 2 &= 0(x - 0) \\ y &= 2 \end{aligned}$$

At $(2, 0)$:

$m = \dfrac{dy}{dx}$ is undefined.



The tangent line is $x = 2$.

40.

$$\begin{aligned} x^2y - 8 &= -4y \\ x^2y + 4y &= 8 \\ y(x^2 + 4) &= 8 \\ y &= \frac{8}{x^2 + 4} = 8(x^2 + 4)^{-1} \\ \frac{dy}{dx} &= 8(-1)(x^2 + 4)^{-2}(2x) \\ \frac{dy}{dx} &= -\frac{16x}{(x^2 + 4)^2} \end{aligned}$$

At $(-2, 1)$:

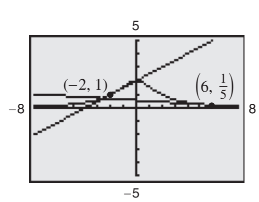
$$m = \frac{dy}{dx} = -\frac{16(-2)}{\left((-2)^2 + 4\right)^2} = \frac{32}{64} = \frac{1}{2}$$

$$\begin{aligned} y - 1 &= \frac{1}{2}(x - (-2)) \\ y &= \frac{1}{2}x + 2 \end{aligned}$$

At $\left(6, \dfrac{1}{5}\right)$:

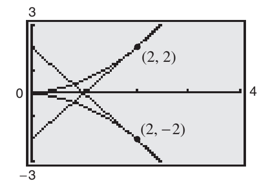
$$m = \frac{dy}{dx} = -\frac{16(6)}{\left[(6)^2 + 4\right]^2} = -\frac{96}{1600} = -\frac{3}{50}$$

$$\begin{aligned} y - \frac{1}{5} &= -\frac{3}{50}(x - 6) \\ y - \frac{1}{5} &= -\frac{3}{50}x + \frac{9}{25} \\ y &= -\frac{3}{50}x + \frac{14}{25} \end{aligned}$$



41.

$$\begin{aligned} y^2 &= \frac{x^3}{4 - x} \\ 2y\frac{dy}{dx} &= \frac{(4 - x)(3x^2) - (x^3)(-1)}{(4 - x)^2} \\ 2y\frac{dy}{dx} &= \frac{2x^2(6 - x)}{(4 - x)^2} \\ \frac{dy}{dx} &= \frac{x^2(6 - x)}{y(4 - x)^2} \end{aligned}$$



At $(2, 2)$:

$$\begin{aligned} m &= 2 \\ y - 2 &= 2(x - 2) \\ y &= 2x - 2 \end{aligned}$$

At $(2, -2)$:

$$\begin{aligned} m &= -2 \\ y + 2 &= -2(x - 2) \\ y &= -2x + 2 \end{aligned}$$

42.

$$\begin{aligned} x + y^3 &= 6xy^3 - 1 \\ y^3 - 6xy^3 &= -1 - x \\ y^3(1 - 6x) &= -(1 + x) \\ y^3 &= \frac{x + 1}{6x - 1} \\ 3y^2\frac{dy}{dx} &= \frac{(6x - 1)(1) - (x + 1)(6)}{(6x - 1)^2} \\ 3y^2\frac{dy}{dx} &= \frac{6x - 1 - 6x - 6}{(6x - 1)^2} \\ \frac{dy}{dx} &= -\frac{7}{3y^2(6x - 1)^2} \end{aligned}$$

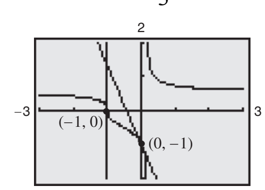
At $(-1, 0)$:

$m = \dfrac{dy}{dx}$ is undefined. The tangent line is $x = -1$.

At $(0, -1)$:

$$m = \frac{dy}{dx} = -\frac{7}{3}$$

$$\begin{aligned} y - (-1) &= -\frac{7}{3}(x - 0) \\ y &= -\frac{7}{3}x - 1 \end{aligned}$$



43. $p = \dfrac{2}{0.00001x^3 + 0.1x},\ x \geq 0$

$$0.00001x^3 + 0.1x = \frac{2}{p}$$

$$0.00003x^2\frac{dx}{dp} + 0.1\frac{dx}{dp} = -\frac{2}{p^2}$$

$$\left(0.00003x^2 + 0.1\right)\frac{dx}{dp} = -\frac{2}{p^2}$$

$$\frac{dx}{dp} = -\frac{2}{p^2\left(0.00003x^2 + 0.1\right)}$$

44. $p = \dfrac{4}{0.000001x^2 + 0.05x + 1},\ x \geq 0$

$$0.000001x^2 + 0.05x + 1 = \frac{4}{p}$$

$$0.000002x\frac{dx}{dp} + 0.05\frac{dx}{dp} = -\frac{4}{p^2}$$

$$(0.000002x + 0.05)\frac{dx}{dp} = -\frac{4}{p^2}$$

$$\frac{dx}{dp} = -\frac{4}{p^2(0.000002x + 0.05)}$$

45. $p = \sqrt{\dfrac{200 - x}{2x}},\ 0 < x \leq 200$

$$2xp^2 = 200 - x$$

$$2x(2p) + p^2\left(2\frac{dx}{dp}\right) = -\frac{dx}{dp}$$

$$\left(2p^2 + 1\right)\frac{dx}{dp} = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

46. $p = \sqrt{\dfrac{500 - x}{2x}},\ 0 < x \leq 500$

$$2xp^2 = 500 - x$$

$$2x(2p) + p^2\left(2\frac{dx}{dp}\right) = -\frac{dx}{dp}$$

$$\frac{dx}{dp}\left(2p^2 + 1\right) = -4xp$$

$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

47. (a) $100x^{0.75}y^{0.25} = 135{,}540$

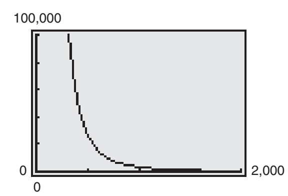
$$100x^{0.75}\left(0.25y^{-0.75}\frac{dy}{dx}\right) + y^{0.25}\left(75x^{-0.25}\right) = 0$$

$$\frac{25x^{0.75}}{y^{0.75}} \cdot \frac{dy}{dx} = -\frac{75y^{0.25}}{x^{0.25}}$$

$$\frac{dy}{dx} = -\frac{3y}{x}$$

When $x = 1500$ and $y = 1000$, $\dfrac{dy}{dx} = -2$.

(b)



If more labor is used, then less capital is available.
If more capital is used, then less labor is available.

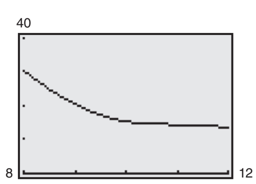
48. (a) As price increases, the demand decreases.

(b) For $x > 0$, the rate of change of demand, x, with respect to the price, p, is always decreasing; that is, for $x > 0$, $\dfrac{dx}{dp}$ is never increasing.

49. (a) $y^2 - 35{,}892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$

$$y^2 = -27.0021t^3 + 888.789t^2 - 9753.25t + 35{,}892.5$$

$$y = \pm\sqrt{-27.0021t^3 + 888.789t^2 - 9753.25t + 35{,}892.5}$$



The numbers of cases of Chickenpox decreases from 2008 to 2012.

(b) It appears that the number of reported cases was decreasing at the greatest rate during 2008, $t = 8$.

(c) $$y^2 - 35{,}892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$

$$2y\frac{dy}{dt} = -81.0063t^2 + 1777.578t - 9753.25t$$

$$y' = \frac{dy}{dt} = \frac{-81.0063t^2 + 1777.578t - 9753.25}{2y}$$

t	8	9	10	11	12
y	30.40	20.51	15.39	14.51	13.40
y'	−11.79	−7.71	−2.54	−0.06	−3.26

The table of values for y' agrees with the answer in part (b) when the greatest value of y' is -11.79 thousand cases per year.

