2 FUNCTIONS, LIMITS, AND THE DERIVATIVE

2.1 Functions and Their Graphs

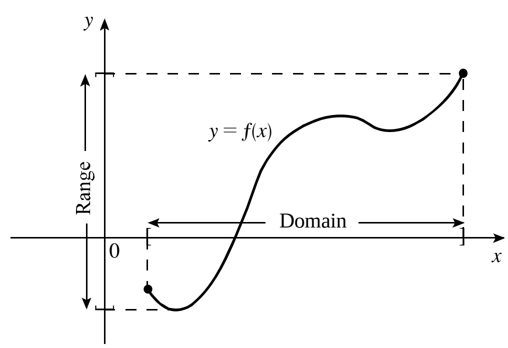
Concept Questions page 59

1. a. A function is a rule that associates with each element in a set A exactly one element in a set B.

b. The domain of a function f is the set of all elements x in the set such that $f(x)$ is an element in B. The range of f is the set of all elements $f(x)$ whenever x is an element in its domain.

c. An independent variable is a variable in the domain of a function f. The dependent variable is $y = f(x)$.

2. a. The graph of a function f is the set of all ordered pairs (x, y) such that $y = f(x)$, x being an element in the domain of f.



b. Use the vertical line test to determine if every vertical line intersects the curve in at most one point. If so, then the curve is the graph of a function.

3. a. Yes, every vertical line intersects the curve in at most one point.

b. No, a vertical line intersects the curve at more than one point.

c. No, a vertical line intersects the curve at more than one point.

d. Yes, every vertical line intersects the curve in at most one point.

4. The domain is $[1, 3) \cup [3, 5)$ and the range is $\left[\frac{1}{2}, 2\right) \cup (2, 4]$.

Exercises page 59

1. $f(x) = 5x + 6$. Therefore $f(3) = 5(3) + 6 = 21$, $f(-3) = 5(-3) + 6 = -9$, $f(a) = 5(a) + 6 = 5a + 6$, $f(-a) = 5(-a) + 6 = -5a + 6$, and $f(a + 3) = 5(a + 3) + 6 = 5a + 15 + 6 = 5a + 21$.

2. $f(x) = 4x - 3$. Therefore, $f(4) = 4(4) - 3 = 16 - 3 = 13$, $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 3 = 1 - 3 = -2$, $f(0) = 4(0) - 3 = -3$, $f(a) = 4(a) - 3 = 4a - 3$, $f(a + 1) = 4(a + 1) - 3 = 4a + 1$.

3. $g(x) = 3x^2 - 6x - 3$, so $g(0) = 3(0) - 6(0) - 3 = -3$, $g(-1) = 3(-1)^2 - 6(-1) - 3 = 3 + 6 - 3 = 6$, $g(a) = 3(a)^2 - 6(a) - 3 = 3a^2 - 6a - 3$, $g(-a) = 3(-a)^2 - 6(-a) - 3 = 3a^2 + 6a - 3$, and $g(x+1) = 3(x+1)^2 - 6(x+1) - 3 = 3\left(x^2 + 2x + 1\right) - 6x - 6 - 3 = 3x^2 + 6x + 3 - 6x - 9 = 3x^2 - 6$.

4. $h(x) = x^3 - x^2 + x + 1$, so $h(-5) = (-5)^3 - (-5)^2 + (-5) + 1 = -125 - 25 - 5 + 1 = -154$, $h(0) = (0)^3 - (0)^2 + 0 + 1 = 1$, $h(a) = a^3 - (a)^2 + a + 1 = a^3 - a^2 + a + 1$, and $h(-a) = (-a)^3 - (-a)^2 + (-a) + 1 = -a^3 - a^2 - a + 1$.

5. $f(x) = 2x + 5$, so $f(a+h) = 2(a+h) + 5 = 2a + 2h + 5$, $f(-a) = 2(-a) + 5 = -2a + 5$, $f\left(a^2\right) = 2\left(a^2\right) + 5 = 2a^2 + 5$, $f(a - 2h) = 2(a - 2h) + 5 = 2a - 4h + 5$, and $f(2a - h) = 2(2a - h) + 5 = 4a - 2h + 5$

6. $g(x) = -x^2 + 2x$, $g(a+h) = -(a+h)^2 + 2(a+h) = -a^2 - 2ah - h^2 + 2a + 2h$, $g(-a) = -(-a)^2 + 2(-a) = -a^2 - 2a = -a(a+2)$, $g\left(\sqrt{a}\right) = -\left(\sqrt{a}\right)^2 + 2\left(\sqrt{a}\right) = -a + 2\sqrt{a}$, $a + g(a) = a - a^2 + 2a = -a^2 + 3a = -a(a-3)$, and $\dfrac{1}{g(a)} = \dfrac{1}{-a^2 + 2a} = -\dfrac{1}{a(a-2)}$.

7. $s(t) = \dfrac{2t}{t^2 - 1}$. Therefore, $s(4) = \dfrac{2(4)}{(4)^2 - 1} = \dfrac{8}{15}$, $s(0) = \dfrac{2(0)}{0^2 - 1} = 0$, $s(a) = \dfrac{2(a)}{a^2 - 1} = \dfrac{2a}{a^2 - 1}$; $s(2+a) = \dfrac{2(2+a)}{(2+a)^2 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 4 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 3}$, and $s(t+1) = \dfrac{2(t+1)}{(t+1)^2 - 1} = \dfrac{2(t+1)}{t^2 + 2t + 1 - 1} = \dfrac{2(t+1)}{t(t+2)}$.

8. $g(u) = (3u - 2)^{3/2}$. Therefore, $g(1) = [3(1) - 2]^{3/2} = (1)^{3/2} = 1$, $g(6) = [3(6) - 2]^{3/2} = 16^{3/2} = 4^3 = 64$, $g\left(\frac{11}{3}\right) = \left[3\left(\frac{11}{3}\right) - 2\right]^{3/2} = (9)^{3/2} = 27$, and $g(u+1) = [3(u+1) - 2]^{3/2} = (3u+1)^{3/2}$.

9. $f(t) = \dfrac{2t^2}{\sqrt{t-1}}$. Therefore, $f(2) = \dfrac{2\left(2^2\right)}{\sqrt{2-1}} = 8$, $f(a) = \dfrac{2a^2}{\sqrt{a-1}}$, $f(x+1) = \dfrac{2(x+1)^2}{\sqrt{(x+1)-1}} = \dfrac{2(x+1)^2}{\sqrt{x}}$, and $f(x-1) = \dfrac{2(x-1)^2}{\sqrt{(x-1)-1}} = \dfrac{2(x-1)^2}{\sqrt{x-2}}$.

10. $f(x) = 2 + 2\sqrt{5-x}$. Therefore, $f(-4) = 2 + 2\sqrt{5-(-4)} = 2 + 2\sqrt{9} = 2 + 2(3) = 8$, $f(1) = 2 + 2\sqrt{5-1} = 2 + 2\sqrt{4} = 2 + 4 = 6$, $f\left(\frac{11}{4}\right) = 2 + 2\left(5 - \frac{11}{4}\right)^{1/2} = 2 + 2\left(\frac{9}{4}\right)^{1/2} = 2 + 2\left(\frac{3}{2}\right) = 5$, and $f(x+5) = 2 + 2\sqrt{5-(x+5)} = 2 + 2\sqrt{-x}$.

11. Because $x = -2 \le 0$, we calculate $f(-2) = (-2)^2 + 1 = 4 + 1 = 5$. Because $x = 0 \le 0$, we calculate $f(0) = (0)^2 + 1 = 1$. Because $x = 1 > 0$, we calculate $f(1) = \sqrt{1} = 1$.

12. Because $x = -2 < 2$, $g(-2) = -\frac{1}{2}(-2) + 1 = 1 + 1 = 2$. Because $x = 0 < 2$, $g(0) = -\frac{1}{2}(0) + 1 = 0 + 1 = 1$. Because $x = 2 \ge 2$, $g(2) = \sqrt{2-2} = 0$. Because $x = 4 \ge 2$, $g(4) = \sqrt{4-2} = \sqrt{2}$.

13. Because $x = -1 < 1$, $f(-1) = -\frac{1}{2}(-1)^2 + 3 = \frac{5}{2}$. Because $x = 0 < 1$, $f(0) = -\frac{1}{2}(0)^2 + 3 = 3$. Because $x = 1 \ge 1$, $f(1) = 2\left(1^2\right) + 1 = 3$. Because $x = 2 \ge 1$, $f(2) = 2\left(2^2\right) + 1 = 9$.

14. Because $x = 0 \le 1$, $f(0) = 2 + \sqrt{1-0} = 2 + 1 = 3$. Because $x = 1 \le 1$, $f(1) = 2 + \sqrt{1-1} = 2 + 0 = 2$. Because $x = 2 > 1$, $f(2) = \dfrac{1}{1-2} = \dfrac{1}{-1} = -1$.

15. **a.** $f(0) = -2$.

b. **(i)** $f(x) = 3$ when $x \approx 2$. **(ii)** $f(x) = 0$ when $x = 1$.

c. $[0, 6]$

d. $[-2, 6]$

16. **a.** $f(7) = 3$. **b.** $x = 4$ and $x = 6$. **c.** $x = 2$; 0. **d.** $[-1, 9]$; $[-2, 6]$.

17. $g(2) = \sqrt{2^2 - 1} = \sqrt{3}$, so the point $\left(2, \sqrt{3}\right)$ lies on the graph of g.

18. $f(3) = \dfrac{3+1}{\sqrt{3^2+7}} + 2 = \dfrac{4}{\sqrt{16}} + 2 = \dfrac{4}{4} + 2 = 3$, so the point $(3, 3)$ lies on the graph of f.

19. $f(-2) = \dfrac{|-2-1|}{-2+1} = \dfrac{|-3|}{-1} = -3$, so the point $(-2, -3)$ does lie on the graph of f.

20. $h(-3) = \dfrac{|-3+1|}{(-3)^3+1} = \dfrac{2}{-27+1} = -\dfrac{2}{26} = -\dfrac{1}{13}$, so the point $\left(-3, -\frac{1}{13}\right)$ does lie on the graph of h.

21. Because the point $(1, 5)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(1) = 2(1)^2 - 4(1) + c = 5$, or $c = 7$.

22. Because the point $(2, 4)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(2) = 2\sqrt{9-(2)^2} + c = 4$, or $c = 4 - 2\sqrt{5}$.

23. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

24. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

25. $f(x)$ is not defined at $x = 0$ and so the domain of f is $(-\infty, 0) \cup (0, \infty)$.

26. $g(x)$ is not defined at $x = 1$ and so the domain of g is $(-\infty, 1) \cup (1, \infty)$.

27. $f(x)$ is a real number for all values of x. Note that $x^2 + 1 \ge 1$ for all x. Therefore, the domain of f is $(-\infty, \infty)$.

28. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $x - 5 \ge 0$ or $x \ge 5$, and the domain is $[5, \infty)$.

29. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $5 - x \ge 0$, or $-x \ge -5$ and so $x \le 5$. (Recall that multiplying by -1 reverses the sign of an inequality.) Therefore, the domain of f is $(-\infty, 5]$.

30. Because $2x^2 + 3$ is always greater than zero, the domain of g is $(-\infty, \infty)$.

31. The denominator of f is zero when $x^2 - 1 = 0$, or $x = \pm 1$. Therefore, the domain of f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

32. The denominator of f is equal to zero when $x^2 + x - 2 = (x+2)(x-1) = 0$; that is, when $x = -2$ or $x = 1$. Therefore, the domain of f is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

33. f is defined when $x + 3 \geq 0$, that is, when $x \geq -3$. Therefore, the domain of f is $[-3, \infty)$.

34. g is defined when $x - 1 \geq 0$; that is when $x \geq 1$. Therefore, the domain of f is $[1, \infty)$.

35. The numerator is defined when $1 - x \geq 0$, $-x \geq -1$ or $x \leq 1$. Furthermore, the denominator is zero when $x = \pm 2$. Therefore, the domain is the set of all real numbers in $(-\infty, -2) \cup (-2, 1]$.

36. The numerator is defined when $x - 1 \geq 0$, or $x \geq 1$, and the denominator is zero when $x = -2$ and when $x = 3$. So the domain is $[1, 3) \cup (3, \infty)$.

37. a. The domain of f is the set of all real numbers.

b. $f(x) = x^2 - x - 6$, so

$f(-3) = (-3)^2 - (-3) - 6 = 9 + 3 - 6 = 6$,

$f(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$,

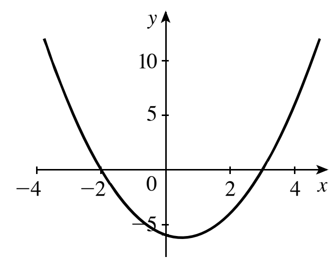
$f(-1) = (-1)^2 - (-1) - 6 = 1 + 1 - 6 = -4$,

$f(0) = (0)^2 - (0) - 6 = -6$,

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{2}{4} - \frac{24}{4} = -\frac{25}{4}$, $f(1) = (1)^2 - 1 - 6 = -6$,

$f(2) = (2)^2 - 2 - 6 = 4 - 2 - 6 = -4$, and $f(3) = (3)^2 - 3 - 6 = 9 - 3 - 6 = 0$.

c.



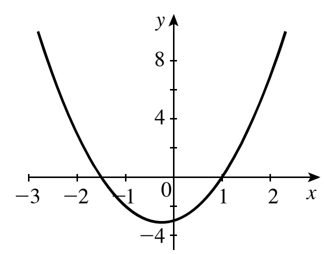
38. $f(x) = 2x^2 + x - 3$.

a. Because $f(x)$ is a real number for all values of x, the domain of f is $(-\infty, \infty)$.

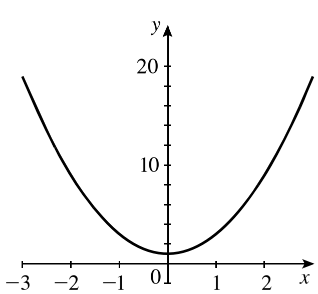
b.

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2	3
y	12	3	-2	-3	-3	0	7	18

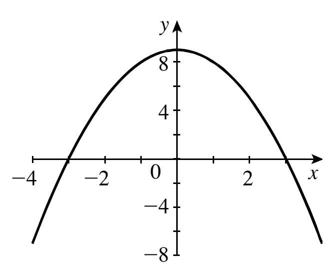
c.



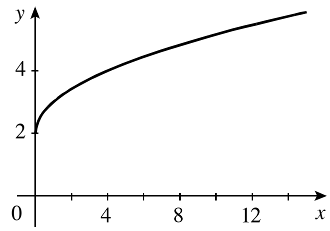
39. $f(x) = 2x^2 + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$.



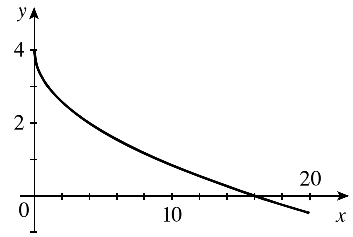
40. $f(x) = 9 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 9]$.



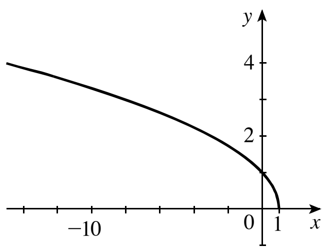
41. $f(x) = 2 + \sqrt{x}$ has domain $[0, \infty)$ and range $[2, \infty)$.



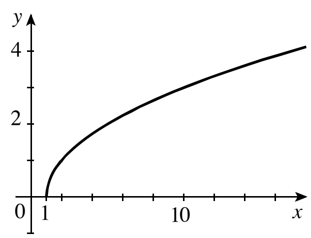
42. $g(x) = 4 - \sqrt{x}$ has domain $[0, \infty)$ and range $(-\infty, 4]$.



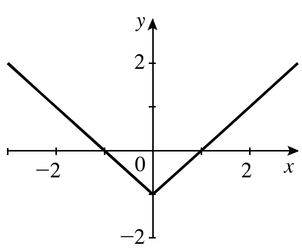
43. $f(x) = \sqrt{1 - x}$ has domain $(-\infty, 1]$ and range $[0, \infty)$



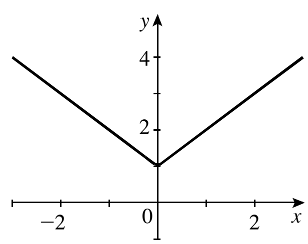
44. $f(x) = \sqrt{x - 1}$ has domain $(1, \infty)$ and range $[0, \infty)$.



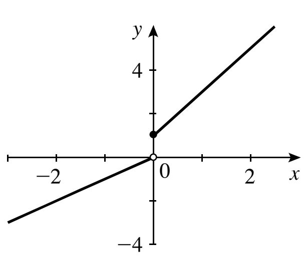
45. $f(x) = |x| - 1$ has domain $(-\infty, \infty)$ and range $[-1, \infty)$.



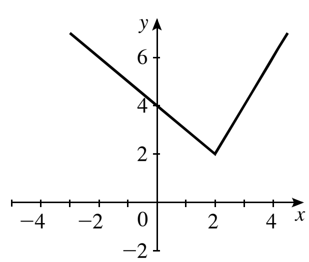
46. $f(x) = |x| + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$



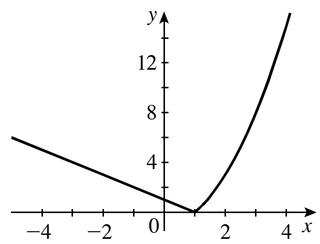
47. $f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$ has domain $(-\infty, \infty)$ and range $(-\infty, 0) \cup [1, \infty)$.



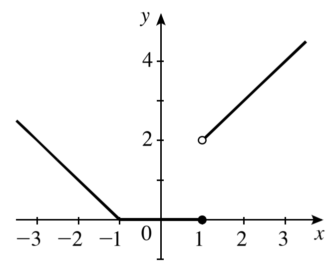
48. For $x < 2$, the graph of f is the half-line $y = 4 - x$. For $x \geq 2$, the graph of f is the half-line $y = 2x - 2$. f has domain $(-\infty, \infty)$ and range $[2, \infty)$.



49. If $x \leq 1$, the graph of f is the half-line $y = -x + 1$. For $x > 1$, we calculate a few points: $f(2) = 3$, $f(3) = 8$, and $f(4) = 15$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



50. If $x < -1$ the graph of f is the half-line $y = -x - 1$. For $-1 \leq x \leq 1$, the graph consists of the line segment $y = 0$. For $x > 1$, the graph is the half-line $y = x + 1$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



51. Each vertical line cuts the given graph at exactly one point, and so the graph represents y as a function of x.

52. Because the y-axis, which is a vertical line, intersects the graph at two points, the graph does not represent y as a function of x.

53. Because there is a vertical line that intersects the graph at three points, the graph does not represent y as a function of x.

54. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

55. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

56. The y-axis intersects the circle at *two* points, and this shows that the circle is not the graph of a function of x.

57. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

58. A vertical line containing a line segment comprising the graph cuts it at infinitely many points and so the graph does not define y as a function of x.

59. The circumference of a circle with a 5-inch radius is given by $C(5) = 2\pi(5) = 10\pi$, or 10π inches.

60. $V(2.1) = \frac{4}{3}\pi(2.1)^3 \approx 38.79$, $V(2) = \frac{4}{3}\pi(8) \approx 33.51$, and so $V(2.1) - V(2) = 38.79 - 33.51 = 5.28$ is the amount by which the volume of a sphere of radius 2.1 exceeds the volume of a sphere of radius 2.

61. $C(0) = 6$, or 6 billion dollars; $C(50) = 0.75(50) + 6 = 43.5$, or 43.5 billion dollars; and $C(100) = 0.75(100) + 6 = 81$, or 81 billion dollars.

62. The child should receive $D(4) = \frac{2}{25}(500)(4) = 160$, or 160 mg.

63. **a.** From $t = 0$ through $t = 5$, that is, from the beginning of 2001 until the end of 2005.

b. From $t = 5$ through $t = 9$, that is, from the beginning of 2006 until the end of 2010.

c. The average expenditures were the same at approximately $t = 5.2$, that is, in the year 2006. The level of expenditure on each service was approximately \$900.