Section 2.8 Related Rates

Skills Warm Up

1. $A = \pi r^2$

2. $V = \frac{4}{3}\pi r^3$

3. $SA = 6s^2$

4. $V = s^3$

5. $V = \frac{1}{3}\pi r^2 h$

6. $A = \frac{1}{2}bh$

7.
$$\begin{aligned} x^2 + y^2 &= 9 \\ \frac{d}{dx}\left[x^2 + y^2\right] &= \frac{d}{dx}[9] \\ 2x + 2y\frac{dy}{dx} &= 0 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= \frac{-2x}{2y} \\ &= \frac{-x}{y} \end{aligned}$$

8.
$$\begin{aligned} 3xy - x^2 &= 6 \\ \frac{d}{dx}\left[3xy - x^2\right] &= \frac{d}{dx}[6] \\ 3y + 3x\frac{dy}{dx} - 2x &= 0 \\ 3x\frac{dy}{dx} &= 2x - 3y \\ \frac{dy}{dx} &= \frac{2x - 3y}{3x} \end{aligned}$$

9.
$$\begin{aligned} x^2 + 2y + xy &= 12 \\ \frac{d}{dx}\left[x^2 + 2y + xy\right] &= \frac{d}{dx}(12) \\ 2x + 2\frac{dy}{dx} + y + x\frac{dy}{dx} &= 0 \\ 2\frac{dy}{dx} + x\frac{dy}{dx} &= -y - 2x \\ \frac{dy}{dx}(2 + x) &= -y - 2x \\ \frac{dy}{dx} &= \frac{-y - 2x}{2 + x} \end{aligned}$$

10.
$$\begin{aligned} x + xy^2 - y^2 &= xy \\ \frac{d}{dx}\left[x + xy^2 - y^2\right] &= \frac{d}{dx}[xy] \\ 1 + y^2 + 2xy\frac{dy}{dx} - 2y\frac{dy}{dx} &= y + x\frac{dy}{dx} \\ 2xy\frac{dy}{dx} - 2y\frac{dy}{dx} - x\frac{dy}{dx} &= y - y^2 - 1 \\ \frac{dy}{dx}(2xy - 2y - x) &= y - y^2 - 1 \\ \frac{dy}{dx} &= \frac{y - y^2 - 1}{2xy - 2y - x} \end{aligned}$$

1. $y = \sqrt{x}$, $\dfrac{dy}{dt} = \dfrac{1}{2}x^{-1/2}\dfrac{dx}{dt} = \dfrac{1}{2\sqrt{x}}\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = 2\sqrt{x}\dfrac{dy}{dt}$

(a) When $x = 4$ and $\dfrac{dx}{dt} = 3$, $\dfrac{dy}{dt} = \left(\dfrac{1}{2\sqrt{4}}\right)(3) = \dfrac{3}{4}$.

(b) When $x = 25$ and $\dfrac{dy}{dt} = 2$, $\dfrac{dx}{dt} = 2\sqrt{25}(2) = 20$.

2. $y = 3x^2 - 5x$, $\dfrac{dy}{dt} = 6x\dfrac{dx}{dt} - 5\dfrac{dx}{dt}$, $\dfrac{dy}{dt} = (6x - 5)\dfrac{dx}{dt}$, $\dfrac{\frac{dy}{dt}}{6x - 5} = \dfrac{dx}{dt}$

(a) When $x = 3$ and $\dfrac{dx}{dt} = 2$, $\dfrac{dy}{dt} = \big(6(3) - 5(2)\big) = 26$.

(b) When $x = 2$ and $\dfrac{dy}{dt} = 4$, $\dfrac{4}{6(2) - 5} = \dfrac{4}{7} = \dfrac{dx}{dt}$.

3. $xy = 4$, $x\dfrac{dy}{dt} + y\dfrac{dx}{dt} = 0$, $\dfrac{dy}{dt} = \left(-\dfrac{y}{x}\right)\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = \left(-\dfrac{x}{y}\right)\dfrac{dy}{dt}$

(a) When $x = 8$, $y = \dfrac{1}{2}$, and $\dfrac{dx}{dt} = 10$, $\dfrac{dy}{dt} = -\dfrac{1/2}{8}(10) = -\dfrac{5}{8}$.

(b) When $x = 1$, $y = 4$, and $\dfrac{dy}{dt} = -6$, $\dfrac{dx}{dt} = -\dfrac{1}{4}(-6) = \dfrac{3}{2}$.

4. $x^2 + y^2 = 25$, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$, $\dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = -\dfrac{y}{x}\dfrac{dy}{dt}$

(a) When $x = 3$, $y = 4$, and $\dfrac{dx}{dt} = 8$, $\dfrac{dy}{dt} = -\dfrac{3}{4}(8) = -6$.

(b) When $x = 4$, $y = 3$, and $\dfrac{dy}{dt} = -2$, $\dfrac{dx}{dt} = -\dfrac{3}{4}(-2) = \dfrac{3}{2}$.

5. $A = \pi r^2$, $\dfrac{dr}{dt} = 3$, $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 6\pi r$

(a) When $r = 6$, $\dfrac{dA}{dt} = 2\pi(6)(3) = 36\pi$ in.^{2}/min.

(b) When $r = 24$, $\dfrac{dA}{dt} = 2\pi(24)(3) = 144\pi$ in.^{2}/min.

6. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dr}{dt} = 3$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt} = 12\pi r^2$

(a) When $r = 9$, $\dfrac{dV}{dt} = 12\pi(9)^2 = 972\pi$ in.^{3}/min.

(b) When $r = 16$, $\dfrac{dV}{dt} = 12\pi(16)^2 = 3072\pi$ in.^{3}/min.

7. $A = \pi r^2$, $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

If $\dfrac{dr}{dt}$ is constant, then $\dfrac{dA}{dt}$ is not constant; $\dfrac{dA}{dt}$ is proportional to r.

8. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

If $\dfrac{dr}{dt}$ is constant, $\dfrac{dV}{dt}$ is *not* constant since it is proportional to the square of r.

9. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dV}{dt} = 10$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$,

$\dfrac{dr}{dt} = \left(\dfrac{1}{4\pi r^2}\right)\dfrac{dV}{dt}$

(a) When $r = 1$, $\dfrac{dr}{dt} = \dfrac{1}{4\pi(1)^2}(10) = \dfrac{5}{2\pi}$ ft/min.

(b) When $r = 2$, $\dfrac{dr}{dt} = \dfrac{1}{4\pi(2)^2}(10) = \dfrac{5}{8\pi}$ ft/min.

10. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt} = 6\pi r^2$

(a) When $r = 6$, $\frac{dV}{dt} = 6\pi(6)^2 = 216\pi$ in.^{3}/min.

(b) When $r = 24$, $\frac{dV}{dt} = 6\pi(24)^2 = 3456\pi$ in.^{3}/min.

11. (a) $\frac{dC}{dt} = 0.75\frac{dx}{dt} = 0.75(150)$

$= 112.5$ dollars per week

(b) $\frac{dR}{dt} = 250\frac{dx}{dt} - \frac{1}{5}x\frac{dx}{dt}$

$= 250(150) - \frac{1}{5}(1000)(150)$

$= 7500$ dollars per week

(c) $P = R - C$

$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 7500 - 112.5$

$= 7387.5$ dollars per week

12. (a) $\frac{dC}{dt} = 1.05\frac{dx}{dt} = 1.05(250) = 262.5$ dollars/week

(b) $\frac{dR}{dt} = \left(500 - \frac{2x}{25}\right)\frac{dx}{dt} = \left(500 - \frac{2(5000)}{25}\right)(250)$

$= 25{,}000$ dollars/week

(c) $P = R - C$

$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 25{,}000 - 262.5$

$= 24{,}737.5$ dollars/week

13. $R = 1200x - x^2$, $\frac{dR}{dt} = 1200\frac{dx}{dt} - 2x\frac{dx}{dt}$,

$\frac{dR}{dt} = (1200 - 2x)\frac{dx}{dt}$

(a) When $\frac{dx}{dt} = 23$ units/day and $x = 300$ units,

$\frac{dR}{dt} = [1200 - 2(300)](23) = \$13{,}800$ per day.

(b) When $\frac{dx}{dt} = 23$ units/day and $x = 450$ units,

$\frac{dR}{dt} = [1200 - 2(450)](23) = \$6900$ per day.