64. **a.** The slope of the straight line passing through $(0, 0.61)$ and $(10, 0.59)$ is $m_1 = \dfrac{0.59 - 0.61}{10 - 0} = -0.002$. Therefore, an equation of the straight line passing through the two points is $y - 0.61 = -0.002\,(t - 0)$ or $y = -0.002t + 0.61$. Next, the slope of the straight line passing through $(10, 0.59)$ and $(20, 0.60)$ is $m_2 = \dfrac{0.60 - 0.59}{20 - 10} = 0.001$, and so an equation of the straight line passing through the two points is $y - 0.59 = 0.001\,(t - 10)$ or $y = 0.001t + 0.58$. The slope of the straight line passing through $(20, 0.60)$ and $(30, 0.66)$ is $m_3 = \dfrac{0.66 - 0.60}{30 - 20} = 0.006$, and so an equation of the straight line passing through the two points is $y - 0.60 = 0.006\,(t - 20)$ or $y = 0.006t + 0.48$. The slope of the straight line passing through $(30, 0.66)$ and $(40.0, 0.78)$ is $m_4 = \dfrac{0.78 - 0.66}{40 - 30} = 0.012$, and so an equation of the straight line passing through the two points is $y = 0.012t + 0.30$. Therefore, a rule for f is $f(t) = \begin{cases} -0.002t + 0.61 & \text{if } 0 \le t \le 10 \\ 0.001t + 0.58 & \text{if } 10 < t \le 20 \\ 0.006t + 0.48 & \text{if } 20 < t \le 30 \\ 0.012t + 0.30 & \text{if } 30 < t \le 40 \end{cases}$

b. The gender gap was expanding between 1960 and 1970 and shrinking between 1970 and 2000.

c. The gender gap was expanding at the rate of 0.002/yr between 1960 and 1970, shrinking at the rate of 0.001/yr between 1970 and 1980, shrinking at the rate of 0.006/yr between 1980 and 1990, and shrinking at the rate of 0.012/yr between 1990 and 2000.

65. **a.** The slope of the straight line passing through the points $(0, 0.58)$ and $(20, 0.95)$ is $m_1 = \dfrac{0.95 - 0.58}{20 - 0} = 0.0185$, so an equation of the straight line passing through these two points is $y - 0.58 = 0.0185\,(t - 0)$ or $y = 0.0185t + 0.58$. Next, the slope of the straight line passing through the points $(20, 0.95)$ and $(30, 1.1)$ is $m_2 = \dfrac{1.1 - 0.95}{30 - 20} = 0.015$, so an equation of the straight line passing through the two points is $y - 0.95 = 0.015\,(t - 20)$ or $y = 0.015t + 0.65$. Therefore, a rule for f is $f(t) = \begin{cases} 0.0185t + 0.58 & \text{if } 0 \le t \le 20 \\ 0.015t + 0.65 & \text{if } 20 < t \le 30 \end{cases}$

b. The ratios were changing at the rates of 0.0185/yr from 1960 through 1980 and 0.015/yr from 1980 through 1990.

c. The ratio was 1 when $t \approx 20.3$. This shows that the number of bachelor's degrees earned by women equaled the number earned by men for the first time around 1983.

66. **a.** $T(x) = 0.06x$

b. $T(200) = 0.06\,(200) = 12$, or \$12.00 and $T(5.65) = 0.06\,(5.65) = 0.34$, or \$0.34.

67. **a.** $I(x) = 1.053x$

b. $I(1520) = 1.053\,(1520) = 1600.56$, or \$1600.56.

68. **a.** The function is linear with y-intercept 1.44 and slope 0.058, so we have $f(t) = 0.058t + 1.44$, $0 \le t \le 9$.

b. The projected spending in 2018 will be $f(9) = 0.058\,(9) + 1.44 = 1.962$, or \$1.962 trillion.

69. $S(r) = 4\pi r^2$.

70. $\frac{4}{3}(\pi)(2r)^3 = \frac{4}{3}\pi 8r^3 = 8\left(\frac{4}{3}\pi r^3\right)$. Therefore, the volume of the tumor is increased by a factor of 8.

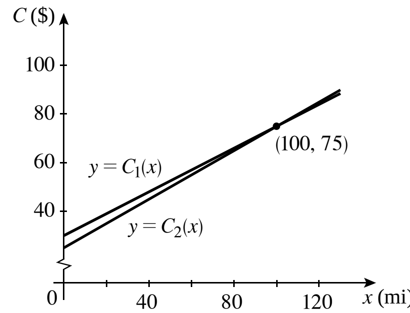
71. a. The median age was changing at the rate of 0.3 years/year.

b. The median age in 2011 was $M(11) = 0.3(11) + 37.9 = 41.2$ (years).

c. The median age in 2015 is projected to be $M(5) = 0.3(15) + 37.9 = 42.4$ (years).

72. a. The daily cost of leasing from Ace is $C_1(x) = 30 + 0.45x$, while the daily cost of leasing from Acme is $C_2(x) = 25 + 0.50x$, where x is the number of miles driven.

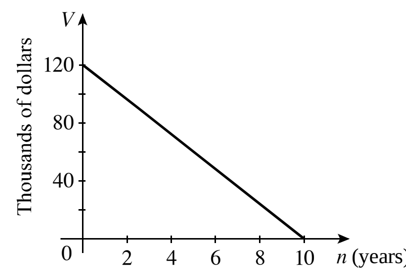
b.



c. The costs are the same when $C_1(x) = C_2(x)$, that is, when $30 + 0.45x = 25 + 0.50x$, $-0.05x = -5$, or $x = 100$. Because $C_1(70) = 30 + 0.45(70) = 61.5$ and $C_2(70) = 25 + 0.50(70) = 60$, and the customer plans to drive less than 70 miles, she should rent from Acme.

73. a. The graph of the function is a straight line passing through $(0, 120000)$ and $(10, 0)$. Its slope is $m = \dfrac{0 - 120{,}000}{10 - 0} = -12{,}000$. The required equation is $V = -12{,}000n + 120{,}000$.

b.



c. $V = -12{,}000(6) + 120{,}000 = 48{,}000$, or \$48,000.

d. This is given by the slope, that is, \$12,000 per year.

74. Here $V = -20{,}000n + 1{,}000{,}000$. The book value in 2012 is given by $V = -20{,}000(15) + 1{,}000{,}000$, or \$700,000. The book value in 2016 is given by $V = -20{,}000(19) + 1{,}000{,}000$, or \$620,000. The book value in 2021 is $V = -20{,}000(24) + 1{,}000{,}000$, or \$520,000.

75. a. The number of incidents in 2009 was $f(0) = 0.46$ (million).

b. The number of incidents in 2013 was $f(4) = 0.2\left(4^2\right) - 0.14(4) + 0.46 = 3.1$ (million).

76. a. The number of passengers in 1995 was $N(0) = 4.6$ (million).

b. The number of passengers in 2010 was $N(15) = 0.011(15)^2 + 0.521(15) + 4.6 = 14.89$ (million).

77. a. The life expectancy of a male whose current age is 65 is $f(65) = 0.0069502(65)^2 - 1.6357(65) + 93.76 \approx 16.80$, or approximately 16.8 years.

b. The life expectancy of a male whose current age is 75 is $f(75) = 0.0069502(75)^2 - 1.6357(75) + 93.76 \approx 10.18$, or approximately 10.18 years.

78. a. $N(t) = 0.00445t^2 + 0.2903t + 9.564$. $N(0) = 9.564$, or 9.6 million people; $N(12) = 0.00445(12)^2 + 0.2903(12) + 9.564 \approx 13.6884$, or approximately 13.7 million people.

b. $N(14) = 0.00445(14)^2 + 0.2903(14) + 9.564 \approx 14.5004$, or approximately 14.5 million people.

79. The projected number in 2030 is $P(20) = -0.0002083(20)^3 + 0.0157(20)^2 - 0.093(20) + 5.2 = 7.9536$, or approximately 8 million.
The projected number in 2050 is $P(40) = -0.0002083(40)^3 + 0.0157(40)^2 - 0.093(40) + 5.2 = 13.2688$, or approximately 13.3 million.

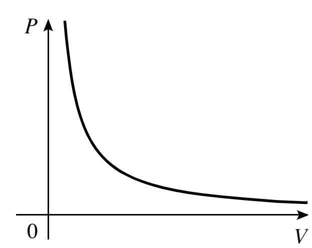
80. $N(t) = -t^3 + 6t^2 + 15t$. Between 8 a.m. and 9 a.m., the average worker can be expected to assemble $N(1) - N(0) = (-1 + 6 + 15) - 0 = 20$, or 20 walkie-talkies. Between 9 a.m. and 10 a.m., we expect that $N(2) - N(1) = \left[-2^3 + 6\left(2^2\right) + 15(2)\right] - (-1 + 6 + 15) = 46 - 20 = 26$, or 26 walkie-talkies can be assembled by the average worker.

81. When the proportion of popular votes won by the Democratic presidential candidate is 0.60, the proportion of seats in the House of Representatives won by Democratic candidates is given by

$$s(0.6) = \frac{(0.6)^3}{(0.6)^3 + (1 - 0.6)^3} = \frac{0.216}{0.216 + 0.064} = \frac{0.216}{0.280} \approx 0.77.$$

82. The amount spent in 2004 was $S(0) = 5.6$, or \$5.6 billion. The amount spent in 2008 was $S(4) = -0.03(4)^3 + 0.2(4)^2 + 0.23(4) + 5.6 = 7.8$, or \$7.8 billion.

83. The domain of the function f is the set of all real positive numbers where $V \neq 0$; that is, $(0, \infty)$.

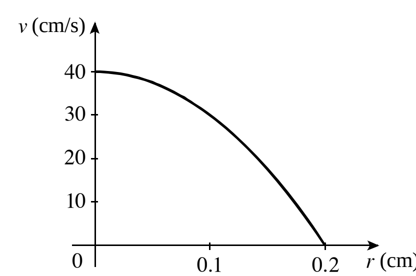


84. **a.** We require that $0.04 - r^2 \geq 0$ and $r \geq 0$. This is true if $0 \leq r \leq 0.2$. Therefore, the domain of v is $[0, 0.2]$.

b. We compute $v(0) = 1000\left[0.04 - (0)^2\right] = 1000(0.04) = 40$,

$$v(0.1) = 1000\left[0.04 - (0.1)^2\right] = 1000(0.04 - 0.01)$$
$$= 1000(0.03) = 30, \text{ and}$$
$$v(0.2) = 1000\left[0.04 - (0.2)^2\right] = 1000(0.04 - 0.04) = 0.$$

c.



d. As the distance r increases, the velocity of the blood decreases.

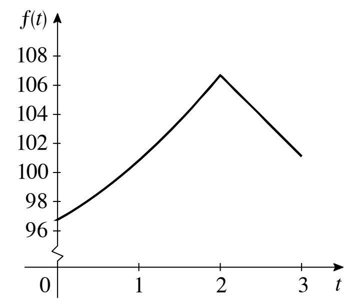
85. **a.** The assets at the beginning of 2002 were \$0.6 trillion. At the beginning of 2003, they were $f(1) = 0.6$, or \$0.6 trillion.

b. The assets at the beginning of 2005 were $f(3) = 0.6(3)^{0.43} \approx 0.96$, or \$0.96 trillion. At the beginning of 2007, they were $f(5) = 0.6(5)^{0.43} \approx 1.20$, or \$1.2 trillion.

86. a. We compute $f(0) = 0.88(0)^2 + 3.21(0) + 96.75 = 96.75$, $f(1) = 0.88(1)^2 + 3.21(1) + 96.75 = 100.84$, and $f(2) = -5.58(2) + 117.85 = 106.69$. We summarize these results in a table.

Year	2006	2007	2008
Rate	96.75	100.84	106.69

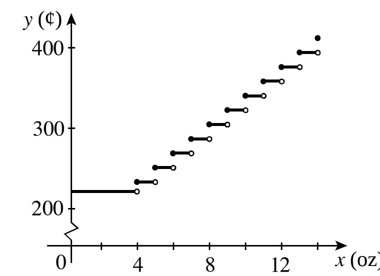
b.



87. a. The domain of f is $(0, 13]$.

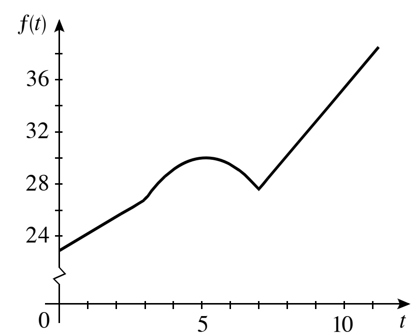
$$f(x) = \begin{cases} 2.32 & \text{if } 0 < x < 4 \\ 2.50 & \text{if } 4 \le x < 5 \\ 2.68 & \text{if } 5 \le x < 6 \\ 2.86 & \text{if } 6 \le x < 7 \\ 3.04 & \text{if } 7 \le x < 8 \\ 3.22 & \text{if } 8 \le x < 9 \\ 3.40 & \text{if } 9 \le x < 10 \\ 3.58 & \text{if } 10 \le x < 11 \\ 3.76 & \text{if } 11 \le x < 12 \\ 3.94 & \text{if } 12 \le x < 13 \\ 4.12 & \text{if } x = 13 \end{cases}$$

b.



88. a. The median age of the U.S. population at the beginning of 1900 was $f(0) = 22.9$, or 22.9 years; at the beginning of 1950 it was $f(5) = -0.7(5)^2 + 7.2(5) + 11.5 = 30$, or 30 years; and at the beginning of 2000 it was $f(10) = 2.6(10) + 9.4 = 35.4$, or 35.4 years.

b.



89. a. The passenger ship travels a distance given by $14t$ miles east and the cargo ship travels a distance of $10(t-2)$ miles north. After two hours have passed, the distance between the two ships is given by $\sqrt{[10(t-2)]^2 + (14t)^2} = \sqrt{296t^2 - 400t + 400}$ miles, so $D(t) = \begin{cases} 14t & \text{if } 0 \le t \le 2 \\ 2\sqrt{74t^2 - 100t + 100} & \text{if } t > 2 \end{cases}$

b. Three hours after the cargo ship leaves port the value of t is 5. Therefore, $D = 2\sqrt{74(5)^2 - 100(5) + 100} \approx 76.16$, or 76.16 miles.

90. True, by definition of a function (page 52).

91. False. Take $f(x) = x^2$, $a = 1$, and $b = -1$. Then $f(1) = 1 = f(-1)$, but $a \neq b$.

92. False. Let $f(x) = x^2$, then take $a = 1$ and $b = 2$. Then $f(a) = f(1) = 1$, $f(b) = f(2) = 4$, and $f(a) + f(b) = 1 + 4 \neq f(a+b) = f(3) = 9$.

93. False. It intersects the graph of a function in at most one point.

94. True. We have $x + 2 \ge 0$ and $2 - x \ge 0$ simultaneously; that is $x \ge -2$ and $x \le 2$. These inequalities are satisfied if $-2 \le x \le 2$.

95. False. Take $f(x) = x^2$ and $k = 2$. Then $f(x) = (2x)^2 = 4x^2 \neq 2x^2 = 2f(x)$.

96. False. Take $f(x) = 2x + 3$ and $c = 2$. Then $f(2x + y) = 2(2x + y) + 3 = 4x + 2y + 3$, but $cf(x) + f(y) = 2(2x + 3) + (2y + 3) = 4x + 2y + 9 \neq f(2x + y)$.

97. False. They are equal everywhere except at $x = 0$, where g is not defined.

98. False. The rule suggests that R takes on the values 0 and 1 when $x = 1$. This violates the uniqueness property that a function must possess.

Using Technology

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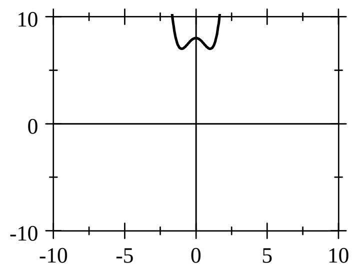
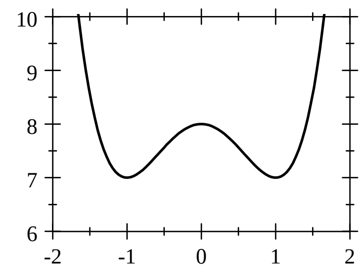
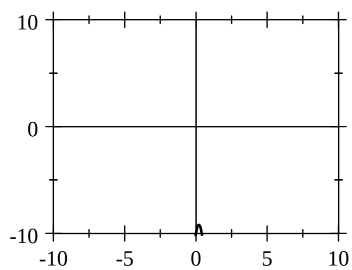
1. a.



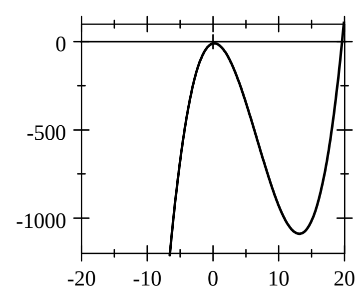
b.



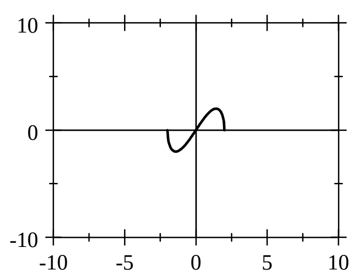
2. a.



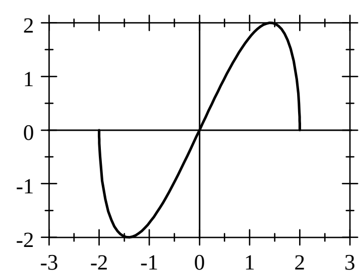
b.



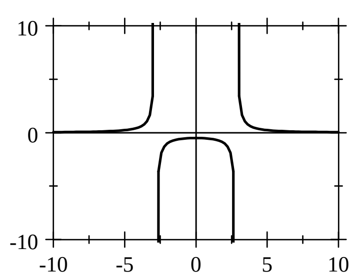
3. a.



b.



4. a.



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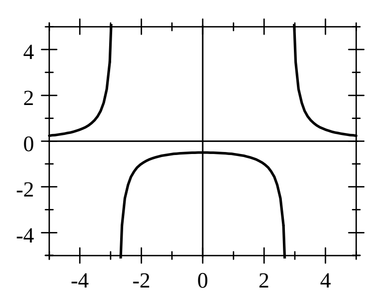

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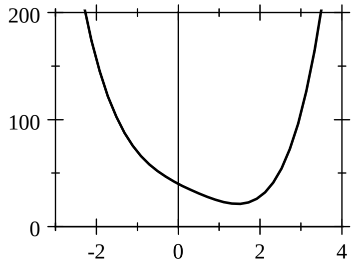

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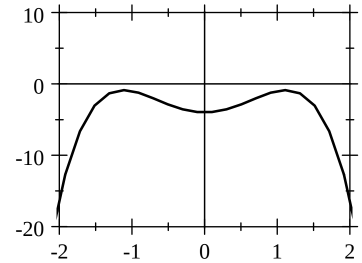

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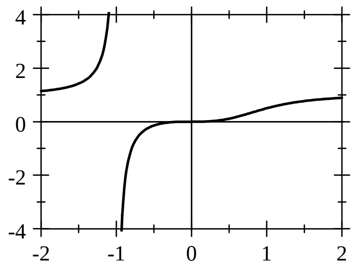

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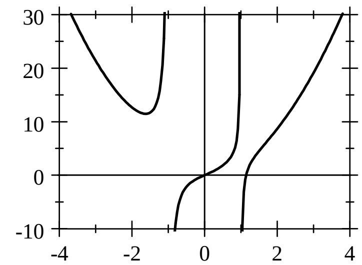


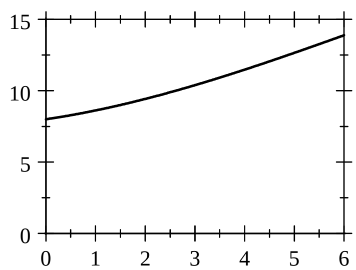
9. $f(2.145) \approx 18.5505$.

10. $f(1.28) \approx 17.3850$.

11. $f(2.41) \approx 4.1616$.