14. $R = 510x - 0.3x^2$, $\frac{dR}{dt} = 510\frac{dx}{dt} - 0.6x\frac{dx}{dt}$,

$\frac{dR}{dt} = (510 - 0.6x)\frac{dx}{dt}$

(a) When $\frac{dx}{dt} = 9$ units/day and $x = 400$ units,

$\frac{dR}{dt} = [510 - 0.6(400)](9) = \$2430$ per day.

(b) When $\frac{dx}{dt} = 9$ units/day and $x = 600$ units,

$\frac{dR}{dt} = [510 - 0.6(600)](9) = \$1350$ per day.

15. $V = x^3$, $\frac{dx}{dt} = 6$, $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

(a) When $x = 2$, $\frac{dV}{dt} = 3(2)^2(6) = 72$ cm^3/sec.

(b) When $x = 10$, $\frac{dV}{dt} = 3(10)^2(6) = 1800$ cm^3/sec.

16. $A = 6x^2$, $\frac{dx}{dt} = 6$, $\frac{dA}{dt} = 12x\frac{dx}{dt}$

(a) When $x = 2$, $\frac{dA}{dt} = 12(2)(6) = 144$ cm^2/sec.

(b) When $x = 10$, $\frac{dA}{dt} = 12(10)(6) = 720$ cm^2/sec.

17. Let x be the distance from the boat to the dock and y be the length of the rope.

$12^2 + x^2 = y^2$

$\frac{dy}{dt} = -4$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

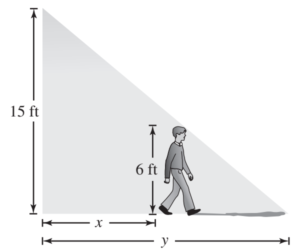
$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$

When $y = 13$, $x = 5$ and

$\frac{dx}{dt} = \frac{13}{5}(-4) = -10.4$ ft/sec.

As $x \to 0$, $\frac{dx}{dt}$ increases.

18.



(a) $\dfrac{15}{6} = \dfrac{y}{y - x} \Rightarrow 15y - 15x = 6y$

$$9y = 15x$$

$$y = \frac{5}{3}x$$

Find $\dfrac{dy}{dt}$ if $\dfrac{dx}{dt} = 5$ ft/sec when $x = 10$ ft.

$$\frac{dy}{dt} = \frac{5}{3}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{3}(5) = \frac{25}{3} \text{ ft/sec}$$

(b) Find $\dfrac{d}{dt}(y - x)$ if $\dfrac{dx}{dt} = 5$ ft/sec and

$\dfrac{dy}{dt} = \dfrac{25}{3}$ ft/sec when $x = 10$ ft.

$$\frac{d}{dt}(y - x) = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3} \text{ ft/sec}$$

19. $x^2 + 6^2 = s^2$

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

When $s = 10$, $x = 8$ and $\dfrac{ds}{dt} = 240$:

$$\frac{dx}{dt} = \frac{10}{8}(-240) = 300 \text{ mi/hr.}$$

20. $s^2 = 90^2 + x^2$, $x = 26$, $\dfrac{dx}{dt} = -30$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt}$$

When $x = 26$,

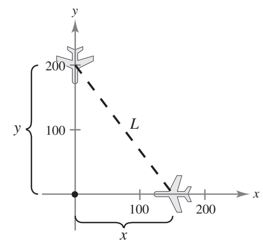
$$\frac{ds}{dt} = \frac{26}{\sqrt{90^2 + 26^2}}(-30) \approx -8.33 \text{ ft/sec.}$$

21. (a) $L^2 = x^2 + y^2$, $\dfrac{dx}{dt} = -450$, $\dfrac{dy}{dt} = -600$, and

$$\frac{dL}{dt} = \frac{x(dx/dt) + y(dy/dt)}{L}$$

When $x = 150$ and $y = 200$, $L = 250$ and

$$\frac{dL}{dt} = \frac{150(-450) + 200(-600)}{250} = -750 \text{ mph.}$$



(b) $t = \dfrac{250}{750} = \dfrac{1}{3}$ hr $= 20$ min

22. $S = 2250 + 50x + 0.35x^2$

$$\frac{dS}{dt} = 50\frac{dx}{dt} + 0.70x\frac{dx}{dt}$$

$$\frac{dS}{dt} = 50(125) + 0.70(1500)(125)$$

$$= \$137{,}500 \text{ per week}$$

23. $V = \pi r^2 h$, $h = 0.08$, $V = 0.08\pi r^2$, $\dfrac{dV}{dt} = 0.16\pi r\dfrac{dr}{dt}$

When $r = 150$ and $\dfrac{dr}{dt} = \dfrac{1}{2}$,

$$\frac{dV}{dt} = 0.16\pi(150)\left(\frac{1}{2}\right) = 12\pi = 37.70 \text{ ft}^3\text{/min.}$$

24. $P = R - C$

$$= xp - C$$

$$= x(50 - 0.01x) - (4000 + 40x - 0.02x^2)$$

$$= 50x - 0.01x^2 - 4000 - 40x + 0.02x^2$$

$$= 0.01x^2 + 10x - 4000$$

$$\frac{dP}{dt} = 0.02x\frac{dx}{dt} + 10\frac{dx}{dt}$$

When $x = 800$ and $\dfrac{dx}{dt} = 25$,

$$\frac{dP}{dt} = 0.02(800)(25) + (10)(25) = \$650\text{/week.}$$

25. $P = R - C = xp - C = x(6000 - 25x) - (2400x + 5200)$

$$= -25x^2 + 3600x - 5200$$

$$\frac{dP}{dt} = -50x\frac{dx}{dt} + 3600\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{3600 - 50x}\frac{dP}{dt}$$

When $x = 44$ and $\frac{dP}{dt} = 5600$, $\frac{dx}{dt} = \frac{1}{3600 - 50(44)}(5600) = 4$ units per week.

26. (a) For supply, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is negative. For demand, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is positive.

(b) For supply, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is positive. For demand, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is negative.

Review Exercises for Chapter 2

1. Slope $\approx \frac{-4}{2} = -2$

2. Slope $\approx \frac{4}{2} = 2$

3. Slope ≈ 0

4. Slope $\approx \frac{-2}{4} = -\frac{1}{2}$

5. Answers will vary. Sample answer:

$t = 8$; slope $\approx$ \$225 million/yr; Revenue was increasing by about \$225 million per year in 2008.

$t = 10$; slope $\approx$ \$350 million/yr; Revenue was increasing by about \$350 million per year in 2010.

6. Answers will vary. Sample answer:

$t = 10$; slope ≈ -20 thousand/year; The number of farms was decreasing by about 20 thousand per year in 2010.

$t = 12$; slope ≈ -10 thousand/year; The number of farms was decreasing by about 10 thousand per year in 2012.

7. Answers will vary. Sample answer:

$t = 1$: $m \approx 65$ hundred thousand visitors/month; The number of visitors to the national park is increasing at about 65,000,000/per month in January.

$t = 8$: $m \approx 0$ visitors/month; The number of visitors to the national park is neither increasing nor decreasing in August.

$t = 12$: $m \approx -1000$ hundred thousand/month; The number of visitors to the national park is decreasing at about 1,000,000,000 visitors per month in December.

8. (a) At t_1, the slope of $g(t)$ is greater than the slope of $f(t)$, so the rafter whose progress is given by $g(t)$ is traveling faster.

(b) At t_2, the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(c) At t_3, the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(d) The rafter whose progress is given by $f(t)$ finishes first. The value of t where $f(t) = 9$ is smaller than the value of t where $g(t) = 9$.