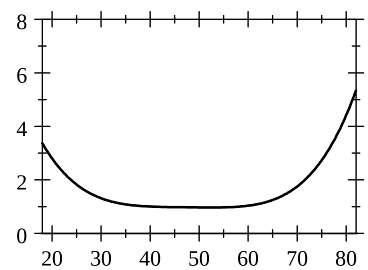
12. $f(0.62) \approx 1.7214$.

13. a.



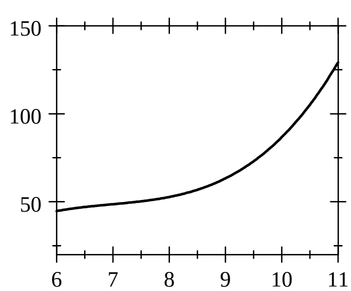
b. The amount spent in the year 2005 was $f(2) \approx 9.42$, or approximately \$9.4 billion. In 2009, it was $f(6) \approx 13.88$, or approximately \$13.9 billion.

14. a.



b. $f(18) = 3.3709$, $f(50) = 0.971$, and $f(80) = 4.4078$.

15. a.



b. $f(6) = 44.7$, $f(8) = 52.7$, and $f(11) = 129.2$.

2.2 The Algebra of Functions

Concept Questions

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1. **a.** $P(x_1) = R(x_1) - C(x_1)$ gives the profit if x_1 units are sold.

   b. $P(x_2) = R(x_2) - C(x_2)$. Because $P(x_2) < 0$, $|R(x_2) - C(x_2)| = -[R(x_2) - C(x_2)]$ gives the loss sustained if x_2 units are sold.

2. **a.** $(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, and $(fg)(x) = f(x)g(x)$; all have domain $A \cap B$. $(f/g)(x) = \dfrac{f(x)}{g(x)}$ has domain $A \cap B$ excluding $x \in A \cap B$ such that $g(x) = 0$.

   b. $(f+g)(2) = f(2) + g(2) = 3 + (-2) = 1$, $(f-g)(2) = f(2) - g(2) = 3 - (-2) = 5$, $(fg)(2) = f(2)g(2) = 3(-2) = -6$, and $(f/g)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{-2} = -\dfrac{3}{2}$

3. **a.** $y = (f+g)(x) = f(x) + g(x)$

   b. $y = (f-g)(x) = f(x) - g(x)$

   c. $y = (fg)(x) = f(x)g(x)$

   d. $y = \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

4. **a.** The domain of $(f \circ g)(x) = f(g(x))$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. The domain of $(g \circ f)(x) = g(f(x))$ is the set of all x in the domain of f such that $f(x)$ is in the domain of g.

   b. $(g \circ f)(2) = g(f(2)) = g(3) = 8$. We cannot calculate $(f \circ g)(3)$ because $(f \circ g)(3) = f(g(3)) = f(8)$, and we don't know the value of $f(8)$.

5. No. Let $A = (-\infty, \infty)$, $f(x) = x$, and $g(x) = \sqrt{x}$. Then $a = -1$ is in A, but $(g \circ f)(-1) = g(f(-1)) = g(-1) = \sqrt{-1}$ is not defined.

6. The required expression is $P = g(f(p))$.

Exercises

page 74

1. $(f+g)(x) = f(x) + g(x) = \left(x^3 + 5\right) + \left(x^2 - 2\right) = x^3 + x^2 + 3$.

2. $(f-g)(x) = f(x) - g(x) = \left(x^3 + 5\right) - \left(x^2 - 2\right) = x^3 - x^2 + 7$.

3. $fg(x) = f(x)g(x) = \left(x^3 + 5\right)\left(x^2 - 2\right) = x^5 - 2x^3 + 5x^2 - 10$.

4. $gf(x) = g(x)f(x) = \left(x^2 - 2\right)\left(x^3 + 5\right) = x^5 - 2x^3 + 5x^2 - 10$.

5. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3 + 5}{x^2 - 2}$.

6. $\dfrac{f-g}{h}(x) = \dfrac{f(x) - g(x)}{h(x)} = \dfrac{x^3 + 5 - \left(x^2 - 2\right)}{2x + 4} = \dfrac{x^3 - x^2 + 7}{2x + 4}$.

7. $\dfrac{fg}{h}(x) = \dfrac{f(x)g(x)}{h(x)} = \dfrac{\left(x^3 + 5\right)\left(x^2 - 2\right)}{2x + 4} = \dfrac{x^5 - 2x^3 + 5x^2 - 10}{2x + 4}$.

8. $fgh(x) = f(x)g(x)h(x) = \left(x^3+5\right)\left(x^2-2\right)(2x+4) = \left(x^5-2x^3+5x^2-10\right)(2x+4)$
$= 2x^6-4x^4+10x^3-20x+4x^5-8x^3+20x^2-40 = 2x^6+4x^5-4x^4+2x^3+20x^2-20x-40.$

9. $(f+g)(x) = f(x)+g(x) = x-1+\sqrt{x+1}.$

10. $(g-f)(x) = g(x)-f(x) = \sqrt{x+1}-(x-1) = \sqrt{x+1}-x+1.$

11. $(fg)(x) = f(x)g(x) = (x-1)\sqrt{x+1}.$

12. $(gf)(x) = g(x)f(x) = \sqrt{x+1}(x-1).$

13. $\dfrac{g}{h}(x) = \dfrac{g(x)}{h(x)} = \dfrac{\sqrt{x+1}}{2x^3-1}.$

14. $\dfrac{h}{g}(x) = \dfrac{h(x)}{g(x)} = \dfrac{2x^3-1}{\sqrt{x+1}}.$

15. $\dfrac{fg}{h}(x) = \dfrac{(x-1)\left(\sqrt{x+1}\right)}{2x^3-1}.$

16. $\dfrac{fh}{g}(x) = \dfrac{(x-1)\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{2x^4-2x^3-x+1}{\sqrt{x+1}}.$

17. $\dfrac{f-h}{g}(x) = \dfrac{x-1-\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{x-2x^3}{\sqrt{x+1}}.$

18. $\dfrac{gh}{g-f}(x) = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-(x-1)} = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-x+1}.$

19. $(f+g)(x) = x^2+5+\sqrt{x}-2 = x^2+\sqrt{x}+3$, $(f-g)(x) = x^2+5-\left(\sqrt{x}-2\right) = x^2-\sqrt{x}+7$, $(fg)(x) = \left(x^2+5\right)\left(\sqrt{x}-2\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+5}{\sqrt{x}-2}.$

20. $(f+g)(x) = \sqrt{x-1}+x^3+1$, $(f-g)(x) = \sqrt{x-1}-x^3-1$, $(fg)(x) = \sqrt{x-1}\left(x^3+1\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{x^3+1}.$

21. $(f+g)(x) = \sqrt{x+3}+\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}+1}{x-1}$, $(f-g)(x) = \sqrt{x+3}-\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}-1}{x-1}$, $(fg)(x) = \sqrt{x+3}\left(\dfrac{1}{x-1}\right) = \dfrac{\sqrt{x+3}}{x-1}$, and $\left(\dfrac{f}{g}\right) = \sqrt{x+3}(x-1).$

22. $(f+g)(x) = \dfrac{1}{x^2+1}+\dfrac{1}{x^2-1} = \dfrac{x^2-1+x^2+1}{\left(x^2+1\right)\left(x^2-1\right)} = \dfrac{2x^2}{\left(x^2+1\right)\left(x^2-1\right)}$,
$(f-g)(x) = \dfrac{1}{x^2+1}-\dfrac{1}{x^2-1} = \dfrac{x^2-1-x^2-1}{\left(x^2+1\right)\left(x^2-1\right)} = -\dfrac{2}{\left(x^2+1\right)\left(x^2-1\right)}$, $(fg)(x) = \dfrac{1}{\left(x^2+1\right)\left(x^2-1\right)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2-1}{x^2+1}.$

23. $(f+g)(x) = \dfrac{x+1}{x-1} + \dfrac{x+2}{x-2} = \dfrac{(x+1)(x-2)+(x+2)(x-1)}{(x-1)(x-2)} = \dfrac{x^2-x-2+x^2+x-2}{(x-1)(x-2)}$
$= \dfrac{2x^2-4}{(x-1)(x-2)} = \dfrac{2\left(x^2-2\right)}{(x-1)(x-2)}$,
$(f-g)(x) = \dfrac{x+1}{x-1} - \dfrac{x+2}{x-2} = \dfrac{(x+1)(x-2)-(x+2)(x-1)}{(x-1)(x-2)} = \dfrac{x^2-x-2-x^2-x+2}{(x-1)(x-2)}$
$= \dfrac{-2x}{(x-1)(x-2)}$,
$(fg)(x) = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{(x+1)(x-2)}{(x-1)(x+2)}$.

24. $(f+g)(x) = x^2+1+\sqrt{x+1}$, $(f-g)(x) = x^2+1-\sqrt{x+1}$, $(fg)(x) = \left(x^2+1\right)\sqrt{x+1}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+1}{\sqrt{x+1}}$.

25. $(f\circ g)(x) = f(g(x)) = f\left(x^2\right) = \left(x^2\right)^2 + x^2 + 1 = x^4 + x^2 + 1$ and
$(g\circ f)(x) = g(f(x)) = g\left(x^2+x+1\right) = \left(x^2+x+1\right)^2$.

26. $(f\circ g)(x) = f(g(x)) = 3\left[g(x)\right]^2 + 2g(x) + 1 = 3(x+3)^2 + 2(x+3) + 1 = 3x^2 + 20x + 34$ and
$(g\circ f)(x) = g(f(x)) = f(x) + 3 = 3x^2 + 2x + 1 + 3 = 3x^2 + 2x + 4$.

27. $(f\circ g)(x) = f(g(x)) = f\left(x^2-1\right) = \sqrt{x^2-1} + 1$ and
$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x}+1\right) = \left(\sqrt{x}+1\right)^2 - 1 = x + 2\sqrt{x} + 1 - 1 = x + 2\sqrt{x}$.

28. $(f\circ g)(x) = f(g(x)) = 2\sqrt{g(x)} + 3 = 2\sqrt{x^2+1} + 3$ and
$(g\circ f)(x) = g(f(x)) = \left[f(x)\right]^2 + 1 = \left(2\sqrt{x}+3\right)^2 + 1 = 4x + 12\sqrt{x} + 10$.

29. $(f\circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} \div \left(\dfrac{1}{x^2}+1\right) = \dfrac{1}{x}\cdot\dfrac{x^2}{x^2+1} = \dfrac{x}{x^2+1}$ and
$(g\circ f)(x) = g(f(x)) = g\left(\dfrac{x}{x^2+1}\right) = \dfrac{x^2+1}{x}$.

30. $(f\circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x-1}\right) = \sqrt{\dfrac{x}{x-1}}$ and
$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x+1}\right) = \dfrac{1}{\sqrt{x+1}-1}\cdot\dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \dfrac{\sqrt{x+1}+1}{x}$.

31. $h(2) = g(f(2))$. But $f(2) = 2^2+2+1 = 7$, so $h(2) = g(7) = 49$.

32. $h(2) = g(f(2))$. But $f(2) = \left(2^2-1\right)^{1/3} = 3^{1/3}$, so $h(2) = g\left(3^{1/3}\right) = 3\left(3^{1/3}\right)^3 + 1 = 3(3) + 1 = 10$.

33. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2(2)+1} = \dfrac{1}{5}$, so $h(2) = g\left(\dfrac{1}{5}\right) = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.

34. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2-1} = 1$, so $g(1) = 1^2 + 1 = 2$.

35. $f(x) = 2x^3 + x^2 + 1$, $g(x) = x^5$.

36. $f(x) = 3x^2 - 4$, $g(x) = x^{-3}$.

37. $f(x) = x^2 - 1$, $g(x) = \sqrt{x}$.

38. $f(x) = (2x - 3)$, $g(x) = x^{3/2}$.

39. $f(x) = x^2 - 1$, $g(x) = \dfrac{1}{x}$.

40. $f(x) = x^2 - 4$, $g(x) = \dfrac{1}{\sqrt{x}}$.

41. $f(x) = 3x^2 + 2$, $g(x) = \dfrac{1}{x^{3/2}}$.

42. $f(x) = \sqrt{2x+1}$, $g(x) = \dfrac{1}{x} + x$.

43. $f(a+h) - f(a) = [3(a+h) + 4] - (3a + 4) = 3a + 3h + 4 - 3a - 4 = 3h$.

44. $f(a+h) - f(a) = -\frac{1}{2}(a+h) + 3 - \left(-\frac{1}{2}a + 3\right) = -\frac{1}{2}a - \frac{1}{2}h + 3 + \frac{1}{2}a - 3 = -\frac{1}{2}h$.

45. $f(a+h) - f(a) = 4 - (a+h)^2 - \left(4 - a^2\right) = 4 - a^2 - 2ah - h^2 - 4 + a^2 = -2ah - h^2 = -h(2a+h)$.

46.
$$\begin{aligned} f(a+h) - f(a) &= \left[(a+h)^2 - 2(a+h) + 1\right] - \left(a^2 - 2a + 1\right) \\ &= a^2 + 2ah + h^2 - 2a - 2h + 1 - a^2 + 2a - 1 = h(2a + h - 2). \end{aligned}$$

47.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[(a+h)^2 + 1\right] - \left(a^2 + 1\right)}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} = \frac{2ah + h^2}{h} \\ &= \frac{h(2a+h)}{h} = 2a + h. \end{aligned}$$

48.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[2(a+h)^2 - (a+h) + 1\right] - \left(2a^2 - a + 1\right)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - a - h + 1 - 2a^2 + a - 1}{h} = \frac{4ah + 2h^2 - h}{h} = 4a + 2h - 1. \end{aligned}$$

49.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[(a+h)^3 - (a+h)\right] - \left(a^3 - a\right)}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a - h - a^3 + a}{h} \\ &= \frac{3a^2h + 3ah^2 + h^3 - h}{h} = 3a^2 + 3ah + h^2 - 1. \end{aligned}$$

50.
$$\begin{aligned} \frac{f(a+h) - f(a)}{h} &= \frac{\left[2(a+h)^3 - (a+h)^2 + 1\right] - \left(2a^3 - a^2 + 1\right)}{h} \\ &= \frac{2a^3 + 6a^2h + 6ah^2 + 2h^3 - a^2 - 2ah - h^2 + 1 - 2a^3 + a^2 - 1}{h} \\ &= \frac{6a^2h + 6ah^2 + 2h^3 - 2ah - h^2}{h} = 6a^2 + 6ah + 2h^2 - 2a - h. \end{aligned}$$

51.
$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{1}{a+h} - \dfrac{1}{a}}{h} = \frac{\dfrac{a - (a+h)}{a(a+h)}}{h} = -\frac{1}{a(a+h)}.$$

52.
$$\frac{f(a+h) - f(a)}{h} = \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \frac{(a+h) - a}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{1}{\sqrt{a+h} + \sqrt{a}}.$$

53. $F(t)$ represents the total revenue for the two restaurants at time t.

54. $F(t)$ represents the net rate of growth of the species of whales in year t.

55. $f(t)g(t)$ represents the dollar value of Nancy's holdings at time t.

56. $f(t)/g(t)$ represents the unit cost of the commodity at time t.

57. $g \circ f$ is the function giving the amount of carbon monoxide pollution from cars in parts per million at time t.

58. $f \circ g$ is the function giving the revenue at time t.

59. $C(x) = 0.6x + 12{,}100$.

60. **a.** $h(t) = f(t) - g(t) = (3t + 69) - (-0.2t + 13.8) = 3.2t + 55.2$, $0 \le t \le 5$.

b. $f(5) = 3(5) + 69 = 84$, $g(5) = -0.2(5) + 13.8 = 12.8$, and $h(5) = 3.2(5) + 55.2 = 71.2$.
Since $f(5) - g(5) = 84 - 12.8 = 71.2$, we see that $h(5)$ is indeed equal to $f(5) - g(5)$.

61. $D(t) = (D_2 - D_1)(t) = D_2(t) - D_1(t) = \left(0.035t^2 + 0.21t + 0.24\right) - \left(0.0275t^2 + 0.081t + 0.07\right)$

$\approx 0.0075t^2 + 0.129t + 0.17$.

The function D gives the difference in year t between the deficit without the \$160 million rescue package and the deficit with the rescue package.

62. **a.** $(g \circ f)(0) = g(f(0)) = g(0.64) = 26$, so the mortality rate of motorcyclists in the year 2000 was 26 per 100 million miles traveled.

b. $(g \circ f)(6) = g(f(6)) = g(0.51) = 42$, so the mortality rate of motorcyclists in 2006 was 42 per 100 million miles traveled.

c. Between 2000 and 2006, the percentage of motorcyclists wearing helmets had dropped from 64 to 51, and as a consequence, the mortality rate of motorcyclists had increased from 26 million miles traveled to 42 million miles traveled.

63. **a.** $(g \circ f)(1) = g(f(1)) = g(406) = 23$. So in 2002, the percentage of reported serious crimes that end in arrests or in the identification of suspects was 23.

b. $(g \circ f)(6) = g(f(6)) = g(326) = 18$. In 2007, 18% of reported serious crimes ended in arrests or in the identification of suspects.

c. Between 2002 and 2007, the total number of detectives had dropped from 406 to 326 and as a result, the percentage of reported serious crimes that ended in arrests or in the identification of suspects dropped from 23 to 18.

64. **a.** $C(x) = 0.000003x^3 - 0.03x^2 + 200x + 100{,}000$.

b. $P(x) = R(x) - C(x) = -0.1x^2 + 500x - \left(0.000003x^3 - 0.03x^2 + 200x + 100{,}000\right)$

$= -0.000003x^3 - 0.07x^2 + 300x - 100{,}000$.

c. $P(1500) = -0.000003(1500)^3 - 0.07(1500)^2 + 300(1500) - 100{,}000 = 182{,}375$, or \$182,375.

65. a. $C(x) = V(x) + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20{,}000$.

b. $P(x) = R(x) - C(x) = -0.02x^2 + 150x - 0.000001x^3 + 0.01x^2 - 50x - 20{,}000$

$$= -0.000001x^3 - 0.01x^2 + 100x - 20{,}000.$$

c. $P(2000) = -0.000001(2000)^3 - 0.01(2000)^2 + 100(2000) - 20{,}000 = 132{,}000$, or \$132,000.

66. a. $D(t) = R(t) - S(t)$

$$= \left(0.023611t^3 - 0.19679t^2 + 0.34365t + 2.42\right) - \left(-0.015278t^3 + 0.11179t^2 + 0.02516t + 2.64\right)$$

$$= 0.038889t^3 - 0.30858t^2 + 0.31849t - 0.22,\ 0 \le t \le 6.$$

b. $S(3) = 3.309084$, $R(3) = 2.317337$, and $D(3) = -0.991747$, so the spending, revenue, and deficit are approximately \$3.31 trillion, \$2.32 trillion, and \$0.99 trillion, respectively.

c. Yes: $R(3) - S(3) = 2.317337 - 3.308841 = -0.991504 = D(3)$.

67. a. $h(t) = f(t) + g(t) = \left(4.389t^3 - 47.833t^2 + 374.49t + 2390\right) + \left(13.222t^3 - 132.524t^2 + 757.9t + 7481\right)$

$$= 17.611t^3 - 180.357t^2 + 1132.39t + 9871,\ 1 \le t \le 7.$$

b. $f(6) = 3862.976$ and $g(6) = 10{,}113.488$, so $f(6) + g(6) = 13{,}976.464$. The worker's contribution was approximately \$3862.98, the employer's contribution was approximately \$10,113.49, and the total contributions were approximately \$13,976.46.

c. $h(6) = 13{,}976 = f(6) + g(6)$, as expected.

68. a. $N(r(t)) = \dfrac{7}{1 + 0.02\left(\dfrac{5t + 75}{t + 10}\right)^2}$.

b. $N(r(0)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 0 + 75}{0 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{75}{10}\right)^2} \approx 3.29$, or 3.29 million units.

$N(r(12)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 12 + 75}{12 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{135}{22}\right)^2} \approx 3.99$, or 3.99 million units.

$N(r(18)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 18 + 75}{18 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{165}{28}\right)^2} \approx 4.13$, or 4.13 million units.

69. a. The occupancy rate at the beginning of January is $r(0) = \frac{10}{81}(0)^3 - \frac{10}{3}(0)^2 + \frac{200}{9}(0) + 55 = 55$, or 55%.
$r(5) = \frac{10}{81}(5)^3 - \frac{10}{3}(5)^2 + \frac{200}{9}(5) + 55 \approx 98.2$, or approximately 98.2%.

b. The monthly revenue at the beginning of January is $R(55) = -\frac{3}{5000}(55)^3 + \frac{9}{50}(55)^2 \approx 444.68$, or approximately \$444,700.
The monthly revenue at the beginning of June is $R(98.2) = -\frac{3}{5000}(98.2)^3 + \frac{9}{50}(98.2)^2 \approx 1167.6$, or approximately \$1,167,600.

70. $N(t) = 1.42 \cdot x(t) = \dfrac{1.42 \cdot 7(t+10)^2}{(t+10)^2 + 2(t+15)^2} = \dfrac{9.94(t+10)^2}{(t+10)^2 + 2(t+15)^2}$. The number of jobs created 6 months from now will be $N(6) = \dfrac{9.94(16)^2}{(16)^2 + 2(21)^2} \approx 2.24$, or approximately 2.24 million jobs. The number of jobs created 12 months from now will be $N(12) = \dfrac{9.94(22)^2}{(22)^2 + 2(27)^2} \approx 2.48$, or approximately 2.48 million jobs.

71. a. $s = f + g + h = (f + g) + h = f + (g + h)$. This suggests we define the sum s by $s(x) = (f + g + h)(x) = f(x) + g(x) + h(x)$.

b. Let f, g, and h define the revenue (in dollars) in week t of three branches of a store. Then its total revenue (in dollars) in week t is $s(t) = (f + g + h)(t) = f(t) + g(t) + h(t)$.

72. a. $(h \circ g \circ f)(x) = h(g(f(x)))$

b. Let t denote time. Suppose f gives the number of people at time t in a town, g gives the number of cars as a function of the number of people in the town, and h gives the amount of carbon monoxide in the atmosphere. Then $(h \circ g \circ f)(t) = h(g(f(t)))$ gives the amount of carbon monoxide in the atmosphere at time t.

73. True. $(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x)$.

74. False. Let $f(x) = x + 2$ and $g(x) = \sqrt{x}$. Then $(g \circ f)(x) = \sqrt{x+2}$ is defined at $x = -1$, But $(f \circ g)(x) = \sqrt{x} + 2$ is not defined at $x = -1$.

75. False. Take $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Then $(g \circ f)(x) = \sqrt{x} + 1$, but $(f \circ g)(x) = \sqrt{x+1}$.

76. False. Take $f(x) = x + 1$. Then $(f \circ f)(x) = f(f(x)) = x + 2$, but $f^2(x) = [f(x)]^2 = (x+1)^2 = x^2 + 2x + 1$.

77. True. $(h \circ (g \circ f))(x) = h((g \circ f)(x)) = h(g(f(x)))$ and $((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$.

78. False. Take $h(x) = \sqrt{x}$, $g(x) = x$, and $f(x) = x^2$. Then $(h \circ (g + f))(x) = h\left(x + x^2\right) = \sqrt{x + x^2} \neq ((h \circ g) + (h \circ f))(x) = h(g(x)) + h(f(x)) = \sqrt{x} + \sqrt{x^2}$.

2.3 Functions and Mathematical Models

Concept Questions

page 88

1. See page 78 of the text. Answers will vary.

2. a. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where $a_n \neq 0$ and n is a positive integer. An example is $P(x) = 4x^3 - 3x^2 + 2$.

b. $R(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials with $Q(x) \neq 0$. An example is $R(x) = \dfrac{3x^4 - 2x^2 + 1}{x^2 + 3x + 5}$.

3. a. A demand function $p = D(x)$ gives the relationship between the unit price of a commodity p and the quantity x demanded. A supply function $p = S(x)$ gives the relationship between the unit price of a commodity p and the quantity x the supplier will make available in the marketplace.

b. Market equilibrium occurs when the quantity produced is equal to the quantity demanded. To find the market equilibrium, we solve the equations $p = D(x)$ and $p = S(x)$ simultaneously.

Exercises page 88

1. Yes. $2x + 3y = 6$ and so $y = -\frac{2}{3}x + 2$.

2. Yes. $4y = 2x + 7$ and so $y = \frac{1}{2}x + \frac{7}{4}$.

3. Yes. $2y = x + 4$ and so $y = \frac{1}{2}x + 2$.

4. Yes. $3y = 2x - 8$ and so $y = \frac{2}{3}x - \frac{8}{3}$.

5. Yes. $4y = 2x + 9$ and so $y = \frac{1}{2}x + \frac{9}{4}$.

6. Yes. $6y = 3x + 7$ and so $y = \frac{1}{2}x + \frac{7}{6}$.

7. No, because of the term x^2.

8. No, because of the term $\sqrt{x}$.

9. f is a polynomial function in x of degree 6.

10. f is a rational function.

11. Expanding $G(x) = 2\left(x^2 - 3\right)^3$, we have $G(x) = 2x^6 - 18x^4 + 54x^2 - 54$, and we conclude that G is a polynomial function of degree 6 in x.

12. We can write $H(x) = \dfrac{2}{x^3} + \dfrac{5}{x^2} + 6 = \dfrac{2 + 5x + 6x^3}{x^3}$ and conclude that H is a rational function.

13. f is neither a polynomial nor a rational function.

14. f is a rational function.

15. $f(0) = 2$ gives $f(0) = m(0) + b = b = 2$. Next, $f(3) = -1$ gives $f(3) = m(3) + b = -1$. Substituting $b = 2$ in this last equation, we have $3m + 2 = -1$, or $3m = -3$, and therefore, $m = -1$ and $b = 2$.

16. $f(2) = 4$ gives $f(2) = 2m + b = 4$. We also know that $m = -1$. Therefore, we have $2(-1) + b = 4$ and so $b = 6$.

17. a. $C(x) = 8x + 40{,}000$.

b. $R(x) = 12x$.

c. $P(x) = R(x) - C(x) = 12x - (8x + 40{,}000) = 4x - 40{,}000$.

d. $P(8000) = 4(8000) - 40{,}000 = -8000$, or a loss of \$8000. $P(12{,}000) = 4(12{,}000) - 40{,}000 = 8000$, or a profit of \$8000.

18. a. $C(x) = 14x + 100{,}000$.

b. $R(x) = 20x$.

c. $P(x) = R(x) - C(x) = 20x - (14x + 100{,}000) = 6x - 100{,}000$.

d. $P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$, or a loss of \$28,000.
$P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$, or a profit of \$20,000.

19. The individual's disposable income is $D = (1 - 0.28) \cdot 60{,}000 = 43{,}200$, or \$43,200.

20. The child should receive $D(0.4) = \dfrac{(0.4)(500)}{1.7} \approx 117.65$, or approximately 118 mg.

21. The child should receive $D(4) = \left(\dfrac{4+1}{24}\right)(500) \approx 104.17$, or approximately 104 mg.

22. a. The graph of f passes through the points $P_1(0, 17.5)$ and $P_2(10, 10.3)$. Its slope is $\dfrac{10.3 - 17.5}{10 - 0} = -0.72$. An equation of the line is $y - 17.5 = -0.72(t - 0)$ or $y = -0.72t + 17.5$, so the linear function is $f(t) = -0.72t + 17.5$.

b. The rate was decreasing at 0.72% per year.

c. The percentage of high school students who drink and drive at the beginning of 2014 is projected to be $f(13) = -0.72(13) + 17.5 = 8.14$, or 8.14%.

23. a. The slope of the graph of f is a line with slope -13.2 passing through the point $(0, 400)$, so an equation of the line is $y - 400 = -13.2(t - 0)$ or $y = -13.2t + 400$, and the required function is $f(t) = -13.2t + 400$.

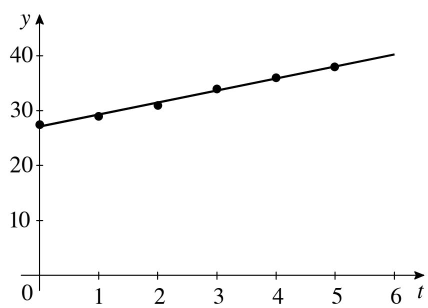
b. The emissions cap is projected to be $f(2) = -13.2(2) + 400 = 373.6$, or 373.6 million metric tons of carbon dioxide equivalent.

24. a. The graph of f is a line through the points $P_1(0, 0.7)$ and $P_2(20, 1.2)$, so it has slope $\dfrac{1.2 - 0.7}{20 - 0} = 0.025$. Its equation is $y - 0.7 = 0.025(t - 0)$ or $y = 0.025t + 0.7$. The required function is thus $f(t) = 0.025t + 0.7$.

b. The projected annual rate of growth is the slope of the graph of f, that is, 0.025 billion per year, or 25 million per year.

c. The projected number of boardings per year in 2022 is $f(10) = 0.025(10) + 0.7 = 0.95$, or 950 million boardings per year.

25. a.



b. The projected revenue in 2010 is $f(6) = 2.19(6) + 27.12 = 40.26$, or \$40.26 billion.

c. The rate of increase is the slope of the graph of f, that is, 2.19 (billion dollars per year).

26. Two hours after starting work, the average worker will be assembling at the rate of $f(2) = -\frac{3}{2}(2)^2 + 6(2) + 10 = 16$, or 16 phones per hour.

27. $P(28) = -\frac{1}{8}(28)^2 + 7(28) + 30 = 128$, or \$128,000.

28. a. The amount paid out in 2010 was $S(0) = 0.72$, or \$0.72 trillion (or \$720 billion).

b. The amount paid out in 2030 is projected to be $S(3) = 0.1375(3)^2 + 0.5185(3) + 0.72 = 3.513$, or \$3.513 trillion.

29. a. The average time spent per day in 2009 was $f(0) = 21.76$ (minutes).

b. The average time spent per day in 2013 is projected to be $f(4) = 2.25(4)^2 + 13.41(4) + 21.76 = 111.4$ (minutes).

30. a. The GDP in 2011 was $G(0) = 15$, or \$15 trillion.

b. The projected GDP in 2015 is $G(4) = 0.064(4)^2 + 0.473(4) + 15.0 = 17.916$, or \$17.916 trillion.

31. a. The GDP per capita in 2000 was $f(10) = 1.86251(10)^2 - 28.08043(10) + 884 = 789.4467$, or \$789.45.

b. The GDP per capita in 2030 is projected to be $f(40) = 1.86251(40)^2 - 28.08043(40) + 884 = 2740.7988$, or \$2740.80.

32. **a.** The number of enterprise IM accounts in 2006 is given by $N(0) = 59.7$, or 59.7 million.

b. The number of enterprise IM accounts in 2010, assuming a continuing trend, is given by $N(4) = 2.96(4)^2 + 11.37(4) + 59.7 = 152.54$ million.

33. $S(6) = 0.73(6)^2 + 15.8(6) + 2.7 = 123.78$ million kilowatt-hr.
$S(8) = 0.73(8)^2 + 15.8(8) + 2.7 = 175.82$ million kilowatt-hr.

34. The U.S. public debt in 2005 was $f(0) = 8.246$, or \$8.246 trillion. The public debt in 2008 was $f(3) = -0.03817(3)^3 + 0.4571(3)^2 - 0.1976(3) + 8.246 = 10.73651$, or approximately \$10.74 trillion.

35. The percentage who expected to work past age 65 in 1991 was $f(0) = 11$, or 11%. The percentage in 2013 was $f(22) = 0.004545(22)^3 - 0.1113(22)^2 + 1.385(22) + 11 = 35.99596$, or approximately 36%.

36. $N(0) = 0.7$ per 100 million vehicle miles driven. $N(7) = 0.0336(7)^3 - 0.118(7)^2 + 0.215(7) + 0.7 = 7.9478$ per 100 million vehicle miles driven.

37. **a.** Total global mobile data traffic in 2009 was $f(0) = 0.06$, or 60,000 terabytes.

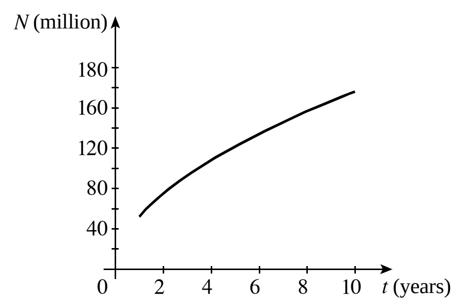
b. The total in 2014 will be $f(5) = 0.021(5)^3 + 0.015(5)^2 + 0.12(5) + 0.06 = 3.66$, or 3.66 million terabytes.

38. $L = \dfrac{1 + 0.05D}{D}$. If $D = 20$, then $L = \dfrac{1 + 0.05(20)}{20} = 0.10$, or 10%.

39. **a.** We first construct a table.

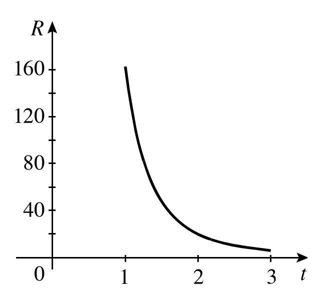
t	$N(t)$
1	52
2	75
3	93
4	109
5	122

t	$N(t)$
6	135
7	146
8	157
9	167
10	177



b. The number of viewers in 2012 is given by $N(10) = 52(10)^{0.531} \approx 176.61$, or approximately 177 million viewers.

40. **a.**



$R(1) = 162.8(1)^{-3.025} = 162.8$, $R(2) = 162.8(2)^{-3.025} \approx 20.0$, and $R(3) = 162.8(3)^{-3.025} \approx 5.9$.

b. The infant mortality rates in 1900, 1950, and 2000 are 162.8, 20.0, and 5.9 per 1000 live births, respectively.

41. $N(5) = 0.0018425(10)^{2.5} \approx 0.58265$, or approximately 0.583 million. $N(13) = 0.0018425(18)^{2.5} \approx 2.5327$, or approximately 2.5327 million.

42. **a.** $S(0) = 4.3(0 + 2)^{0.94} \approx 8.24967$, or approximately \$8.25 billion.

b. $S(8) = 4.3(8 + 2)^{0.94} \approx 37.45$, or approximately \$37.45 billion.

43. a. We are given that $f(1) = 5240$ and $f(4) = 8680$. This leads to the system of equations $a + b = 5240$, $11a + b = 8680$. Solving, we find $a = 344$ and $b = 4896$.

b. From part (a), we have $f(t) = 344t + 4896$, so the approximate per capita costs in 2005 were $f(5) = 344(5) + 4896 = 6616$, or \$6616.

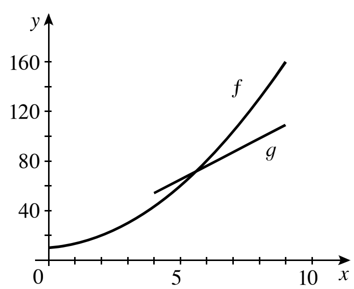
44. a. The given data imply that $R(40) = 50$, that is, $\dfrac{100(40)}{b+40} = 50$, so $50(b + 40) = 4000$, or $b = 40$. Therefore, the required response function is $R(x) = \dfrac{100x}{40+x}$.

b. The response will be $R(60) = \dfrac{100(60)}{40+60} = 60$, or approximately 60 percent.

45. a. $f(0) = 6.85$, $g(0) = 16.58$. Because $g(0) > f(0)$, we see that more film cameras were sold in 2001 (when $t = 0$).

b. We solve the equation $f(t) = g(t)$, that is, $3.05t + 6.85 = -1.85t + 16.58$, so $4.9t = 9.73$ and $t = 1.99 \approx 2$. So sales of digital cameras first exceed those of film cameras in approximately 2003.

46. a.



b. $5x^2 + 5x + 30 = 33x + 30$, so $5x^2 - 28x = 0$, $x(5x - 28) = 0$, and $x = 0$ or $x = \frac{28}{5} = 5.6$, representing 5.6 mi/h. $g(x) = 11(5.6) + 10 = 71.6$, or 71.6 mL/lb/min.

c. The oxygen consumption of the walker is greater than that of the runner.

47. a. We are given that $T = aN + b$ where a and b are constants to be determined. The given conditions imply that $70 = 120a + b$ and $80 = 160a + b$. Subtracting the first equation from the second gives $10 = 40a$, or $a = \frac{1}{4}$. Substituting this value of a into the first equation gives $70 = 120\left(\frac{1}{4}\right) + b$, or $b = 40$. Therefore, $T = \frac{1}{4}N + 40$.

b. Solving the equation in part (a) for N, we find $\frac{1}{4}N = T - 40$, or $N = f(t) = 4T - 160$. When $T = 102$, we find $N = 4(102) - 160 = 248$, or 248 times per minute.

48. a. $f(0) = 3173$ gives $c = 3173$, $f(4) = 6132$ gives $16a + 4b + c = 6132$, and $f(6) = 7864$ gives $36a + 6b + c = 1864$. Solving, we find $a \approx 21.0417$, $b \approx 655.5833$, and $c = 3173$.

b. From part (a), we have $f(t) = 21.0417t^2 + 655.5833t + 3173$, so the number of farmers' markets in 2014 is projected to be $f(8) = 21.0417(8)^2 + 655.5833(8) + 3173 = 9764.3352$, or approximately 9764.

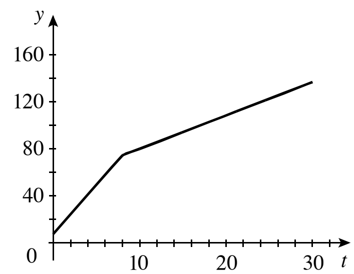
49. a. We have $f(0) = c = 1547$, $f(2) = 4a + 2b + c = 1802$, and $f(4) = 16a + 4b + c = 2403$. Solving this system of equations gives $a = 43.25$, $b = 41$, and $c = 1547$.

b. From part (a), we have $f(t) = 43.25t^2 + 41t + 1547$, so the number of craft-beer breweries in 2014 is projected to be $f(6) = 43.25(6)^2 + 41(6) + 1547 = 3350$.

50. The slope of the line is $m = \dfrac{S-C}{n}$. Therefore, an equation of the line is $y - C = \dfrac{S-C}{n}(t - 0)$. Letting $y = V(t)$, we have $V(t) = C - \dfrac{C-S}{n}t$.

51. Using the formula given in Exercise 50, we have
$V(2) = 100{,}000 - \dfrac{100{,}000 - 30{,}000}{5}(2) = 100{,}000 - \dfrac{70{,}000}{5}(2) = 72{,}000$, or \$72,000.

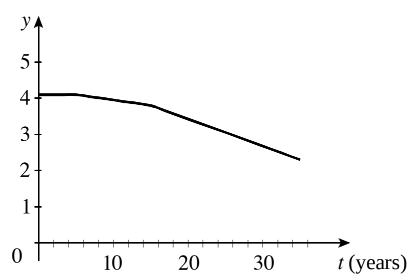
52. a.



b. $f(0) = 8.37(0) + 7.44 = 7.44$, or \$7.44/kilo.
$f(20) = 2.84(20) + 51.68 = 108.48$, or \$108.48/kilo.

53. The total cost by 2011 is given by $f(1) = 5$, or \$5 billion. The total cost by 2015 is given by $f(5) = -0.5278\left(5^3\right) + 3.012\left(5^2\right) + 49.23(5) - 103.29 = 152.185$, or approximately \$152 billion.

54. a.

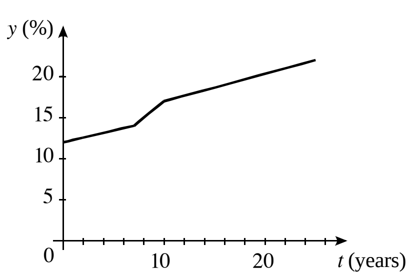


b. At the beginning of 2005, the ratio will be $f(10) = -0.03(10) + 4.25 = 3.95$. At the beginning of 2020, the ratio will be $f(25) = -0.075(25) + 4.925 = 3.05$.

c. The ratio is constant from 1995 to 2000.

d. The decline of the ratio is greatest from 2010 through 2030. It is $\dfrac{f(35) - f(15)}{35 - 15} = \dfrac{2.3 - 3.8}{20} = -0.075$.