9. $f(x) = -3x - 5; (-2, 1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3(x + \Delta x) - 5 - (-3x - 5)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3\Delta x}{\Delta x} = -3$$

$$f'(-2) = -3$$

10. $f(x) = 7x + 3; (-1, -4)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7(x + \Delta x) + 3 - (7x + 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7\Delta x}{\Delta x} = 7$$

$$f'(-1) = 7$$

11. $f(x) = x^2 + 9; (3, 18)$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 9 - (x^2 + 9)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 9 - x^2 - 9}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x
\end{aligned}
$$

$f'(3) = 2(3) = 6$

12. $f(x) = x^2 - 7x; (1, -6)$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 7(x + \Delta x) - (x^2 - 7x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7x - 7\Delta x - x^2 + 7x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 7\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 7)}{\Delta x} \\
&= \lim_{\Delta x \to 0} (2x + \Delta x - 7) = 2x - 7
\end{aligned}
$$

$f'(1) = 2(1) - 7 = -5$

13. $f(x) = \sqrt{x + 9}; (-5, 2)$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 9} - \sqrt{x + 9}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 9) - (x + 9)}{\Delta x\left[\sqrt{x + \Delta x + 9} + \sqrt{x + 9}\right]} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} = \frac{1}{2\sqrt{x + 9}}
\end{aligned}
$$

$f'(-5) = \dfrac{1}{4}$

14. $f(x) = \sqrt{x - 1}; (10, 3)$

$$
\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 1} - \sqrt{x - 1}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1) - (x - 1)}{\Delta x\left[\sqrt{x + \Delta x - 1} + \sqrt{x - 1}\right]} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} = \frac{1}{2\sqrt{x - 1}}
\end{aligned}
$$

$f'(10) = \dfrac{1}{6}$

15. $f(x) = \dfrac{1}{x-5}; (6, 1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x-5} - \dfrac{1}{x-5}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x-5) - (x+\Delta x-5)}{\Delta x(x+\Delta x-5)(x-5)}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-5)(x-5)} = -\frac{1}{(x-5)^2}$$

$f'(6) = -1$

16. $f(x) = \dfrac{1}{x+6}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x+6} - \dfrac{1}{x+6}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x+6) - (x+\Delta x+6)}{\Delta x\left[(x+\Delta x+6)(x+6)\right]}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x+6)(x+6)}$$
$$= -\frac{1}{(x+6)^2}$$

$$f'(-3) = -\frac{1}{(-2+6)^2} = -\frac{1}{16}$$

17. $f(x) = 9x + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[9(x+\Delta x)+1\right] - (9x+1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{9x + 9\Delta x + 1 - 9x - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{9\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 9 = 9$$

18. $f(x) = 1 - 4x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[1 - 4(x+\Delta x)\right] - (1-4x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1 - 4x - 4\Delta x - 1 + 4x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-4\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} -4 = -4$$

19. $f(x) = -\dfrac{1}{2}x^2 + 2x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[-\frac{1}{2}(x+\Delta x)^2 + 2(x+\Delta x)\right] - \left(-\frac{1}{2}x^2 + 2x\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}x^2 - x(\Delta x) - \frac{1}{2}(\Delta x)^2 + 2x + 2\Delta x + \frac{1}{2}x^2 - 2x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(-x - \frac{1}{2}\Delta x + 2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(-x - \tfrac{1}{2}\Delta x + 2\right) = -x + 2$$

20. $f(x) = 3x^2 - \frac{1}{4}x$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[3(x + \Delta x)^2 - \frac{1}{4}(x + \Delta x)\right] - \left(3x^2 - \frac{1}{4}x\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}x - \frac{1}{4}\Delta x - 3x^2 + \frac{1}{4}x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x\left(6x + 3(\Delta x) - \frac{1}{4}\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(6x + 3(\Delta x) - \frac{1}{4}\right) = 6x - \frac{1}{4}
\end{aligned}$$

21. $f(x) = \sqrt{x - 5}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 5} - \sqrt{x - 5}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 5) - (x - 5)}{\Delta x\left(\sqrt{x + \Delta x - 5} + \sqrt{x - 5}\right)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}} = \frac{1}{2\sqrt{x - 5}}
\end{aligned}$$

22. $f(x) = \sqrt{x} + 3$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[\sqrt{x + \Delta x} + 3\right] - \left(\sqrt{x} + 3\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
\end{aligned}$$

23. $f(x) = \frac{5}{x}$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{5}{x + \Delta x} - \dfrac{5}{x}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\dfrac{5x - 5(x + \Delta x)}{x(x + \Delta x)}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x\left[x(x + \Delta x)\right]} \\
&= \lim_{\Delta x \to 0} -\frac{5}{x(x + \Delta x)} = -\frac{5}{x^2}
\end{aligned}$$

24. $f(x) = \dfrac{1}{x+4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x+4} - \dfrac{1}{x+4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{(x+4)-(x+\Delta x+4)}{(x+4)(x+\Delta x+4)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x\left[(x+4)(x+\Delta x+4)\right]}$$

$$= \lim_{\Delta x \to 0} -\frac{1}{(x+4)(x+\Delta x+4)} = -\frac{1}{(x+4)^2}$$

25. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a vertical tangent line.

26. y is not differentiable at $x = 0$. At $(0, 3)$, the graph has a node.

27. y is not differentiable at $x = 0$. The function is discontinuous at $x = 0$.

28. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a cusp.

29. $y = -6$
$y' = 0$

30. $f(x) = 5$
$f'(x) = 0$

31. $f(x) = x^7$
$f'(x) = 7x^6$

32. $h(x) = \dfrac{1}{x^{-4}}$
$h(x) = x^{-4}$
$h'(x) = -4x^{-5}$
$h'(x) = \dfrac{-4}{x^5}$

33. $f(x) = 4x^2$
$f'(x) = 8x$

34. $g(t) = 8t^6$
$g'(t) = 48t^5$

35. $f(x) = \dfrac{5x^3}{4}$
$f'(x) = \dfrac{15x^2}{4}$

36. $y = 3x^{2/3}$
$y' = 2x^{-1/3}$
$y' = \dfrac{2}{x^{1/3}}$

37. $g(x) = 2x^4 + 3x^2$
$g'(x) = 8x^3 + 6x$

38. $f(x) = 6x^2 - 4x$
$f'(x) = 12x - 4$

39. $y = x^2 + 6x - 7$
$y' = 2x + 6$

40. $y = 2x^4 - 3x^3 + x$
$y' = 8x^3 - 9x^2 + 1$

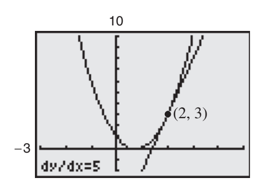
41. $f(x) = 2x^{-1/2}$; $(4, 1)$
$f'(x) = -x^{-3/2}$
$f'(4) = -(4)^{-3/2} = -0.125$

42. $y = \dfrac{3}{2x} + 3$; $\left(\dfrac{1}{2}, 6\right)$
$y = \dfrac{3}{2}x^{-1} + 3$
$y' = -\dfrac{3}{2}x^{-2} = -\dfrac{3}{2x^2}$
$y'\left(\dfrac{1}{2}\right) = \dfrac{3}{2\left(\frac{1}{2}\right)^2} = 6$

43. $g(x) = x^3 - 4x^2 - 6x + 8$; $(-1, 9)$
$g'(x) = 3x^2 - 8x - 6$
$g'(-1) = 3(-1)^2 - 8(-1) - 6 = 5$

44. $y = 2x^4 - 5x^3 + 6x^2 - x$; $(1, 2)$
$y' = 8x^3 - 15x^2 + 12x - 1$
$y'(1) = 8(1)^3 - 15(1)^2 + 12(1) - 1 = 4$

45. $f'(x) = 4x - 3$
$f'(2) = 5$
$y - 3 = 5(x - 2)$
$y = 5x - 7$

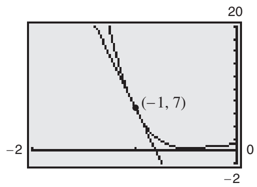


46. $y' = 44x^3 - 10x$

$y'(-1) = -34$

$y - 7 = -34(x + 1)$

$y = -34x - 27$



47. $f(x) = \sqrt{x} - \dfrac{1}{\sqrt{x}} = x^{1/2} - x^{-1/2}$

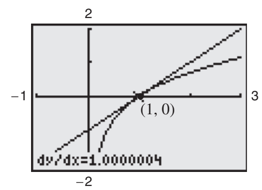
$$f'(x) = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

$$= \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$$

$f'(1) = 1$

$y - 0 = 1(x - 1)$

$y = x - 1$



48. $f(x) = \sqrt[3]{x} - x = x^{1/3} - x$

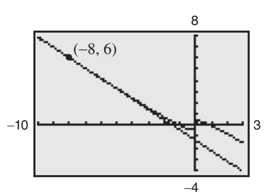
$$f'(x) = \frac{1}{3}x^{-2/3} - 1 = \frac{1}{3\sqrt[3]{x^2}} - 1$$

$$f'(-8) = \frac{1}{3\sqrt[3]{(-8)^2}} - 1 = \frac{1}{12} - 1 = -\frac{11}{12}$$

$$y - 6 = -\frac{11}{12}(x + 8)$$

$$y - 6 = -\frac{11}{12}x - \frac{22}{3}$$

$$y = -\frac{11}{12}x - \frac{4}{3}$$



49. $R = -0.5972t^3 + 51.187t^2 - 485.54t + 2199.0$

$$\frac{dR}{dt} = R'(t) = -1.7916t^2 + 102.374t - 485.54$$

(a) 2008: $R'(8) = m \approx 218.8$

2010: $R'(10) = m \approx 359.0$

(b) Results should be similar.

(c) The slope shows the rate at which sales were increasing or decreasing in that particular year, or value of t.

In 2008, the revenue was increasing about \$218.8 million per year, and in 2010, revenue was increasing about \$359.0 million per year.

50. $N = 0.2083t^4 - 7.954t^3 + 11.96t^2 - 706.5t + 3891$

$$\frac{dN}{dt} = N'(t) = 0.8332t^3 - 23.862t^2 + 23.92t - 706.5$$

(a) 2010: $N'(10) = m \approx -2020.33$

2012: $N'(12) = m \approx -2415.85$

(b) Results should be similar.

(c) The slope shows the rate at which the number of farms was increasing or decreasing in that particular year, or value of t.

In 2010, the number of farms was decreasing about 2020.33 thousand per year, and in 2012, the number of farms was decreasing about 2415.85 thousand per year.

51. $f(t) = 4t + 3; [-3, 1]$

Average rate of change:

$$\frac{f(1) - f(-3)}{1 - (-3)} = \frac{7 - (-9)}{4} = 4$$

Instantaneous rate of change:

$f'(t) = 4$

$f'(1) = 4$

$f'(-3) = 4$

52. $f(x) = x^2 + 3x - 4; [0, 1]$

Average rate of change: $\dfrac{f(1) - f(0)}{1 - 0} = \dfrac{0 - (-4)}{1} = 4$

Instantaneous rate of change:

$f'(x) = 2x + 3$

$f'(1) = 5$

$f'(0) = 3$

53. $f(x) = x^{2/3}; [1, 8]$

Average rate of change: $\dfrac{f(8) - f(1)}{8 - 1} = \dfrac{4 - 1}{7} = \dfrac{3}{7}$

Instantaneous rate of change:

$$f'(x) = \frac{2}{3x^{1/3}}$$

$$f'(8) = \frac{1}{3}$$

$$f'(1) = \frac{2}{3}$$

54. $f(x) = x^3 - x^2 + 3; [-2, 2]$

Average rate of change: $\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{7 - (-9)}{4} = 4$

Instantaneous rate of change: $f'(x) = 3x^2 - 2x$

$f'(2) = 8$

$f'(-2) = 16$

55. $s(t) = -16t^2 - 30t + 600$

(a) Average velocity $= \dfrac{s(3) - s(1)}{3 - 1} = \dfrac{366 - 554}{2} = -94$ ft/sec

(b) $v(t) = s'(t) = -32t - 30$

$v(1) = -62$ ft/sec

$v(3) = -126$ ft/sec

(c)
$$\begin{aligned} s(t) &= 0 \\ -16t^2 - 30 + 600 &= 0 \\ 16t^2 + 30t - 600 &= 0 \\ t &= \frac{-(30) \pm \sqrt{(30)^2 - 4(16)(-600)}}{2(16)} = \frac{-30 \pm \sqrt{39{,}300}}{32} \\ t &\approx 5.26 \text{ sec} \end{aligned}$$

(d) $v(t) = s'(5.26) = -32(5.26) - 30 = -198.32$ ft/sec

56. (a) $s(t) = -16t^2 + 276$

$v(t) = s'(t) = -32t$

(b) Average velocity $= \dfrac{s(2) - 2(0)}{2 - 0}$

$= \dfrac{212 - 276}{2}$

$= -32$ ft/sec

(c) $v(t) = -32t$

$v(2) = -64$ ft /sec

$v(3) = -96$ ft /sec

(d)
$$\begin{aligned} s(t) &= 0 \\ -16t^2 + 276 &= 0 \\ 16t^2 &= 276 \\ t^2 &= \frac{276}{16} \\ t &\approx 4.15 \text{ sec} \end{aligned}$$

(e) $v(4.15) = -132.8$ ft/sec

57. $C = 2500 + 320x$

$\dfrac{dC}{dx} = 320$

58. $C = 24{,}000 + 450x - x^2, 0 \le x \le 225$

$\dfrac{dC}{dx} = 450 - 2x$

59. $C = 370 + 2.55\sqrt{x} = 370 + 2.25x^{1/2}$

$\dfrac{dC}{dx} = \dfrac{1}{2}(2.55)(x^{-1/2}) = \dfrac{1.275}{\sqrt{x}}$

60. $C = 475 + 5.25x^{2/3}$

$\dfrac{dC}{dx} = 5.25\left(\dfrac{2}{3}x^{-1/3}\right) = \dfrac{3.5}{\sqrt[3]{x}}$

61. $R = 150x - 0.6x^2$

$\dfrac{dR}{dx} = 150 - 1.2x$

62. $R = 150x - \dfrac{3}{4}x^2$

$\dfrac{dR}{dx} = 150 - \dfrac{3}{2}x$

63. $R = -4x^3 + 2x^2 + 100x$

$\dfrac{dR}{dx} = -12x^2 + 4x + 100$

64. $R = 4x + 10x^{1/2}$

$$\frac{dR}{dx} = 4 + \frac{5}{x^{1/2}}$$

65. $P = -0.0002x^3 + 6x^2 - x - 2000$

$$\frac{dP}{dx} = -0.0006x^2 + 12x - 1$$

66. $P = -\frac{1}{15}x^3 + 4000x^2 - 120x - 144{,}000$

$$\frac{dP}{dx} = -\frac{1}{5}x^2 + 8000x - 120$$

67. $P = -0.05x^2 + 20x - 1000$

(a) Find $\frac{dP}{dx}$ when $x = 100$.

$$\frac{dP}{dx} = -0.1x + 20 = P'(x)$$

When $x = 100$, $\frac{dP}{dx} = P'(100) = \$10$.

(b) Find $\frac{\Delta P}{\Delta x}$ for $100 \le x \le 101$.

$$\frac{P(101) - P(100)}{101 - 100} = 509.95 - 500 = \$9.95$$

(c) Parts (a) and (b) differ by only $0.05.

68. $P = -0.021t^2 + 2.77t + 148.9$

(a) $P(0) = 148.9$

$P(4) = 159.644$

$P(8) = 169.716$

$P(12) = 179.116$

$P(16) = 187.844$

$P(20) = 195.9$

$P(23) = 201.501$

These values are the populations in millions for Brazil from 1990 to 2013.

(b) $\frac{dP}{dt} = -0.042t + 2.77 = P'(t)$

(c) $P'(0) = 2.77$

$P'(4) = 2.602$

$P'(8) = 2.434$

$P'(12) = 2.266$

$P'(16) = 2.098$

$P'(20) = 1.93$

$P'(23) = 1.804$

These are the rates at which the population of Brazil is changing in millions per year from 1990 to 2013.