55. a.

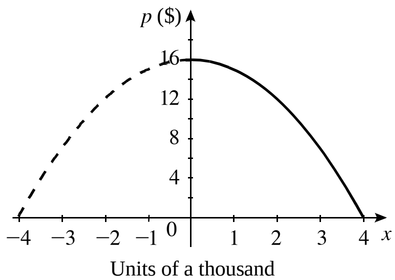


b. $f(5) = \frac{2}{7}(5) + 12 = \frac{10}{7} + 12 \approx 13.43$, or approximately 13.43%. $f(25) = \frac{1}{3}(25) + \frac{41}{3} = 22$, or 22%.

56. a. $f(0) = 5.6$ and $g(0) = 22.5$. Because $g(0) > f(0)$, we conclude that more VCRs than DVD players were sold in 2001.

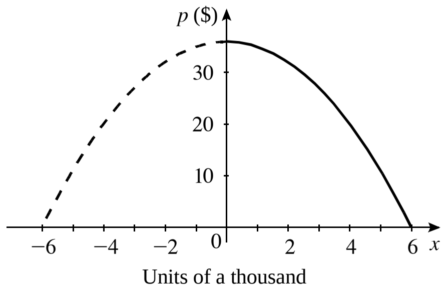
b. We solve the equations $f(t) = g(t)$ over each of the subintervals. $5.6 + 5.6t = -9.6t + 22.5$ for $0 \le t \le 1$. We solve to find $15.2t = 16.9$, so $t \approx 1.11$. This is outside the range for t, so we reject it. $5.6 + 5.6t = -0.5t + 13.4$ for $1 < t \le 2$, so $6.1t = 7.8$, and thus $t \approx 1.28$. So sales of DVD players first exceed those of VCRs at $t \approx 1.3$, or in early 2002.

57. a.



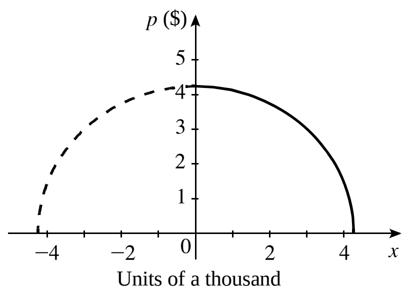
b. If $p = 7$, we have $7 = -x^2 + 16$, or $x^2 = 9$, so that $x = \pm 3$. Therefore, the quantity demanded when the unit price is \$7 is 3000 units.

58. a.



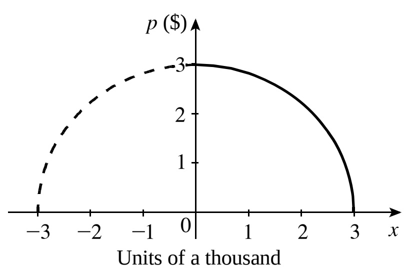
b. If $p = 11$, we have $11 = -x^2 + 36$, or $x^2 = 25$, so that $x = \pm 5$. Therefore, the quantity demanded when the unit price is \$11 is 5000 units.

59. a.



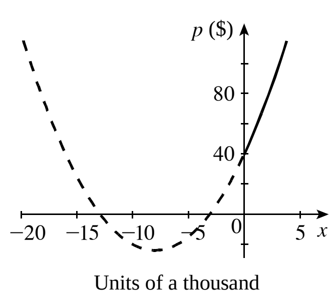
b. If $p = 3$, then $3 = \sqrt{18 - x^2}$, and $9 = 18 - x^2$, so that $x^2 = 9$ and $x = \pm\, 3$. Therefore, the quantity demanded when the unit price is \$3 is 3000 units.

60. a.



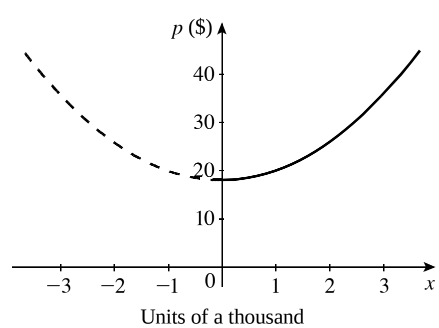
b. If $p = 2$, then $2 = \sqrt{9 - x^2}$, and $4 = 9 - x^2$, so that $x^2 = 5$ and and $x = \pm\sqrt{5}$, or $x \approx \pm 2.236$, Therefore, the quantity demanded when the unit price is \$2 is approximately 2236 units.

61. a.



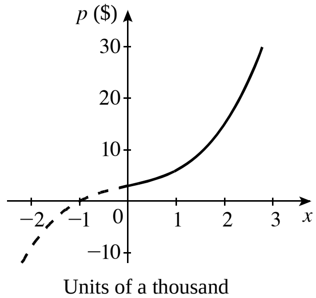
b. If $x = 2$, then $p = 2^2 + 16\,(2) + 40 = 76$, or \$76.

62. a.



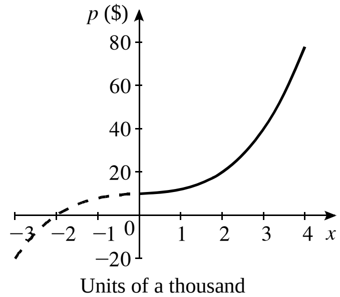
b. If $x = 2$, then $p = 2\,(2)^2 + 18 = 26$, or \$26.

63. a.



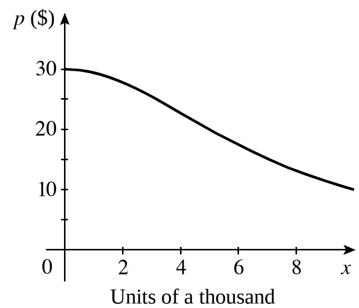
b. $p = 2^3 + 2(2) + 3 = 15$, or \$15.

64. a.



b. $p = 2^3 + 2 + 10 = 20$, or \$20.

65. a.

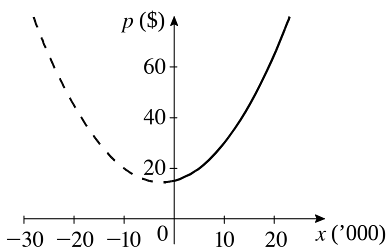


b. Substituting $x = 10$ into the demand function, we have

$$p = \frac{30}{0.02(10)^2 + 1} = \frac{30}{3} = 10, \text{ or } \$10.$$

66. Substituting $x = 6$ and $p = 8$ into the given equation gives $8 = \sqrt{-36a + b}$, or $-36a + b = 64$. Next, substituting $x = 8$ and $p = 6$ into the equation gives $6 = \sqrt{-64a + b}$, or $-64a + b = 36$. Solving the system $\begin{cases} -36a + b = 64 \\ -64a + b = 36 \end{cases}$ for a and b, we find $a = 1$ and $b = 100$. Therefore the demand equation is $p = \sqrt{-x^2 + 100}$. When the unit price is set at \$7.50, we have $7.5 = \sqrt{-x^2 + 100}$, or $56.25 = -x^2 + 100$ from which we deduce that $x \approx \pm 6.614$. Thus, the quantity demanded is approximately 6614 units.

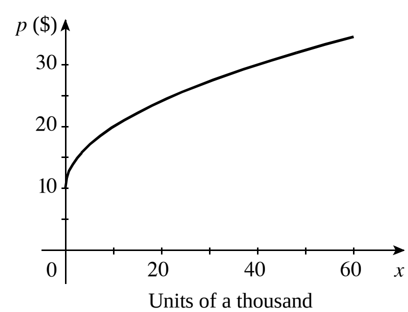
67. a.



b. If $x = 5$, then

$p = 0.1(5)^2 + 0.5(5) + 15 = 20$, or \$20.

68. Substituting $x = 10{,}000$ and $p = 20$ into the given equation yields $20 = a\sqrt{10{,}000} + b = 100a + b$. Next, substituting $x = 62{,}500$ and $p = 35$ into the equation yields $35 = a\sqrt{62{,}500} + b = 250a + b$. Subtracting the first equation from the second yields $15 = 150a$, or $a = \frac{1}{10}$. Substituting this value of a into the first equation gives $b = 10$. Therefore, the required equation is $p = \frac{1}{10}\sqrt{x} + 10$. Substituting $x = 40{,}000$ into the supply equation yields $p = \frac{1}{10}\sqrt{40{,}000} + 10 = 30$, or \$30.



69. a. We solve the system of equations $p = cx + d$ and $p = ax + b$. Substituting the first equation into the second gives $cx + d = ax + d$, so $(c - a)x = b - d$ and $x = \dfrac{b-d}{c-a}$. Because $a < 0$ and $c > 0$, $c - a \neq 0$ and x is well-defined. Substituting this value of x into the second equation, we obtain $p = a\left(\dfrac{b-d}{c-a}\right) + b = \dfrac{ab - ad + bc - ab}{c-a} = \dfrac{bc - ad}{c-a}$. Therefore, the equilibrium quantity is $\dfrac{b-d}{c-a}$ and the equilibrium price is $\dfrac{bc-ad}{c-a}$.

b. If c is increased, the denominator in the expression for x increases and so x gets smaller. At the same time, the first term in the first equation for p decreases and so p gets larger. This analysis shows that if the unit price for producing the product is increased, then the equilibrium quantity decreases while the equilibrium price increases.

c. If b is decreased, the numerator of the expression for x decreases while the denominator stays the same. Therefore, x decreases. The expression for p also shows that p decreases. This analysis shows that if the (theoretical) upper bound for the unit price of a commodity is lowered, then both the equilibrium quantity and the equilibrium price drop.

70. We solve the system of equations $p = -x^2 - 2x + 100$ and $p = 8x + 25$. Thus, $-x^2 - 2x + 100 = 8x + 25$, or $x^2 + 10x - 75 = 0$. Factoring this equation, we have $(x + 15)(x - 5) = 0$. Therefore, $x = -15$ or $x = 5$. Rejecting the negative root, we have $x = 5$, and the corresponding value of p is $p = 8(5) + 25 = 65$. We conclude that the equilibrium quantity is 5000 and the equilibrium price is \$65.

71. We solve the equation $-2x^2 + 80 = 15x + 30$, or $2x^2 + 15x - 50 = 0$ for x. Thus, $(2x - 5)(x + 10) = 0$, and so $x = \frac{5}{2}$ or $x = -10$. Rejecting the negative root, we have $x = \frac{5}{2}$. The corresponding value of p is $p = -2\left(\frac{5}{2}\right)^2 + 80 = 67.5$. We conclude that the equilibrium quantity is 2500 and the equilibrium price is \$67.50.

72. We solve the system $\begin{cases} p = 60 - 2x^2 \\ p = x^2 + 9x + 30 \end{cases}$ Equating the right-hand sides, we have $x^2 + 9x + 30 = 60 - 2x^2$, so $3x^2 + 9x - 30 = 0$, $x^2 + 3x - 10 = 0$, and $(x + 5)(x - 2) = 0$, giving $x = -5$ or $x = 2$. We take $x = 2$. The corresponding value of p is 52, so the equilibrium quantity is 2000 and the equilibrium price is \$52.

73. Solving both equations for x, we have $x = -\frac{11}{3}p + 22$ and $x = 2p^2 + p - 10$. Equating the right-hand sides, we have $-\frac{11}{3}p + 22 = 2p^2 + p - 10$, or $-11p + 66 = 6p^2 + 3p - 30$, and so $6p^2 + 14p - 96 = 0$. Dividing this last equation by 2 and then factoring, we have $(3p + 16)(p - 3) = 0$, so $p = 3$ is the only valid solution. The corresponding value of x is $2(3)^2 + 3 - 10 = 11$. We conclude that the equilibrium quantity is 11,000 and the equilibrium price is \$3.

74. Equating the right-hand sides of the two equations, we have $0.1x^2 + 2x + 20 = -0.1x^2 - x + 40$, so $0.2x^2 + 3x - 20 = 0$, $2x^2 + 30x - 200 = 0$, $x^2 + 15x - 100 = 0$, and $(x + 20)(x - 5) = 0$. Therefore the only valid solution is $x = 5$. Substituting $x = 5$ into the first equation gives $p = -0.1(25) - 5 + 40 = 32.5$. Therefore, the equilibrium quantity is 500 tents (x is measured in hundreds) and the equilibrium price is \$32.50.

75. Equating the right-hand sides of the two equations, we have $144 - x^2 = 48 + \frac{1}{2}x^2$, so $288 - 2x^2 = 96 + x^2$, $3x^2 = 192$, and $x^2 = 64$. Therefore, $x = \pm 8$. We take $x = 8$, and the corresponding value of p is $144 - 8^2 = 80$. We conclude that the equilibrium quantity is 8000 tires and the equilibrium price is \$80.

76. Because there is 80 feet of fencing available, $2x + 2y = 80$, so $x + y = 40$ and $y = 40 - x$. Then the area of the garden is given by $f = xy = x(40 - x) = 40x - x^2$. The domain of f is $[0, 40]$.

77. The area of Juanita's garden is 250 ft^2. Therefore $xy = 250$ and $y = \dfrac{250}{x}$. The amount of fencing needed is given by $2x + 2y$. Therefore, $f = 2x + 2\left(\dfrac{250}{x}\right) = 2x + \dfrac{500}{x}$. The domain of f is $x > 0$.

78. The volume of the box is given by area of the base times the height of the box. Thus, $V = f(x) = (15 - 2x)(8 - 2x)x$.

79. Because the volume of the box is the area of the base times the height of the box, we have $V = x^2 y = 20$. Thus, we have $y = \dfrac{20}{x^2}$. Next, the amount of material used in constructing the box is given by the area of the base of the box, plus the area of the four sides, plus the area of the top of the box; that is, $A = x^2 + 4xy + x^2$. Then, the cost of constructing the box is given by $f(x) = 0.30x^2 + 0.40x \cdot \dfrac{20}{x^2} + 0.20x^2 = 0.5x^2 + \dfrac{8}{x}$, where $f(x)$ is measured in dollars and $f(x) > 0$.

80. Because the perimeter of a circle is $2\pi r$, we know that the perimeter of the semicircle is πx. Next, the perimeter of the rectangular portion of the window is given by $2y + 2x$, so the perimeter of the Norman window is $\pi x + 2y + 2x$ and $\pi x + 2y + 2x = 28$, or $y = \frac{1}{2}(28 - \pi x - 2x)$. Because the area of the window is given by $2xy + \frac{1}{2}\pi x^2$, we see that $A = 2xy + \frac{1}{2}\pi x^2$. Substituting the value of y found earlier, we see that

$$A = f(x) = x(28 - \pi x - 2x) + \tfrac{1}{2}\pi x^2 = \tfrac{1}{2}\pi x^2 + 28x - \pi x^2 - 2x^2 = 28x - \tfrac{\pi}{2}x^2 - 2x^2$$
$$= 28x - \left(\frac{\pi}{2} + 2\right)x^2.$$

81. The average yield of the apple orchard is 36 bushels/tree when the density is 22 trees/acre. Let x be the unit increase in tree density beyond 22. Then the yield of the apple orchard in bushels/acre is given by $(22 + x)(36 - 2x)$.

82. $xy = 50$ and so $y = \dfrac{50}{x}$. The area of the printed page is $A = (x - 1)(y - 2) = (x - 1)\left(\dfrac{50}{x} - 2\right) = -2x + 52 - \dfrac{50}{x}$, so the required function is $f(x) = -2x + 52 - \dfrac{50}{x}$. We must have $x > 0$, $x - 1 \geq 0$, and and $\dfrac{50}{x} - 2 \geq 2$. The last inequality is solved as follows: $\dfrac{50}{x} \geq 4$, so $\dfrac{x}{50} \leq \dfrac{1}{4}$, so $x \leq \frac{50}{4} = \frac{25}{2}$. Thus, the domain is $\left[1, \frac{25}{2}\right]$.

83. **a.** Let x denote the number of bottles sold beyond 10,000 bottles. Then $P(x) = (10{,}000 + x)(5 - 0.0002x) = -0.0002x^2 + 3x + 50{,}000$.

b. He can expect a profit of $P(6000) = -0.0002\left(6000^2\right) + 3(6000) + 50{,}000 = 60{,}800$, or \$60,800.

84. **a.** Let x denote the number of people beyond 20 who sign up for the cruise. Then the revenue is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12{,}000$.

b. $R(40) = -4\left(40^2\right) + 520(40) + 12{,}000 = 26{,}400$, or \$26,400.

c. $R(60) = -4\left(60^2\right) + 520(60) + 12{,}000 = 28{,}800$, or \$28,800.

85. False. $f(x) = 3x^{3/4} + x^{1/2} + 1$ is not a polynomial function. The powers of x must be nonnegative integers.

86. True. If $P(x)$ is a polynomial function, then $P(x) = \frac{P(x)}{1}$ and so it is a rational function. The converse is false. For example, $R(x) = \frac{x+1}{x-1}$ is a rational function that is not a polynomial.

87. False. $f(x) = x^{1/2}$ is not defined for negative values of x.

88. False. A power function has the form x^r, where r is a real number.

Using Technology page 98

1. $(-3.0414, 0.1503)$, $(3.0414, 7.4497)$.

2. $(-5.3852, 9.8007)$, $(5.3852, -4.2007)$.

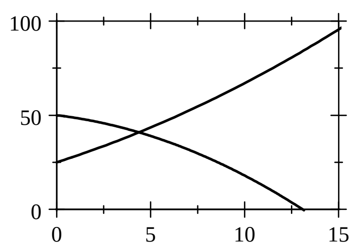
3. $(-2.3371, 2.4117)$, $(6.0514, -2.5015)$.

4. $(-2.5863, -0.3585)$, $(6.1863, -4.5694)$.

5. $(-1.0219, -6.3461)$, $(1.2414, -1.5931)$, and $(5.7805, 7.9391)$.

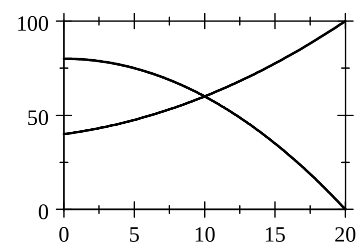
6. $(-0.0484, 2.0609)$, $(2.0823, 2.8986)$, and $(4.9661, 1.1405)$.

7. a.



b. 438 wall clocks; \$40.92.

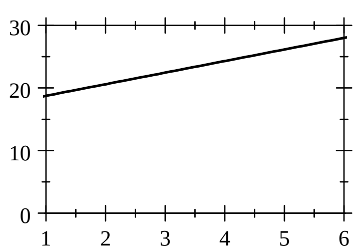
8. a.



b. 1000 cameras; \$60.00.

9. a. $f(t) = 1.85t + 16.9$.

b.



c.

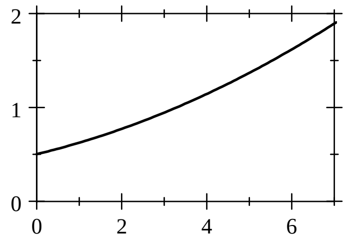
t	y
1	18.8
2	20.6
3	22.5
4	24.3
5	26.2
6	28.0

These values are close to the given data.

d. $f(8) = 1.85(8) + 16.9 = 31.7$ gallons.

10. **a.** $f(t) = 0.0128t^2 + 0.109t + 0.50.$

b.



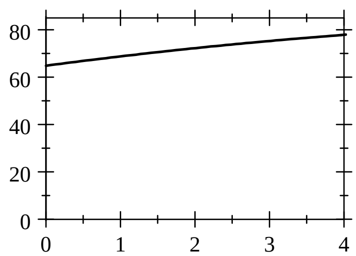
c.

t	y
0	0.50
3	0.94
6	1.61
7	1.89

These values are close to the given data.

11. **a.** $f(t) = -0.221t^2 + 4.14t + 64.8.$

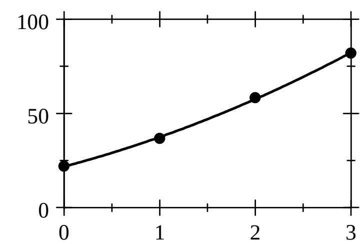
b.



c. 77.8 million

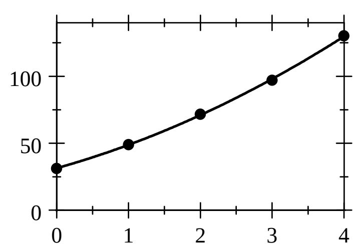
12. **a.** $f(t) = 2.25x^2 + 13.41x + 21.76.$

b.