69. $f(x) = x^3(5 - 3x^2) = 5x^3 - 3x^5$

$$f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2)$$

Simple Power Rule

70. $f(x) = 4x^2(2x^2 - 5) = 8x^4 - 20x^2$

$$f'(x) = 32x^3 - 40x = 8x(4x^2 - 5)$$

Simple Power Rule

71. $y = (4x - 3)(x^3 - 2x^2)$

$$\begin{aligned} y' &= (4x - 3)(3x^2 - 4x) + 4(x^3 - 2x^2) \\ &= 12x^3 - 25x^2 + 12x + 4x^3 - 8x^2 \\ &= 16x^3 - 33x^2 + 12x \end{aligned}$$

Product Rule and Simple Power Rule

72. $s = \left(4 - \frac{1}{t^2}\right)(t^2 - 3t) = (4 - t^{-2})(t^2 - 3t)$

$$\begin{aligned} s' &= (4 - t^{-2})(2t - 3) + (t^2 - 3t)(2t^{-3}) \\ &= 8t - 12 - 2t^{-1} + 3t^{-2} + 2t^{-1} - 6t^{-2} \\ &= 8t - 12 - 3t^{-2} \end{aligned}$$

Product Rule and Simple Power Rule

73. $g(x) = \frac{x}{x + 3}$

$$g'(x) = \frac{(x + 3)(1) - x(1)}{(x + 3)^2}$$

$$g'(x) = \frac{3}{(x + 3)^2}$$

Quotient Rule and Simple Power Rule

74. $f(x) = \frac{2 - 5x}{3x + 1}$

$$f'(x) = \frac{(3x + 1)(-5) - (2 - 5x)(3)}{(3x + 1)^2}$$

$$f'(x) = \frac{-15x - 5 - 6 + 15x}{(3x + 1)^2}$$

$$f'(x) = -\frac{11}{(3x + 1)^2}$$

Quotient Rule and Simple Power Rule

75. $f(x) = \dfrac{6x - 5}{x^2 + 1}$`

$$f'(x) = \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{6 + 10x - 6x^2}{(x^2 + 1)^2}$$

$$= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}$$

Quotient Rule and Simple Power Rule

76. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^3 + x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2 - 1)^2}$$

$$= \frac{-x^2 - 1}{(x^2 - 1)^2} = -\frac{x^2 + 1}{(x^2 - 1)^2}$$

Quotient Rule and Simple Power Rule

77. $f(x) = (5x^2 + 2)^3$

$$f'(x) = 3(5x^2 + 2)^2(10x)$$

$$= 30x(5x^2 + 2)^2$$

General Power Rule

78. $f(x) = \sqrt[3]{x^2 - 1} = (x^2 - 1)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 - 1)^{-2/3}(2x) = \frac{2x}{3(x^2 - 1)^{2/3}}$$

General Power Rule

79. $h(x) = \dfrac{2}{\sqrt{x + 1}} = 2(x + 1)^{-1/2}$

$$h'(x) = 2\left(-\frac{1}{2}\right)(x + 1)^{-3/2}$$

$$= -\frac{1}{(x + 1)^{3/2}}$$

General Power Rule

80. $g(x) = \dfrac{6}{(3x^2 - 5x)^4} = 6(3x^2 - 5x)^{-4}$

$$g'(x) = 6(-4)(3x^2 - 5x)^{-5}(6x - 5)$$

$$= -\frac{24(6x - 5)}{(3x^2 - 5x)^5}$$

General Power Rule

81. $g(x) = x\sqrt{x^2 + 1} = x(x^2 + 1)^{1/2}$

$$g'(x) = x\left[\frac{1}{2}(x^2 + 1)^{-1/2}(2x)\right] + (1)(x^2 + 1)^{1/2}$$

$$= (x^2 + 1)^{-1/2}\left[x^2 + (x^2 + 1)\right]$$

$$= \frac{2x^2 + 1}{\sqrt{x^2 + 1}}$$

Product and General Power Rule

82. $g(t) = \dfrac{t}{(1 - t)^3}$

$$g'(t) = \frac{(1 - t)^3(1) - t(3)(1 - t)^2(-1)}{(1 - t)^6}$$

$$= \frac{(1 - t)^3 + 3t(1 - t)^2}{(1 - t)^6}$$

$$= \frac{(1 - t) + 3t}{(1 - t)^4} = \frac{2t + 1}{(1 - t)^4}$$

Quotient Rule and General Power Rule

83. $f(x) = x(1 - 4x^2)^2$

$$f'(x) = x(2)(1 - 4x^2)(-8x) + (1 - 4x^2)^2$$

$$= -16x^2(1 - 4x^2) + (1 - 4x^2)^2$$

$$= (1 - 4x^2)\left[-16x^2 + (1 - 4x^2)\right]$$

$$= (1 - 4x^2)(1 - 20x^2)$$

Product and General Power Rule

84. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5 = (x^2 + x^{-1})^5$

$$f'(x) = 5(x^2 + x^{-1})^4(2x - x^{-2})$$

$$= 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

General Power Rule

85. $h(x) = \left[x^2(2x+3)\right]^3 = x^6(2x+3)^3$

$$h'(x) = x^6\left[3(2x+3)^2(2)\right] + 6x^5(2x+3)^3$$
$$= 6x^5(2x+3)^2\left[x + (2x+3)\right]$$
$$= 18x^5(2x+3)^2(x+1)$$

Product and General Power Rule

86. $f(x) = \left[(x-2)(x+4)\right]^2$

$$f'(x) = 2\left[(x-2)(x+4)\right]\left[(x-2) + (x+4)\right]$$
$$= 2(x-2)(x+4)(2x+2)$$
$$= 4(x-2)(x+4)(x+1)$$

Product and General Power Rule

87. $f(x) = x^2(x-1)^5$

$$f'(x) = 5x^2(x-1)^4 + 2x(x-1)^5$$
$$= x(x-1)^4\left[5x + 2(x-1)\right]$$
$$= x(x-1)^4(7x-2)$$

Product and General Power Rule

88. $f(s) = s^3(s^2-1)^{5/2}$

$$f'(s) = s^3\left(\tfrac{5}{2}\right)(s^2-1)^{3/2}(2s) + 3s^2(s^2-1)^{5/2}$$
$$= s^2(s^2-1)^{3/2}\left[5s^2 + 3(s^2-1)\right]$$
$$= s^2(s^2-1)^{3/2}(8s^2-3)$$

Product and General Power Rule

89. $h(t) = \dfrac{\sqrt{3t+1}}{(1-3t)^2} = \dfrac{(3t+1)^{1/2}}{(1-3t)^2}$

$$h'(t) = \frac{(1-3t)^2(1/2)(3t+1)^{-1/2}(3) - (3t+1)^{1/2}(2)(1-3t)(-3)}{(1-3t)^4}$$
$$= \frac{(3t+1)^{-1/2}\left[(1-3t)(3/2) + (3t+1)6\right]}{(1-3t)^3}$$
$$= \frac{3(9t+5)}{2\sqrt{3t+1}(1-3t)^3}$$

Quotient and General Power Rule

90. $g(x) = \left(\dfrac{3x+1}{x^2+1}\right)^2 = \dfrac{(3x+1)^2}{(x^2+1)^2}$

$$g'(x) = \frac{(x^2+1)^2(2)(3x+1)(3) - (3x+1)^2 2(x^2+1)2x}{(x^2+1)^4}$$
$$= \frac{6(x^2+1)^2(3x+1) - 4x(3x+1)^2(x^2+1)}{(x^2+1)^4}$$
$$= \frac{2(3x+1)(x^2+1)\left[3(x^2+1) - 2x(3x+1)\right]}{(x^2+1)^4}$$
$$= \frac{2(3x+1)(x^2+1)(-3x^2-2x+3)}{(x^2+1)^4}$$
$$= \frac{2(3x+1)(-3x^2-2x+3)}{(x^2+1)^3}$$

Quotient and General Power Rule.

91. $T = \dfrac{1300}{t^2 + 2t + 25} = 1300\left(t^2 + 2t + 25\right)^{-1}$

$T'(t) = -1300\left(t^2 + 2t + 25\right)^{-2}(2t + 2)$

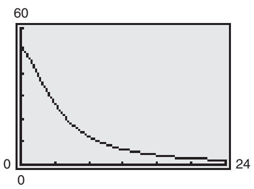
$= -\dfrac{2600(t + 1)}{\left(t^2 + 2t + 25\right)^2}$

(a) $T'(1) = -\dfrac{325}{49} \approx -6.63°\text{F/hr}$

$T'(3) = -\dfrac{13}{2} \approx -6.5°\text{F/hr}$

$T'(5) = -\dfrac{13}{3} \approx -4.33°\text{F/hr}$

$T'(10) = -\dfrac{1144}{841} \approx -1.36°\text{F/hr}$

(b) 

The rate of decrease is approaching zero.

92. When $L = 12$,

$V = \dfrac{L}{16}(D - 4)^2 = \dfrac{12}{16}(D - 4)^2 = \dfrac{3}{4}(D - 4)^2$

$\dfrac{dV}{dD} = \dfrac{3}{2}(D - 4).$

(a) When $D = 8$, $\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(8 - 4) = 6$ board ft/in.

(b) When $D = 16$,
$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(16 - 4) = 18$ board ft/in.

(c) When $D = 24$,
$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(24 - 4) = 30$ board ft/in.

(d) When $D = 36$,
$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(36 - 4) = 48$ board ft/in.

93. $f(x) = 3x^2 + 7x + 1$

$f'(x) = 6x + 7$

$f''(x) = 6$

94. $f'(x) = 5x^4 - 6x^2 + 2x$

$f''(x) = 20x^3 - 12x + 2$

$f'''(x) = 60x^2 - 12 = 12\left(5x^2 - 1\right)$

95. $f'''(x) = -\dfrac{3}{x^4} = -3x^{-4}$

$f^{(4)}(x) = 12x^{-5}$

$f^{(5)}(x) = -60x^{-6}$

$f^{(6)}(x) = 360x^{-7} = \dfrac{360}{x^7}$

96. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2}$

$f''(x) = -\dfrac{1}{4}x^{-3/2}$

$f'''(x) = \dfrac{3}{8}x^{-5/2}$

$f^{(4)}(x) = -\dfrac{15}{16}x^{-7/2} = -\dfrac{15}{16x^{7/2}}$

97. $f'(x) = 8x^{5/2}$

$f''(x) = 20x^{3/2}$

$f'''(x) = 30x^{1/2}$

$f^{(4)}(x) = 15x^{-1/2} = \dfrac{15}{x^{1/2}}$

98. $f''(x) = 9\sqrt[3]{x} = 9x^{1/3}$

$f'''(x) = 3x^{-2/3}$

$f^{(4)}(x) = -2x^{-5/3}$

$f^{(5)}(x) = \dfrac{10}{3}x^{-8/3} = \dfrac{10}{3x^{8/3}}$

99. $f(x) = x^2 + \dfrac{3}{x} = x^2 + 3x^{-1}$

$f'(x) = 2x - 3x^{-2}$

$f''(x) = 2 + 6x^{-3} = 2 + \dfrac{6}{x^3}$

100. $f'''(x) = 20x^4 - \dfrac{2}{x^3} = 20x^4 - 2x^{-3}$

$f^{(4)}(x) = 80x^3 + 6x^{-4}$

$f^{(5)}(x) = 240x^2 - 24x^{-5} = 240x^2 - \dfrac{24}{x^5}$

101. (a) $s(t) = -16t^2 + 5t + 30$

$v(t) = s'(t) = -32t + 5$

$a(t) = v'(t) = s''(t) = -32$

(b) $s(t) = 0 = -16t^2 + 5t + 30$

Using the Quadratic Formula, $t \approx 1.534$ seconds.

(c) $v(t) = s'(t) = -32t + 5$

$v(1.534) \approx -44.09$ ft/sec

(d) $a(t) = v'(t) = -32$ ft/sec^2

102. $s(t) = \dfrac{1}{t^2 + 2t + 1} = (t + 1)^{-2}$

$v(t) = s'(t) = -2(t + 1)^{-3} = -\dfrac{2}{(t + 1)^3}$

$a(t) = v'(t) = 6(t + 1)^{-4} = \dfrac{6}{(t + 1)^4}$

103.
$$x^2 + 3xy + y^3 = 10$$
$$2x + 3x\frac{dy}{dx} + 3y + 3y^2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(3x + 3y^2) = -2x - 3y$$
$$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2} = -\frac{2x + 3y}{3(x + y^2)}$$

104.
$$x^2 + 9xy + y^2 = 0$$
$$2x + 9y + 9x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$
$$(9x + 2y)\frac{dy}{dx} = -2x - 9y$$
$$\frac{dy}{dx} = \frac{-2x - 9y}{9x + 2y} = -\frac{2x + 9y}{9x + 2y}$$

105. $y^2 - x^2 + 8x - 9y - 1 = 0$
$$2y\frac{dy}{dx} - 2x + 8 - 9\frac{dy}{dx} = 0$$
$$(2y - 9)\frac{dy}{dx} = 2x - 8$$
$$\frac{dy}{dx} = \frac{2x - 8}{2y - 9}$$

106. $y^2 + x^2 - 6y - 2x - 5 = 0$
$$2y\frac{dy}{dx} + 2x - 6\frac{dy}{dx} - 2 = 0$$
$$\frac{dy}{dx}(2y - 6) = 2 - 2x$$
$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$$

107.
$$y^2 = x - y$$
$$2y\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$2y\frac{dy}{dx} + \frac{dy}{dx} = 1$$
$$(2y + 1)\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{2y + 1}$$

At $(2, 1)$, $\dfrac{dy}{dx} = \dfrac{1}{3}$.
$$y - 1 = \frac{1}{3}(x - 2)$$
$$y = \frac{1}{3}x + \frac{1}{3}$$

108.
$$2x^{1/3} + 3y^{1/2} = 10$$
$$\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{4y^{1/2}}{9x^{2/3}}$$

At $(8, 4)$, $\dfrac{dy}{dx} = -\dfrac{2}{9}$.
$$y - 4 = -\frac{2}{9}(x - 8)$$
$$y = -\frac{2}{9}x + \frac{52}{9}$$

109. $y^2 - 2x = xy$
$$2y\frac{dy}{dx} - 2 = x\frac{dy}{dx} + y$$
$$(2y - x)\frac{dy}{dx} = y + 2$$
$$\frac{dy}{dx} = \frac{y + 2}{2y - x}$$

At $(1, 2)$, $\dfrac{dy}{dx} = \dfrac{4}{3}$.
$$y - 2 = \frac{4}{3}(x - 1)$$
$$y = \frac{4}{3}x + \frac{2}{3}$$

110.
$$y^3 - 2x^2y + 3xy^2 = -1$$
$$3y^2\frac{dy}{dx} - 2x^2\frac{dy}{dx} - 4xy + 6xy\frac{dy}{dx} + 3y^2 = 0$$
$$\frac{dy}{dx}\left(3y^2 - 2x^2 + 6xy\right) = 4xy - 3y^2$$
$$\frac{dy}{dx} = \frac{4xy - 3y^2}{3y^2 - 2x^2 + 6xy}$$

At $(0, -1)$, $\dfrac{dy}{dx} = -1$.

$$y + 1 = -1(x - 0)$$
$$y = -x - 1$$

111. $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

(a) Find $\dfrac{dA}{dt}$ when $r = 3$ in. and $\dfrac{dr}{dt} = 2$ in./min.

$$\frac{dA}{dt} = 2\pi(3)(2) = 12\pi \text{ in.}^2/\text{min} \approx 37.7 \text{ in.}^2/\text{min}$$

(b) Find $\dfrac{dA}{dt}$ when $r = 10$ in. and $\dfrac{dr}{dt} = 2$ in./min.

$$\frac{dA}{dt} = 2\pi(10)(2) = 40\pi \text{ in.}^2/\text{min} \approx 125.7 \text{ in.}^2/\text{min}$$

112. $P = 375x - 1.5x^2$

$$\frac{dP}{dt} = 375\frac{dx}{dt} - 3.0x\frac{dx}{dt}$$
$$\frac{dP}{dt} = (375 - 3.0x)\frac{dx}{dt}$$

(a) $\dfrac{dP}{dt} = \left[375 - 3.0(50)\right](2) = \$450/\text{day}$

(b) $\dfrac{dP}{dt} = \left[375 - 3.0(100)\right](2) = \$150/\text{day}$

113. Let b be the horizontal distance of the water and h be the depth of the water at the deep end.

Then $b = 8h$ for $0 \le h \le 5$.

$$V = \frac{1}{2}bh(20) = 10bh = 10(8h)h = 80h^2$$
$$\frac{dV}{dt} = 160h\frac{dh}{dt}$$
$$\frac{dh}{dt} = \frac{1}{160h}\frac{dV}{dt} = \frac{1}{160h}(10) = \frac{1}{16h}$$

When $h = 4$, $\dfrac{dh}{dt} = \dfrac{1}{16(4)} = \dfrac{1}{64}$ ft/min.