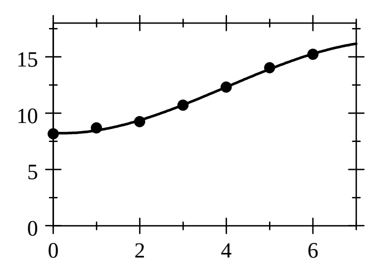
13. **a.** $f(t) = 2.4t^2 + 15t + 31.4.$

b.



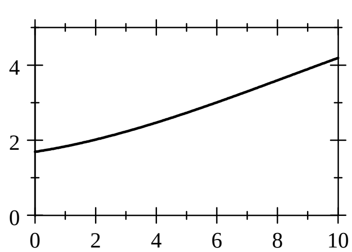
14. **a.** $f(t) = -0.038167t^3 + 0.45713t^2 - 0.19758t + 8.2457.$

b.



15. **a.** $f(t) = -0.00081t^3 + 0.0206t^2 + 0.125t + 1.69.$

b.



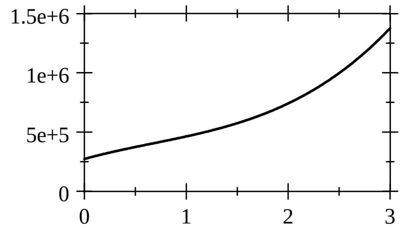
c.

t	y
1	1.8
5	2.7
10	4.2

The revenues were \$1.8 trillion in 2001, \$2.7 trillion in 2005, and \$4.2 trillion in 2010.

16. a. $y = 44{,}560t^3 - 89{,}394t^2 + 234{,}633t + 273{,}288$.

b.

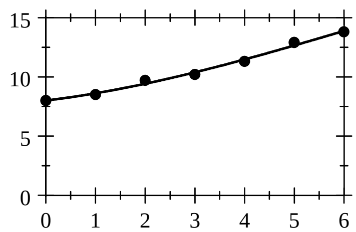


c.

t	$f(t)$
0	273,288
1	463,087
2	741,458
3	1,375,761

17. a. $f(t) = -0.0056t^3 + 0.112t^2 + 0.51t + 8$.

b.

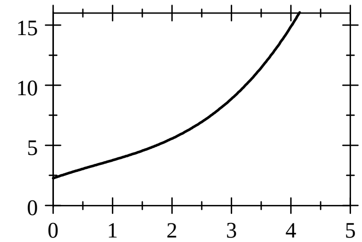


c.

t	0	3	6
$f(t)$	8	10.4	13.9

18. a. $f(t) = 0.2t^3 - 0.45t^2 + 1.75t + 2.26$.

b.

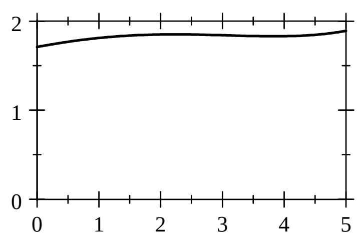


c.

t	0	1	2	3	4
$f(t)$	2.3	3.8	5.6	8.9	14.9

19. a. $f(t) = 0.00125t^4 - 0.0051t^3 - 0.0243t^2 + 0.129t + 1.71$.

b.



c.

t	0	1	2	3	4	5
$f(t)$	1.71	1.81	1.85	1.84	1.83	1.89

d. The average amount of nicotine in 2005 is $f(6) = 2.128$, or approximately 2.13 mg/cigarette.

20. $A(t) = 0.000008140t^4 - 0.00043833t^3 - 0.0001305t^2 + 0.02202t + 2.612$.

2.4 Limits

Concept Questions

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1. The values of $f(x)$ can be made as close to 3 as we please by taking x sufficiently close to $x = 2$.

2. a. Nothing. Whether $f(3)$ is defined or not does not depend on $\lim_{x\to 3} f(x)$.

b. Nothing. $\lim_{x\to 2} f(x)$ has nothing to do with the value of f at $x = 2$.

3. a. $\lim_{x\to 4} \sqrt{x}\left(2x^2+1\right) = \lim_{x\to 4}\left(\sqrt{x}\right)\lim_{x\to 4}\left(2x^2+1\right)$ (Property 4)

$= \sqrt{4}\left[2\,(4)^2+1\right]$ (Properties 1 and 3)

$= 66$

b. $\lim_{x\to 1}\left(\dfrac{2x^2+x+5}{x^4+1}\right)^{3/2} = \left(\lim_{x\to 1}\dfrac{2x^2+x+5}{x^4+1}\right)^{3/2}$ (Property 1)

$= \left(\dfrac{2+1+5}{1+1}\right)^{3/2}$ (Properties 2, 3, and 5)

$= 4^{3/2} = 8$

4. A limit that has the form $\lim_{x\to a}\dfrac{f(x)}{g(x)} = \dfrac{0}{0}$. For example, $\lim_{x\to 3}\dfrac{x^2-9}{x-3}$.

5. $\lim_{x\to\infty} f(x) = L$ means $f(x)$ can be made as close to L as we please by taking x sufficiently large. $\lim_{x\to-\infty} f(x) = M$ means $f(x)$ can be made as close to M as we please by taking negative x as large as we please in absolute value.

Exercises

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1. $\lim_{x\to -2} f(x) = 3$.

2. $\lim_{x\to 1} f(x) = 2$.

3. $\lim_{x\to 3} f(x) = 3$.

4. $\lim_{x\to 1} f(x)$ does not exist. If we consider any value of x to the right of $x = 1$, we find that $f(x) = 3$. On the other hand, if we consider values of x to the left of $x = 1$, $f(x) \le 1.5$, and we conclude that $f(x)$ does not approach a fixed number as x approaches 1.

5. $\lim_{x\to -2} f(x) = 3$.

6. $\lim_{x\to -2} f(x) = 3$.

7. The limit does not exist. If we consider any value of x to the right of $x = -2$, $f(x) \le 0$. If we consider values of x to the left of $x = -2$, $f(x) \ge 0$. Because $f(x)$ does not approach any one number as x approaches $x = -2$, we conclude that the limit does not exist.

8. The limit does not exist.

9.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.61	4.9601	4.9960	5.004	5.0401	5.41

$\lim_{x\to 2}\left(x^2+1\right) = 5$.

10.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.62	0.9602	0.996002	1.004002	1.0402	1.42

$\lim_{x\to 1}\left(2x^2-1\right)=1.$

11.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

12.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

13.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	100	10,000	1,000,000	1,000,000	10,000	100

The limit does not exist.

14.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	−10	−100	−1000	1000	100	10

The limit does not exist.

15.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.9	2.99	2.999	3.001	3.01	3.1

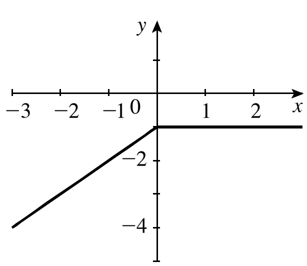
$\lim_{x\to 1}\dfrac{x^2+x-2}{x-1}=3.$

16.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1	1	1	1	1	1

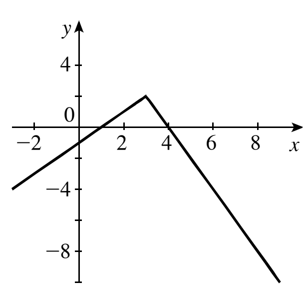
$\lim_{x\to 1}\dfrac{x-1}{x-1}=1.$

17.



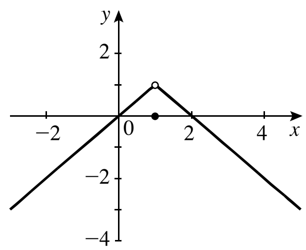
$\lim_{x\to 0} f(x)=-1.$

18.



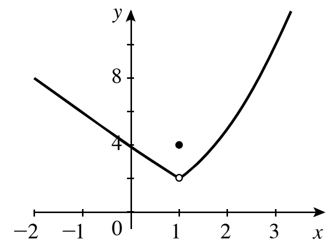
$\lim_{x\to 3} f(x)=2.$

19.



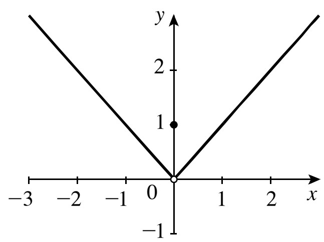
$\lim_{x \to 1} f(x) = 1.$

20.



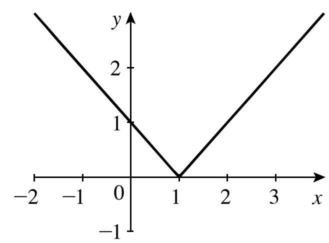
$\lim_{x \to 1} f(x) = 2.$

21.



$\lim_{x \to 0} f(x) = 0.$

22.



$\lim_{x \to 1} f(x) = 0.$

23. $\lim_{x \to 2} 3 = 3.$

24. $\lim_{x \to -2} -3 = -3.$

25. $\lim_{x \to 3} x = 3.$

26. $\lim_{x \to -2} -3x = -3(-2) = 6.$

27. $\lim_{x \to 1} \left(1 - 2x^2\right) = 1 - 2(1)^2 = -1.$

28. $\lim_{t \to 3} \left(4t^2 - 2t + 1\right) = 4(3)^2 - 2(3) + 1 = 31.$

29. $\lim_{x \to 1} \left(2x^3 - 3x^2 + x + 2\right) = 2(1)^3 - 3(1)^2 + 1 + 2 = 2.$

30. $\lim_{x \to 0} \left(4x^5 - 20x^2 + 2x + 1\right) = 4(0)^5 - 20(0)^2 + 2(0) + 1 = 1.$

31. $\lim_{s \to 0} \left(2s^2 - 1\right)(2s + 4) = (-1)(4) = -4.$

32. $\lim_{x \to 2} \left(x^2 + 1\right)\left(x^2 - 4\right) = \left(2^2 + 1\right)\left(2^2 - 4\right) = 0.$

33. $\lim_{x \to 2} \frac{2x + 1}{x + 2} = \frac{2(2) + 1}{2 + 2} = \frac{5}{4}.$

34. $\lim_{x \to 1} \frac{x^3 + 1}{2x^3 + 2} = \frac{1^3 + 1}{2\left(1^3\right) + 2} = \frac{2}{4} = \frac{1}{2}.$

35. $\lim_{x \to 2} \sqrt{x + 2} = \sqrt{2 + 2} = 2.$

36. $\lim_{x \to -2} \sqrt[3]{5x + 2} = \sqrt[3]{5(-2) + 2} = \sqrt[3]{-8} = -2.$

37. $\lim_{x \to -3} \sqrt{2x^4 + x^2} = \sqrt{2(-3)^4 + (-3)^2} = \sqrt{162 + 9} = \sqrt{171} = 3\sqrt{19}.$

38. $\lim_{x \to 2} \sqrt{\frac{2x^3 + 4}{x^2 + 1}} = \sqrt{\frac{2(8) + 4}{4 + 1}} = 2.$

39. $\lim_{x \to -1} \frac{\sqrt{x^2 + 8}}{2x + 4} = \frac{\sqrt{(-1)^2 + 8}}{2(-1) + 4} = \frac{\sqrt{9}}{2} = \frac{3}{2}.$

40. $\lim_{x \to 3} \frac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}} = \frac{3\sqrt{3^2 + 7}}{2(3) - \sqrt{2(3) + 3}} = \frac{12}{3} = 4.$

41. $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a}f(x)-\lim_{x\to a}g(x)$
$=3-4=-1.$

42. $\lim_{x\to a}2f(x)=2(3)=6.$

43. $\lim_{x\to a}\left[2f(x)-3g(x)\right]=\lim_{x\to a}2f(x)-\lim_{x\to a}3g(x)$
$=2(3)-3(4)=-6.$

44. $\lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x)=3\cdot 4$
$=12.$

45. $\lim_{x\to a}\sqrt{g(x)}=\lim_{x\to a}\sqrt{4}=2.$

46. $\lim_{x\to a}\sqrt[3]{5f(x)+3g(x)}=\sqrt[3]{5(3)+3(4)}=\sqrt[3]{27}=3.$

47. $\lim_{x\to a}\dfrac{2f(x)-g(x)}{f(x)g(x)}=\dfrac{2(3)-(4)}{(3)(4)}=\dfrac{2}{12}=\dfrac{1}{6}.$

48. $\lim_{x\to a}\dfrac{g(x)-f(x)}{f(x)+\sqrt{g(x)}}=\dfrac{4-3}{3+2}=\dfrac{1}{5}.$

49. $\lim_{x\to 1}\dfrac{x^2-1}{x-1}=\lim_{x\to 1}\dfrac{(x-1)(x+1)}{x-1}=\lim_{x\to 1}(x+1)$
$=1+1=2.$

50. $\lim_{x\to -2}\dfrac{x^2-4}{x+2}=\lim_{x\to -2}\dfrac{(x-2)(x+2)}{x+2}$
$=\lim_{x\to -2}(x-2)=-2-2=-4.$

51. $\lim_{x\to 0}\dfrac{x^2-x}{x}=\lim_{x\to 0}\dfrac{x(x-1)}{x}=\lim_{x\to 0}(x-1)$
$=0-1=-1.$

52. $\lim_{x\to 0}\dfrac{2x^2-3x}{x}=\lim_{x\to 0}\dfrac{x(2x-3)}{x}=\lim_{x\to 0}(2x-3)$
$=-3.$

53. $\lim_{x\to -5}\dfrac{x^2-25}{x+5}=\lim_{x\to -5}\dfrac{(x+5)(x-5)}{x+5}$
$=\lim_{x\to -5}(x-5)=-10.$

54. $\lim_{b\to -3}\dfrac{b+1}{b+3}$ does not exist.

55. $\lim_{x\to 1}\dfrac{x}{x-1}$ does not exist.

56. $\lim_{x\to 2}\dfrac{x+2}{x-2}$ does not exist.

57. $\lim_{x\to -2}\dfrac{x^2-x-6}{x^2+x-2}=\lim_{x\to -2}\dfrac{(x-3)(x+2)}{(x+2)(x-1)}=\lim_{x\to -2}\dfrac{x-3}{x-1}=\dfrac{-2-3}{-2-1}=\dfrac{5}{3}.$

58. $\lim_{z\to 2}\dfrac{z^3-8}{z-2}=\lim_{z\to 2}\dfrac{(z-2)\left(z^2+2z+4\right)}{z-2}=\lim_{z\to 2}\left(z^2+2z+4\right)=2^2+2(2)+4=12.$

59. $\lim_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\dfrac{x-1}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{2}.$

60. $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to 4}\sqrt{x}+2=2+2=4.$

61. $\lim_{x\to 1}\dfrac{x-1}{x^3+x^2-2x}=\lim_{x\to 1}\dfrac{x-1}{x(x-1)(x+2)}=\lim_{x\to 1}\dfrac{1}{x(x+2)}=\dfrac{1}{3}.$

62. $\lim_{x\to -2}\dfrac{4-x^2}{2x^2+x^3}=\lim_{x\to -2}\dfrac{(2-x)(2+x)}{x^2(2+x)}=\lim_{x\to -2}\dfrac{2-x}{x^2}=\dfrac{2-(-2)}{(-2)^2}=1.$

63. $\lim_{x\to\infty}f(x)=\infty$ (does not exist) and $\lim_{x\to -\infty}f(x)=\infty$ (does not exist).

64. $\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

65. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$.

66. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$.

67. $\lim_{x\to\infty} f(x) = -\infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

68. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = \infty$ (does not exist).

69. $f(x) = \dfrac{1}{x^2+1}$.

x	1	10	100	1000
$f(x)$	0.5	0.009901	0.0001	0.000001

x	-1	-10	-100	-1000
$f(x)$	0.5	0.009901	0.0001	0.000001

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$.

70. $f(x) = \dfrac{2x}{x+1}$.

x	1	10	100	1000
$f(x)$	1	1.818	1.980	1.998

x	-5	-10	-100	-1000
$f(x)$	2.5	2.222	2.020	2.002

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$.

71. $f(x) = 3x^3 - x^2 + 10$.

x	1	5	10	100	1000
$f(x)$	12	360	2910	2.99×10^6	2.999×10^9

x	-1	-5	-10	-100	-1000
$f(x)$	6	-390	-3090	-3.01×10^6	-3.0×10^9

$\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

72. $f(x) = \dfrac{|x|}{x}$

x	1	10	100
$f(x)$	1	1	1

x	-1	-10	-100
$f(x)$	-1	-1	-1

$\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

73. $\displaystyle\lim_{x\to\infty} \frac{3x+2}{x-5} = \lim_{x\to\infty} \frac{3+\dfrac{2}{x}}{1-\dfrac{5}{x}} = \frac{3}{1} = 3.$

74. $\lim_{x\to-\infty}\dfrac{4x^2-1}{x+2}=\lim_{x\to-\infty}\dfrac{4x-\frac{1}{x}}{1+\frac{2}{x}}=-\infty$; that is, the limit does not exist.

75. $\lim_{x\to-\infty}\dfrac{3x^3+x^2+1}{x^3+1}=\lim_{x\to-\infty}\dfrac{3+\frac{1}{x}+\frac{1}{x^3}}{1+\frac{1}{x^3}}=3.$

76. $\lim_{x\to\infty}\dfrac{2x^2+3x+1}{x^4-x^2}=\lim_{x\to\infty}\dfrac{\frac{2}{x^2}+\frac{3}{x^3}+\frac{1}{x^4}}{1-\frac{1}{x^2}}=0.$

77. $\lim_{x\to-\infty}\dfrac{x^4+1}{x^3-1}=\lim_{x\to-\infty}\dfrac{x+\frac{1}{x^3}}{1-\frac{1}{x^3}}=-\infty$; that is, the limit does not exist.

78. $\lim_{x\to\infty}\dfrac{4x^4-3x^2+1}{2x^4+x^3+x^2+x+1}=\lim_{x\to\infty}\dfrac{4-\frac{3}{x^2}+\frac{1}{x^4}}{2+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}=2.$

79. $\lim_{x\to\infty}\dfrac{x^5-x^3+x-1}{x^6+2x^2+1}=\lim_{x\to\infty}\dfrac{\frac{1}{x}-\frac{1}{x^3}+\frac{1}{x^5}-\frac{1}{x^6}}{1+\frac{2}{x^4}+\frac{1}{x^6}}=0.$

80. $\lim_{x\to\infty}\dfrac{2x^2-1}{x^3+x^2+1}=\lim_{x\to\infty}\dfrac{\frac{2}{x}-\frac{1}{x^3}}{1+\frac{1}{x}+\frac{1}{x^3}}=0.$

81. a. The cost of removing 50% of the pollutant is $C(50)=\dfrac{0.5(50)}{100-50}=0.5$, or \$500,000. Similarly, we find that the cost of removing 60%, 70%, 80%, 90%, and 95% of the pollutant is \$750,000, \$1,166,667, \$2,000,000, \$4,500,000, and \$9,500,000, respectively.

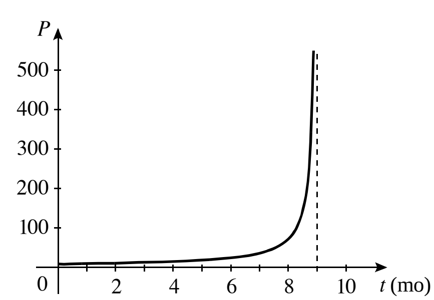
b. $\lim_{x\to100}\dfrac{0.5x}{100-x}=\infty$, which means that the cost of removing the pollutant increases without bound if we wish to remove almost all of the pollutant.

82. a. The number present initially is given by $P(0)=\dfrac{72}{9-0}=8.$

b. As t approaches 9 (remember that $0<t<9$), the denominator approaches 0 while the numerator remains constant at 72. Therefore, $P(t)$ gets larger and larger. Thus,

$$\lim_{t\to9}P(t)=\lim_{t\to9}\frac{72}{9-t}=\infty.$$

c.



83. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} \left(2.2 + \frac{2500}{x}\right) = 2.2$, or \$2.20 per DVD. In the long run, the average cost of producing x DVDs approaches \$2.20/disc.

84. $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \frac{\frac{0.2}{t}}{1+\frac{1}{t^2}} = 0$, which says that the concentration of drug in the bloodstream eventually decreases to zero.

85. **a.** $T(1) = \frac{120}{1+4} = 24$, or \$24 million. $T(2) = \frac{120(2)^2}{8} = 60$, or \$60 million. $T(3) = \frac{120(3)^2}{13} = 83.1$, or \$83.1 million.

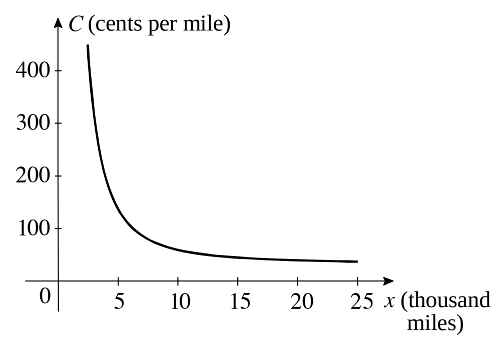
b. In the long run, the movie will gross $\lim_{x\to\infty} \frac{120x^2}{x^2+4} = \lim_{x\to\infty} \frac{120}{1+\frac{4}{x^2}} = 120$, or \$120 million.

86. **a.** The current population is $P(0) = \frac{200}{40} = 5$, or 5000.

b. The population in the long run will be $\lim_{t\to\infty} \frac{25t^2+125t+200}{t^2+5t+40} = \lim_{t\to\infty} \frac{25+\frac{125}{t}+\frac{200}{t^2}}{1+\frac{5}{t}+\frac{40}{t^2}} = 25$, or 25,000.

87. **a.** The average cost of driving 5000 miles per year is $C(5) = \frac{2410}{5^{1.95}} + 32.8 \approx 137.28$, or 137.3 cents per mile. Similarly, we see that the average costs of driving 10, 15, 20, and 25 thousand miles per year are 59.8, 45.1, 39.8, and 37.3 cents per mile, respectively.

b.



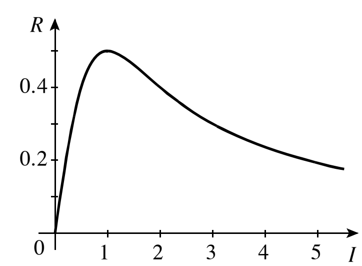
c. It approaches 32.8 cents per mile.

88. **a.** $R(I) = \frac{I}{1+I^2}$

I	0	1	2	3	4	5
$R(I)$	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{4}{17}$	$\frac{5}{26}$

b. $\lim_{I\to\infty} R(I) = \lim_{I\to\infty} \frac{I}{1+I^2} = 0.$

c.



89. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 1$, but $f(1)$ is not defined.

90. True.

91. True. Division by zero is not permitted.

92. False. Let $f(x) = (x-3)^2$ and $g(x) = x-3$. Then $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, but $\lim_{x\to 3} \frac{f(x)}{g(x)} = \lim_{x\to 3} \frac{(x-3)^2}{x-3} = \lim_{x\to 3} (x-3) = 0$.

93. True. Each limit in the sum exists. Therefore, $\lim_{x\to 2} \left(\frac{x}{x+1} + \frac{3}{x-1}\right) = \lim_{x\to 2} \frac{x}{x+1} + \lim_{x\to 2} \frac{3}{x-1} = \frac{2}{3} + \frac{3}{1} = \frac{11}{3}$.

94. False. Neither of the limits $\lim_{x\to 1} \frac{2x}{x-1}$ and $\lim_{x\to 1} \frac{2}{x-1}$ exists.

95. $\lim_{x\to\infty} \frac{ax}{x+b} = \lim_{x\to\infty} \frac{a}{1+\frac{b}{x}} = a$. As the amount of substrate becomes very large, the initial speed approaches the constant a moles per liter per second.

96. Consider the functions $f(x) = 1/x$ and $g(x) = -1/x$. Observe that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \left[f(x) + g(x)\right] = \lim_{x\to 0} 0 = 0$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

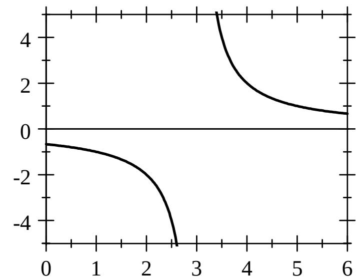
97. Consider the functions $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \left[f(x)\, g(x)\right] = \lim_{x\to 0} (-1) = -1$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

98. Take $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$, and $a = 0$. Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{1}{x} \cdot \frac{x^2}{1} = \lim_{x\to 0} x = a$ exists. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

Using Technology page 121

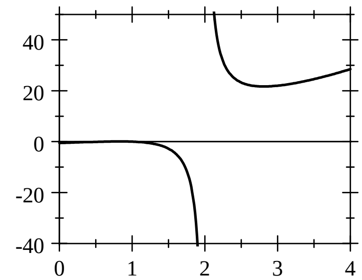
1. 5	**2.** 11	**3.** 3	**4.** 0
5. $\frac{2}{3}$	**6.** $\frac{1}{2}$	**7.** $e^2 \approx 7.38906$	**8.** $\ln 2 \approx 0.693147$

9.



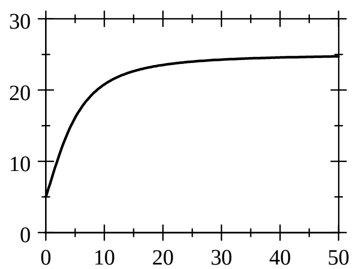
From the graph we see that $f(x)$ does not approach any finite number as x approaches 3.

10.



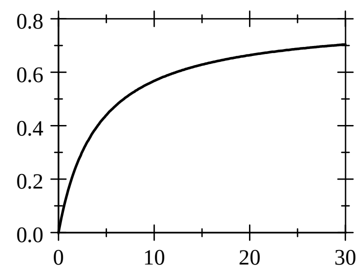
From the graph, we see that $f(x)$ does not approach any finite number as x approaches 2.

11. a.



b. $\lim\limits_{t\to\infty} \dfrac{25t^2 + 125t + 200}{t^2 + 5t + 40} = 25$, so in the long run the population will approach 25,000.

12. a.



b. $\lim\limits_{t\to\infty} \dfrac{0.8t}{t + 4.1} = \lim\limits_{t\to\infty} \dfrac{0.8}{1 + \dfrac{4.1}{t}} = 0.8$.

2.5 One-Sided Limits and Continuity

Concept Questions

page 129

1. $\lim\limits_{x\to3^-} f(x) = 2$ means $f(x)$ can be made as close to 2 as we please by taking x sufficiently close to but to the left of $x = 3$. $\lim\limits_{x\to3^+} f(x) = 4$ means $f(x)$ can be made as close to 4 as we please by taking x sufficiently close to but to the right of $x = 3$.

2. a. $\lim\limits_{x\to1} f(x)$ does not exist because the left- and right-hand limits at $x = 1$ are different.

b. Nothing, because the existence or value of f at $x = 1$ does not depend on the existence (or nonexistence) of the left- or right-hand, or two-sided, limits of f.

3. a. f is continuous at a if $\lim\limits_{x\to a} f(x) = f(a)$.

b. f is continuous on an interval I if f is continuous at each point in I.

4. $f(a) = L = M$.

5. a. f is continuous because the plane does not suddenly jump from one point to another.

b. f is continuous.

c. f is discontinuous because the fare "jumps" after the cab has covered a certain distance or after a certain amount of time has elapsed.

d. f is discontinuous because the rates "jump" by a certain amount (up or down) when it is adjusted at certain times.

6. Refer to page 127 in the text. Answers will vary.

Exercises page 130

1. $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = 2$, so $\lim_{x\to 2} f(x)$ does not exist.

2. $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 5$, so $\lim_{x\to 3} f(x)$ does not exist.

3. $\lim_{x\to -1^-} f(x) = \infty$ and $\lim_{x\to -1^+} f(x) = 2$, so $\lim_{x\to -1} f(x)$ does not exist.

4. $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 3$, so $\lim_{x\to 1} f(x) = 3$.

5. $\lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = 2$, so $\lim_{x\to 1} f(x)$ does not exist.

6. $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = \infty$, so $\lim_{x\to 0} f(x)$ does not exist.

7. $\lim_{x\to 0^-} f(x) = -2$ and $\lim_{x\to 0^+} f(x) = 2$, so $\lim_{x\to 0} f(x)$ does not exist.

8. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 2$.

9. True. **10.** True. **11.** True. **12.** True. **13.** False. **14.** True.

15. True. **16.** True. **17.** False. **18.** True. **19.** True. **20.** False

21. $\lim_{x\to 1^+} (2x+4) = 6$.

22. $\lim_{x\to 1^-} (3x-4) = -1$.

23. $\lim_{x\to 2^-} \dfrac{x-3}{x+2} = \dfrac{2-3}{2+2} = -\dfrac{1}{4}$.

24. $\lim_{x\to 1^+} \dfrac{x+2}{x+1} = \dfrac{1+2}{1+1} = \dfrac{3}{2}$.

25. $\lim_{x\to 0^+} \dfrac{1}{x}$ does not exist because $\dfrac{1}{x} \to \infty$ as $x \to 0$ from the right.

26. $\lim_{x\to 0^-} \dfrac{1}{x} = -\infty$; that is, the limit does not exist.

27. $\lim_{x\to 0^+} \dfrac{x-1}{x^2+1} = \dfrac{-1}{1} = -1$.

28. $\lim_{x\to 2^+} \dfrac{x+1}{x^2-2x+3} = \dfrac{2+1}{4-4+3} = 1$.

29. $\lim_{x\to 0^+} \sqrt{x} = \sqrt{\lim_{x\to 0^+} x} = 0$.

30. $\lim_{x\to 2^+} 2\sqrt{x-2} = 2\cdot 0 = 0$.

31. $\lim_{x\to -2^+} \left(2x+\sqrt{2+x}\right) = \lim_{x\to -2^+} 2x + \lim_{x\to -2^+} \sqrt{2+x} = -4+0 = -4$.

32. $\lim_{x\to -5^+} x\left(1+\sqrt{5+x}\right) = -5\left[1+\sqrt{5+(-5)}\right] = -5$.

33. $\lim_{x\to 1^-} \dfrac{1+x}{1-x} = \infty$, that is, the limit does not exist.

34. $\lim_{x \to 1^+} \frac{1+x}{1-x} = -\infty.$

35. $\lim_{x \to 2^-} \frac{x^2-4}{x-2} = \lim_{x \to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x \to 2^-} (x+2) = 4.$

36. $\lim_{x \to -3^+} \frac{\sqrt{x+3}}{x^2+1} = \frac{0}{10} = 0.$

37. $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x^2 = 0$ and $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 2x = 0.$

38. $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2x+3) = 3$ and $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x+1) = 1.$

39. The function is discontinuous at $x = 0$. Conditions 2 and 3 are violated.

40. The function is not continuous because condition 3 for continuity is not satisfied.

41. The function is continuous everywhere.

42. The function is continuous everywhere.

43. The function is discontinuous at $x = 0$. Condition 3 is violated.

44. The function is not continuous at $x = -1$ because condition 3 for continuity is violated.

45. f is continuous for all values of x.

46. f is continuous for all values of x.

47. f is continuous for all values of x. Note that $x^2 + 1 \geq 1 > 0$.

48. f is continuous for all values of x. Note that $2x^2 + 1 \geq 1 > 0$.

49. f is discontinuous at $x = \frac{1}{2}$, where the denominator is 0. Thus, f is continuous on $\left(-\infty, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$.

50. f is discontinuous at $x = 1$, where the denominator is 0. Thus, f is continuous on $(-\infty, 1)$ and $(1, \infty)$.

51. Observe that $x^2 + x - 2 = (x+2)(x-1) = 0$ if $x = -2$ or $x = 1$, so f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

52. Observe that $x^2 + 2x - 3 = (x+3)(x-1) = 0$ if $x = -3$ or $x = 1$, so, f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -3)$, $(-3, 1)$, and $(1, \infty)$.

53. f is continuous everywhere since all three conditions are satisfied.

54. f is continuous everywhere since all three conditions are satisfied.

55. f is continuous everywhere since all three conditions are satisfied.

56. f is not defined at $x = 1$ and is discontinuous there. It is continuous everywhere else.

57. Because the denominator $x^2 - 1 = (x-1)(x+1) = 0$ if $x = -1$ or 1, we see that f is discontinuous at -1 and 1.

58. The function f is not defined at $x = 1$ and $x = 2$. Therefore, f is discontinuous at 1 and 2.

59. Because $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$ if $x = 1$ or 2, we see that the denominator is zero at these points and so f is discontinuous at these numbers.

60. The denominator of the function f is equal to zero when $x^2 - 2x = x(x - 2) = 0$; that is, when $x = 0$ or $x = 2$. Therefore, f is discontinuous at $x = 0$ and $x = 2$.

61. The function f is discontinuous at $x = 4, 5, 6, \ldots, 13$ because the limit of f does not exist at these points.

62. f is discontinuous at $t = 20$, 40, and 60. When $t = 0$, the inventory stands at 750 reams. The level drops to about 250 reams by the twentieth day at which time a new order of 500 reams arrives to replenish the supply. A similar interpretation holds for the other values of t.

63. Having made steady progress up to $x = x_1$, Michael's progress comes to a standstill at that point. Then at $x = x_2$ a sudden breakthrough occurs and he then continues to solve the problem.

64. The total deposits of Franklin make a jump at each of these points as the deposits of the ailing institutions become a part of the total deposits of the parent company.

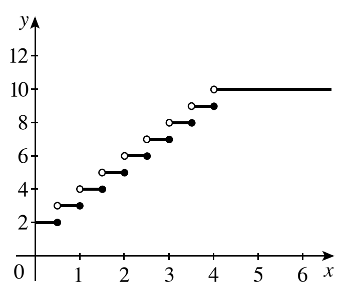
65. Conditions 2 and 3 are not satisfied at any of these points.

66. The function P is discontinuous at $t = 12$, 16, and 28. At $t = 12$, the prime interest rate jumped from $3\frac{1}{2}\%$ to 4%, at $t = 16$ it jumped to $4\frac{1}{2}\%$, and at $t = 28$ it jumped back down to 4%.

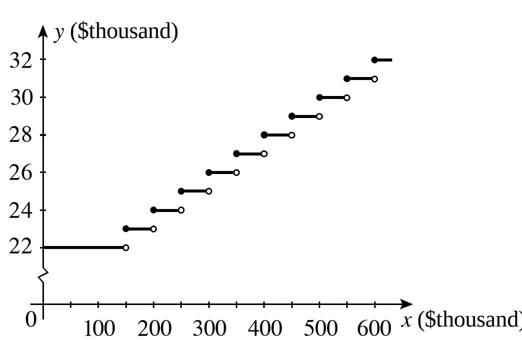
67.

$$f(x) = \begin{cases} 2 & \text{if } 0 < x \le \frac{1}{2} \\ 3 & \text{if } \frac{1}{2} < x \le 1 \\ \vdots & \qquad \vdots \\ 10 & \text{if } x > 4 \end{cases}$$

f is discontinuous at $x = \frac{1}{2}, 1, 1\frac{1}{2}, \ldots, 4$.

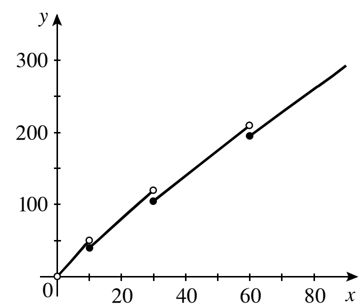


68.



f is discontinuous at $x = 150{,}000$, at $x = 200{,}000$, at $x = 250{,}000$, and so on.

69.



C is discontinuous at $x = 0$, 10, 30, and 60.

70. **a.** $\displaystyle\lim_{v\to u^+} \frac{aLv^3}{v-u} = \infty$. This reflects the fact that when the speed of the fish is very close to that of the current, the energy expended by the fish will be enormous.

b. $\displaystyle\lim_{v\to\infty} \frac{aLv^3}{v-u} = \infty$. This says that if the speed of the fish increases greatly, so does the amount of energy required to swim a distance of L ft.

71. **a.** $\displaystyle\lim_{t\to 0^+} S(t) = \lim_{t\to 0^+} \frac{a}{t} + b = \infty$. As the time taken to excite the tissue is made shorter and shorter, the electric current gets stronger and stronger.

b. $\displaystyle\lim_{t\to\infty} \frac{a}{t} + b = b$. As the time taken to excite the tissue is made longer and longer, the electric current gets weaker and weaker and approaches b.

72. **a.** $\displaystyle\lim_{D\to 0^+} L = \lim_{D\to 0^+} \frac{Y-(1-D)R}{D} = \infty$, so if the investor puts down next to nothing to secure the loan, the leverage approaches infinity.

b. $\displaystyle\lim_{D\to 1^-} L = \lim_{D\to 1^-} \frac{Y-(1-D)R}{D} = Y$, so if the investor puts down all of the money to secure the loan, the leverage is equal to the yield.

73. We require that $f(1) = 1+2 = 3 = \displaystyle\lim_{x\to 1^+} kx^2 = k$, so $k = 3$.

74. Because $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x+2} = \lim_{x\to -2} \frac{(x-2)(x+2)}{x+2} = \lim_{x\to -2}(x-2) = -4$, we define $f(-2) = k = -4$, that is, take $k = -4$.

75. **a.** f is a polynomial of degree 2 and is therefore continuous everywhere, including the interval $[1, 3]$.

b. $f(1) = 3$ and $f(3) = -1$ and so f must have at least one zero in $(1, 3)$.

76. **a.** f is a polynomial of degree 3 and is thus continuous everywhere.

b. $f(0) = 14$ and $f(1) = -23$ and so f has at least one zero in $(0, 1)$.

77. **a.** f is a polynomial of degree 3 and is therefore continuous on $[-1, 1]$.

b. $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -1 - 2 - 3 + 2 = -4$ and $f(1) = 1 - 2 + 3 + 2 = 4$. Because $f(-1)$ and $f(1)$ have opposite signs, we see that f has at least one zero in $(-1, 1)$.

78. f is continuous on $[14, 16]$, $f(14) = 2(14)^{5/3} - 5(14)^{4/3} \approx -6.06$, and $f(16) = 2(16)^{5/3} - 5(16)^{4/3} \approx 1.60$. Thus, f has at least one zero in $(14, 16)$.

79. $f(0) = 6$, $f(3) = 3$, and f is continuous on $[0, 3]$. Thus, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 4$. Solving $f(x) = x^2 - 4x + 6 = 4$, we find $x^2 - 4x + 2 = 0$. Using the quadratic formula, we find that $x = 2 \pm \sqrt{2}$. Because $2 + \sqrt{2}$ does not lie in $[0, 3]$, we see that $x = 2 - \sqrt{2} \approx 0.59$.

80. Because $f(-1) = 3$, $f(4) = 13$, and f is continuous on $[-1, 4]$, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 7$ because $3 < 7 < 13$. Solving $f(x) = x^2 - x + 1 = 7$, we find $x^2 - x - 6 = (x - 3)(x + 2) = 0$, that is, $x = -2$ or 3. Because -2 does not lie in $[-1, 4]$, the required solution is 3.

81. $x^5 + 2x - 7 = 0$

Step	Interval in which a root lies
1	$(1, 2)$
2	$(1, 1.5)$
3	$(1.25, 1.5)$
4	$(1.25, 1.375)$
5	$(1.3125, 1.375)$
6	$(1.3125, 1.34375)$
7	$(1.328125, 1.34375)$
8	$(1.3359375, 1.34375)$

We see that a root is approximately 1.34.

82. $x^3 - x + 1 = 0$

Step	Interval in which a root lies
1	$(-2, -1)$
2	$(-1.5, -1)$
3	$(-1.5, -1.25)$
4	$(-1.375, -1.25)$
5	$(-1.375, -1.3125)$
6	$(-1.34375, -1.3125)$
7	$(-1.328125, -1.3125)$
8	$(-1.328125, -1.3203125)$
9	$(-1.32421875, -1.3203125)$

We see that a root is approximately -1.32.