114.
$$\begin{aligned} P &= R - C \\ &= xp - C \\ &= x(211 - 0.002x) - (30x + 1{,}500{,}000) \\ &= 181x - 0.002x^2 - 1{,}500{,}000 \end{aligned}$$
$$\frac{dP}{dt} = 181\frac{dx}{dt} - 0.004x\frac{dx}{dt}$$
$$\frac{dP}{dt} = (181 - 0.004x)\frac{dx}{dt}$$
$$\frac{dP}{dt} = \left[181 - 0.004(1600)\right](15) = \$2619/\text{week}$$

Chapter 2 Test Yourself

1. $f(x) = x^2 + 3;\ (3, 12)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 3 - (x^2 + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - (x^2 + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x[2x + (\Delta x)]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 2x + \Delta x$$
$$= 2x$$

At $(3, 12)$: $m = 2(3) = 6$

2. $f(x) = \sqrt{x} - 2;\ (4, 0)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - 2 - (\sqrt{x} - 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

At $(4, 0)$: $m = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

3. $f(t) = t^3 + 2t$

$f'(t) = 3t^2 + 2$

4. $f(x) = 4x^2 - 8x + 1$

$f'(x) = 8x - 8$

5. $f(x) = x^{3/2} + 6x^{1/2}$

$$f'(x) = \frac{3}{2}x^{1/2} + 3x^{-1/2} = \frac{3\sqrt{x}}{2} + \frac{3}{\sqrt{x}}$$

6. $f(x) = 5x^2 - \dfrac{3}{x^3} = 5x^2 - 3x^{-3}$

$$f'(x) = 10 + 9x^{-4} = 10x + \frac{9}{x^4}$$

7. $f(x) = (x + 3)(x^2 + 2x)$

$f(x) = x^3 + 5x^2 + 6x$

$f'(x) = 3x^2 + 10x + 6$

(Or use the Product Rule.)

8. $f(x) = \sqrt{x}(5 + x) = 5x^{1/2} + x^{3/2}$

$$f'(x) = \frac{5}{2}x^{-1/2} + \frac{3}{2}x^{1/2} = \frac{5}{2\sqrt{x}} + \frac{3\sqrt{x}}{2}$$

9. $f(x) = (3x^2 + 4)^2$

$$f'(x) = 2(3x^2 + 4)(6x)$$
$$= 36x^3 + 48x$$

10. $f(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2}$

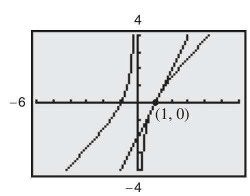
$$f'(x) = \frac{1}{2}(1 - 2x)^{-1/2}(-2)$$
$$= -\frac{1}{\sqrt{1 - 2x}}$$

11. $f(x) = \dfrac{(5x - 1)^3}{x}$

$$f'(x) = \frac{x(3)(5x - 1)^2(5) - (5x - 1)^3}{x^2}$$
$$= \frac{(5x - 1)^2[15x - (5x - 1)]}{x^2}$$
$$= \frac{(5x - 1)^2(10x + 1)}{x^2}$$

12. $f(x) = x - \dfrac{1}{x}$

$$f'(x) = 1 + \frac{1}{x^2}$$
$$f'(1) = 1 + \frac{1}{1^2} = 2$$
$$y - 0 = 2(x - 1)$$
$$y = 2x - 2$$



13. $S = -2.1083t^3 + 70.811t^2 - 777.05t + 2893.6$

(a) $\dfrac{\Delta S}{\Delta t}$ for $10 \le t \le 12$

$$\frac{S(12) - S(10)}{12 - 10} = \frac{122.6416 - 95.9}{2} = \$13.3708 \text{ billion/yr}$$

(b) $S'(t) = -6.3249t^2 + 141.622t - 777.05$

2010: $S'(10) = \$6.68$ billion/yr

2012: $S'(12) = \$11.6284$ billion/yr

(c) The annual sales of CVS Caremark from 2010 to 2012 increased by an average of about \$13.37 billion per year, and the instantaneous rates of change for 2010 and 2012 are \$6.68 billion per year and \$11.63 billion per year, respectively.

14. $P = 1700 - 0.016x$, $C = 715{,}000 + 240x$

Profit = Revenue − Cost

(a) Revenue: $R = xp$

$$R = x(1700 - 0.016x)$$
$$R = 1700x - 0.016x^2$$
$$P = R - C$$
$$P = (1700x - 0.016x^2) - (715{,}000 + 240x)$$
$$P = -0.016x^2 + 1460x - 715{,}000$$

(b) $\dfrac{dP}{dx} = -0.032x + 1460 = P'(x)$

$P'(700) = \$1437.60$

15. $f(x) = 2x^2 + 3x + 1$

$f'(x) = 4x + 3$

$f''(x) = 4$

$f'''(x) = 0$

16. $f(x) = \sqrt{3 - x} = (3 - x)^{1/2}$

$$f'(x) = \frac{1}{2}(3 - x)^{-1/2}(-1) = -\frac{1}{2}(3 - x)^{-1/2}$$
$$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(3 - x)^{-3/2}(-1) = -\frac{1}{4}(3 - x)^{-3/2}$$
$$f'''(x) = -\frac{1}{4}\left(-\frac{3}{2}\right)(3 - x)^{-5/2}(-1)$$
$$= -\frac{3}{8}(3 - x)^{-5/2}$$
$$= -\frac{3}{8(3 - x)^{5/2}}$$

17. $f(x) = \dfrac{2x + 1}{2x - 1}$

$$f'(x) = \frac{(2x - 1)(2) - (2x + 1)(2)}{(2x - 1)^2}$$
$$= \frac{4}{(2x - 1)^2}$$
$$= -4(2x - 1)^{-2}$$
$$f''(x) = 8(2x - 1)^{-3}(2) = 16(2x - 1)^{-3}$$
$$f'''(x) = -48(2x - 1)^{-4}(2) = -\frac{96}{(2x - 1)^4}$$

18. $s(t) = -16t^2 + 30t + 75$

$v(t) = s'(t) = -32t + 30$

$a(t) = v'(t) = s''(t) = -32$

At $t = 2$: $s(2) = 71$ ft

$v(2) = -34$ ft/sec

$a(2) = -32$ ft/sec^2

19. $x + xy = 6$

$$1 + x\frac{dy}{dx} + y = 0$$
$$x\frac{dy}{dx} = -y - 1$$
$$\frac{dy}{dx} = -\frac{y + 1}{x}$$

20. $y^2 + 2x - 2y + 1 = 0$

$$2y\frac{dy}{dx} + 2 - 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(2y - 2) = -2$$
$$\frac{dy}{dx} = -\frac{1}{y - 1}$$

21. $4x^2 - 3y^2 + x^3y = 5$

$$8x - 6y\frac{dy}{dx} + x^3\frac{dy}{dx}3x^2y = 0$$
$$-6y\frac{dy}{dx} + x^3\frac{dy}{dx} = -8x - 3x^2y$$
$$(x^3 - 6y)\frac{dy}{dx} = -(8x + 3x^2y)$$
$$\frac{dy}{dx} = -\frac{8x + 3x^2y}{x^3 - 6y}$$
$$\frac{dy}{dx} = \frac{8x + 3x^2y}{6y - x^3} = \frac{x(8 + 3xy)}{6y - x^3}$$

22. $V = \pi r^2 h = 20\pi r^3$

$$\frac{dV}{dt} = 60\pi r^2 \frac{dr}{dt}$$

(a) $\dfrac{dV}{dt} = 60\pi(0.5)^2(0.25) = 3.75\pi \text{ cm}^3/\text{min}$

(b) $\dfrac{dV}{dt} = 60\pi(1)^2(0.25) = 15\pi \text{ cm}^3/\text{min}$