83. **a.** $h(0) = 4 + 64(0) - 16(0) = 4$ and $h(2) = 4 + 64(2) - 16(4) = 68$.

b. The function h is continuous on $[0, 2]$. Furthermore, the number 32 lies between 4 and 68. Therefore, the Intermediate Value Theorem guarantees that there is at least one value of t in $(0, 2]$ such that $h(t) = 32$, that is, Joan must see the ball at least once during the time the ball is in the air.

c. We solve $h(t) = 4 + 64t - 16t^2 = 32$, obtaining $16t^2 - 64t + 28 = 0$, $4t^2 - 16t + 7 = 0$, and $(2t - 1)(2t - 7) = 0$. Thus, $t = \frac{1}{2}$ or $t = \frac{7}{2}$. Joan sees the ball on its way up half a second after it was thrown and again 3 seconds later when it is on its way down. Note that the ball hits the ground when $t \approx 4.06$, but Joan sees it approximately half a second before it hits the ground.

84. **a.** $f(0) = 100\left(\dfrac{0 + 0 + 100}{0 + 0 + 100}\right) = 100$ and $f(10) = 100\left(\dfrac{100 + 100 + 100}{100 + 200 + 100}\right) = \dfrac{30{,}000}{400} = 75$.

b. Because 80 lies between 75 and 100 and f is continuous on $[75, 100]$, we conclude that there exists some t in $[0, 10]$ such that $f(t) = 80$.

c. We solve $f(t) = 80$; that is, $100\left[\dfrac{t^2 + 10t + 100}{t^2 + 20t + 100}\right] = 80$, obtaining $5\left(t^2 + 10t + 100\right) = 4\left(t^2 + 20t + 100\right)$, and $t^2 - 30t + 100 = 0$. Thus, $t = \dfrac{30 \pm \sqrt{900 - 400}}{2} = \dfrac{30 \pm \sqrt{500}}{2} \approx 3.82$ or 26.18. Because 26.18 lies outside the interval of interest, we reject it. Thus, the oxygen content is 80% approximately 3.82 seconds after the organic waste has been dumped into the pond.

85. False. Take $f(x) = \begin{cases} -1 & \text{if } x < 2 \\ 4 & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$ Then $f(2) = 4$, but $\lim_{x\to 2} f(x)$ does not exist.

86. False. Take $f(x) = \begin{cases} x+3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 3$, but $f(0) = 1$.

87. False. Consider $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ Then $\lim_{x\to 2^+} f(x) = f(2) = 3$, but $\lim_{x\to 2^-} f(x) = 0$.

88. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ 4 & \text{if } x \geq 3 \end{cases}$ Then $\lim_{x\to 3^-} f(x) = 2$ and $\lim_{x\to 3^+} f(x) = 4$, so $\lim_{x\to 3} f(x)$ does not exist.

89. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 5 \\ 3 & \text{if } x > 5 \end{cases}$ Then $f(5)$ is not defined, but $\lim_{x\to 5^-} f(x) = 2$.

90. False. Consider the function $f(x) = x^2 - 1$ on the interval $[-2, 2]$. Here $f(-2) = f(2) = 3$, but f has zeros at $x = -1$ and $x = 1$.

91. False. Let $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$, but $f(0) = 1$.

92. False. Let $f(x) = x$ and let $g(x) = \begin{cases} x & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ Then $\lim_{x\to 1} f(x) = 1 = L$, $g(1) = 2 = M$, $\lim_{x\to 1} g(x) = 1$, and $\lim_{x\to 1} f(x) g(x) = \left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right] = (1)(1) = 1 \neq 2 = LM$.

93. False. Let $f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Then f is continuous for all $x \neq 0$ and $f(0) = 0$, but $\lim_{x\to 0} f(x)$ does not exist.

94. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

95. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

96. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = x^2$.

97. False. Consider $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0 \end{cases}$

98. False. There is no contradiction, because the Intermediate Value Theorem says that there is at least one number c in $[a, b]$ such that $f(c) = M$ if M is a number between $f(a)$ and $f(b)$.

99. a. f is a rational function whose denominator is never zero, and so it is continuous for all values of x.

b. Because the numerator x^2 is nonnegative and the denominator is $x^2 + 1 \geq 1$ for all values of x, we see that $f(x)$ is nonnegative for all values of x.

c. $f(0) = \dfrac{0}{0+1} = \dfrac{0}{1} = 0$, and so f has a zero at $x = 0$. This does not contradict Theorem 5.

100. a. Both $g(x) = x$ and $h(x) = \sqrt{1-x^2}$ are continuous on $[-1, 1]$ and so $f(x) = x - \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

b. $f(-1) = -1$ and $f(1) = 1$, and so f has at least one zero in $(-1, 1)$.

c. Solving $f(x) = 0$, we have $x = \sqrt{1-x^2}$, $x^2 = 1 - x^2$, and $2x^2 = 1$, so $x = \frac{\pm\sqrt{2}}{2}$.

101. a. (i) Repeated use of Property 3 shows that $g(x) = x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$ is a continuous function, since $f(x) = x$ is continuous by Property 1.

(ii) Properties 1 and 5 combine to show that $c \cdot x^n$ is continuous using the results of part (a)(i).

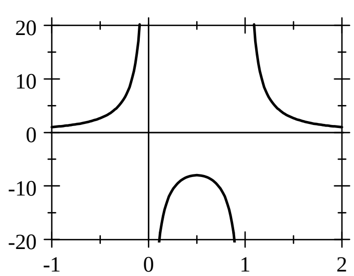
(iii) Each of the terms of $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$ is continuous and so Property 4 implies that p is continuous.

b. Property 6 now shows that $R(x) = \dfrac{p(x)}{q(x)}$ is continuous if $q(a) \neq 0$, since p and q are continuous at $x = a$.

102. Consider the function f defined by $f(x) = \begin{cases} -1 & \text{if } -1 \leq x < 0 \\ 1 & \text{if } 0 \leq x < 1 \end{cases}$ Then $f(-1) = -1$ and $f(1) = 1$, but if we take the number $\frac{1}{2}$, which lies between $y = -1$ and $y = 1$, there is no value of x such that $f(x) = \frac{1}{2}$.

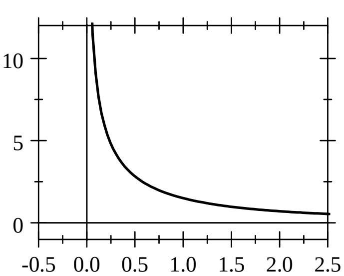
Using Technology page 136

1.



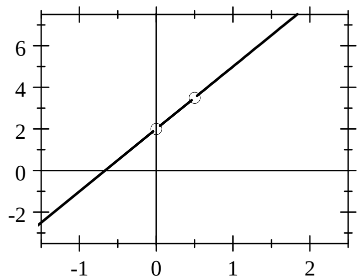
The function is discontinuous at $x = 0$ and $x = 1$.

2.



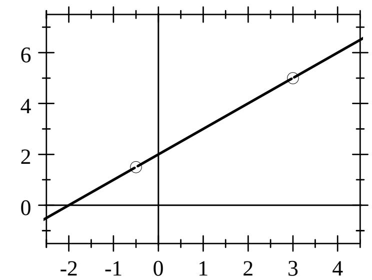
The function is undefined for $x \leq 0$.

3.



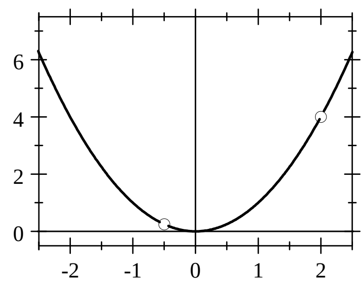
The function is discontinuous at $x = 0$ and $\frac{1}{2}$.

4.



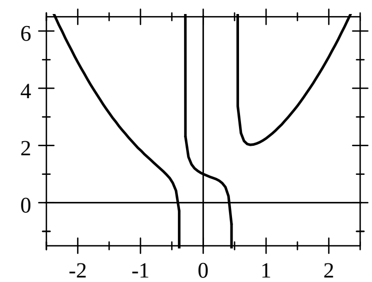
The function is discontinuous at $x = -\frac{1}{2}$ and 3.

5.



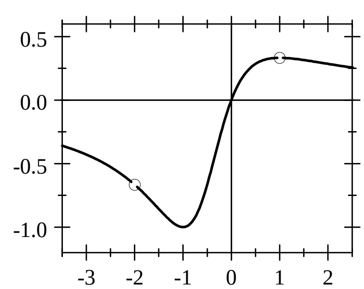
The function is discontinuous at $x = -\frac{1}{2}$ and 2.

6.



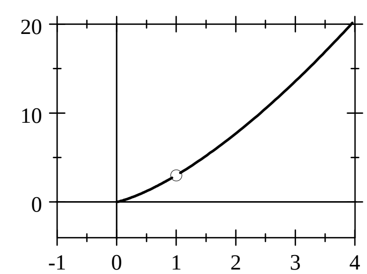
The function is discontinuous at $x = -\frac{1}{3}$ and $\frac{1}{2}$.

7.



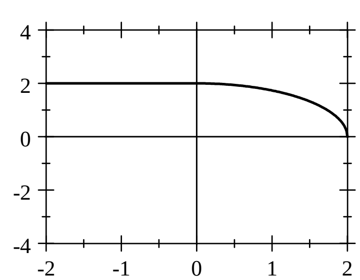
The function is discontinuous at $x = -2$ and 1.

8.

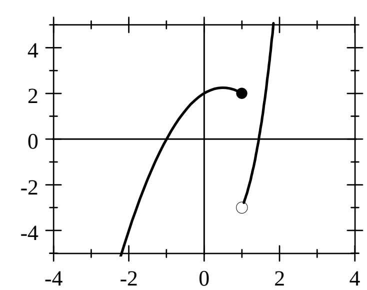


The function is discontinuous only at $x = -1$ and 1.

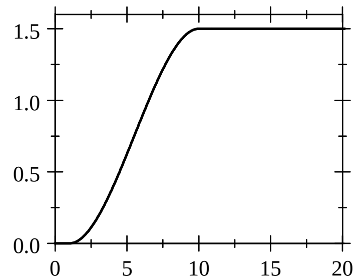
9.



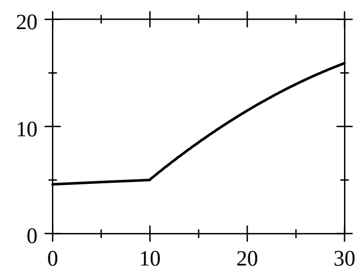
10.



11.



12. a.



b. 4.6%; 8.5%; 15.9%

2.6 The Derivative

Concept Questions

page 148

1. a. $m = \dfrac{f(2+h) - f(2)}{h}$

b. The slope of the tangent line is $\displaystyle\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$.

2. a. The average rate of change is $\dfrac{f(2+h) - f(2)}{h}$.

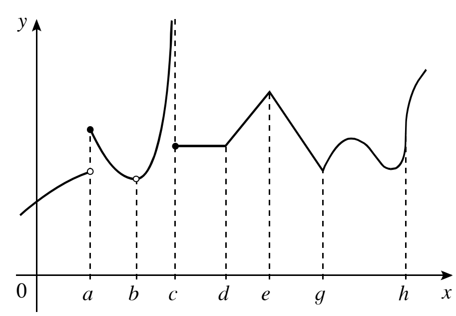
b. The instantaneous rate of change of f at 2 is $\displaystyle\lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$.

c. The expression for the slope of the secant line is the same as that for the average rate of change. The expression for the slope of the tangent line is the same as that for the instantaneous rate of change.

3. a. The expression $\dfrac{f(x+h) - f(x)}{h}$ gives (i) the slope of the secant line passing through the points $(x, f(x))$ and $(x+h, f(x+h))$, and (ii) the average rate of change of f over the interval $[x, x+h]$.

b. The expression $\displaystyle\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ gives (i) the slope of the tangent line to the graph of f at the point $(x, f(x))$, and (ii) the instantaneous rate of change of f at x.

4. Loosely speaking, a function f does not have a derivative at a if the graph of f does not have a tangent line at a, or if the tangent line does exist, but is vertical. In the figure, the function fails to be differentiable at $x = a$, b, and c because it is discontinuous at each of these numbers. The derivative of the function does not exist at $x = d$, e, and g because it has a kink at each point on the graph corresponding to these numbers. Finally, the function is not differentiable at $x = h$ because the tangent line is vertical at $(h, f(h))$.



5. a. $C(500)$ gives the total cost incurred in producing 500 units of the product.

b. $C'(500)$ gives the rate of change of the total cost function when the production level is 500 units.

6. a. $P(5)$ gives the population of the city (in thousands) when $t = 5$.

b. $P'(5)$ gives the rate of change of the city's population (in thousands/year) when $t = 5$.

Exercises page 149

1. The rate of change of the average infant's weight when $t = 3$ is $\frac{7.5}{5}$, or 1.5 lb/month. The rate of change of the average infant's weight when $t = 18$ is $\frac{3.5}{6}$, or approximately 0.58 lb/month. The average rate of change over the infant's first year of life is $\frac{22.5-7.5}{12}$, or 1.25 lb/month.

2. The rate at which the wood grown is changing at the beginning of the 10th year is $\frac{4}{12}$, or $\frac{1}{3}$ cubic meter per hectare per year. At the beginning of the 30th year, it is $\frac{10}{8}$, or 1.25 cubic meters per hectare per year.

3. The rate of change of the percentage of households watching television at 4 p.m. is $\frac{12.3}{4}$, or approximately 3.1 percent per hour. The rate at 11 p.m. is $\frac{-42.3}{2} = -21.15$, that is, it is dropping off at the rate of 21.15 percent per hour.

4. The rate of change of the crop yield when the density is 200 aphids per bean stem is $\frac{-500}{300}$, a decrease of approximately 1.7 kg/4000 m^2 per aphid per bean stem. The rate of change when the density is 800 aphids per bean stem is $\frac{-150}{300}$, a decrease of approximately 0.5 kg/4000 m^2 per aphid per bean stem.

5. a. Car A is travelling faster than Car B at t_1 because the slope of the tangent line to the graph of f is greater than the slope of the tangent line to the graph of g at t_1.

b. Their speed is the same because the slope of the tangent lines are the same at t_2.

c. Car B is travelling faster than Car A.

d. They have both covered the same distance and are once again side by side at t_3.

6. a. At t_1, the velocity of Car A is greater than that of Car B because $f(t_1) > g(t_1)$. However, Car B has greater acceleration because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to f is decreasing as you move across t_1.

b. Both cars have the same velocity at t_2, but the acceleration of Car B is greater than that of Car A because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to the graph of f is decreasing as you move across t_2.

7. a. P_2 is decreasing faster at t_1 because the slope of the tangent line to the graph of g at t_1 is greater than the slope of the tangent line to the graph of f at t_1.

b. P_1 is decreasing faster than P_2 at t_2.

c. Bactericide B is more effective in the short run, but bactericide A is more effective in the long run.

8. a. The revenue of the established department store is decreasing at the slowest rate at $t = 0$.

b. The revenue of the established department store is decreasing at the fastest rate at t_3.

c. The revenue of the discount store first overtakes that of the established store at t_1.

d. The revenue of the discount store is increasing at the fastest rate at t_2 because the slope of the tangent line to the graph of f is greatest at the point $(t_2, f(t_2))$.

9. $f(x) = 13.$

Step 1 $f(x+h) = 13.$

Step 2 $f(x+h) - f(x) = 13 - 13 = 0.$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0.$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 0 = 0.$

10. $f(x) = -6.$

Step 1 $f(x+h) = -6.$

Step 2 $f(x+h) - f(x) = -6 - (-6) = 0.$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0.$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 0 = 0.$

11. $f(x) = 2x + 7.$

Step 1 $f(x+h) = 2(x+h) + 7.$

Step 2 $f(x+h) - f(x) = 2(x+h) + 7 - (2x+7) = 2h.$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2.$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 2 = 2.$

12. $f(x) = 8 - 4x.$

Step 1 $f(x+h) = 8 - 4(x+h) = 8 - 4x - 4h.$

Step 2 $f(x+h) - f(x) = (8 - 4x - 4h) - (8 - 4x) = -4h.$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = -\dfrac{4h}{h} = -4.$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (-4) = -4.$

13. $f(x) = 3x^2.$

Step 1 $f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2.$

Step 2 $f(x+h) - f(x) = \left(3x^2 + 6xh + 3h^2\right) - 3x^2 = 6xh + 3h^2 = h(6x + 3h).$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h)}{h} = 6x + 3h.$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (6x + 3h) = 6x.$

14. $f(x) = -\frac{1}{2}x^2.$

Step 1 $f(x+h) = -\frac{1}{2}(x+h)^2.$

Step 2 $f(x+h) - f(x) = -\frac{1}{2}x^2 - xh - \frac{1}{2}h^2 + \frac{1}{2}x^2 = -h\left(x + \frac{1}{2}h\right).$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-h\left(x + \frac{1}{2}h\right)}{h} = -\left(x + \frac{1}{2}h\right).$

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} -\left(x + \frac{1}{2}h\right) = -x.$

15. $f(x) = -x^2 + 3x$.

Step 1 $f(x+h) = -(x+h)^2 + 3(x+h) = -x^2 - 2xh - h^2 + 3x + 3h$.

Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2 + 3x + 3h\right) - \left(-x^2 + 3x\right) = -2xh - h^2 + 3h$

$$= h(-2x - h + 3).$$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(-2x - h + 3)}{h} = -2x - h + 3$.

Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (-2x - h + 3) = -2x + 3$.

16. $f(x) = 2x^2 + 5x$.

Step 1 $f(x+h) = 2(x+h)^2 + 5(x+h) = 2x^2 + 4xh + 2h^2 + 5x + 5h$.

Step 2 $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 + 5x + 5h - 2x^2 - 5x = h(4x + 2h + 5)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x + 2h + 5)}{h} = 4x + 2h + 5$.

Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (4x + 2h + 5) = 4x + 5$.

17. $f(x) = 2x + 7$.

Step 1 $f(x+h) = 2(x+h) + 7 = 2x + 2h + 7$.

Step 2 $f(x+h) - f(x) = 2x + 2h + 7 - 2x - 7 = 2h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2$.

Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} 2 = 2$.

Therefore, $f'(x) = 2$. In particular, the slope at $x = 2$ is 2. Therefore, an equation of the tangent line is $y - 11 = 2(x - 2)$ or $y = 2x + 7$.

18. $f(x) = -3x + 4$. First, we find $f'(x) = -3$ using the four-step process. Thus, the slope of the tangent line is $f'(-1) = -3$ and an equation is $y - 7 = -3(x + 1)$ or $y = -3x + 4$.

19. $f(x) = 3x^2$.We first compute $f'(x) = 6x$ (see Exercise 13). Because the slope of the tangent line is $f'(1) = 6$, we use the point-slope form of the equation of a line and find that an equation is $y - 3 = 6(x - 1)$, or $y = 6x - 3$.

20. $f(x) = 3x - x^2$.

Step 1 $f(x+h) = 3(x+h) - (x+h)^2 = 3x + 3h - x^2 - 2xh - h^2$.

Step 2 $f(x+h) - f(x) = 3x + 3h - x^2 - 2xh - h^2 - 3x + x^2 = 3h - 2xh - h^2 = h(3 - 2x - h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(3 - 2x - h)}{h} = 3 - 2x - h$.

Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (3 - 2x - h) = 3 - 2x$.

Therefore, $f'(x) = 3 - 2x$. In particular, $f'(-2) = 3 - 2(-2) = 7$. Using the point-slope form of an equation of a line, we find $y + 10 = 7(x + 2)$, or $y = 7x + 4$.

21. $f(x) = -1/x$. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = -\dfrac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\dfrac{1}{x+h} + \dfrac{1}{x} = \dfrac{-x + (x+h)}{x(x+h)} = \dfrac{h}{x(x+h)}$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\frac{h}{x(x+h)}}{h} = \dfrac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{1}{x(x+h)} = \dfrac{1}{x^2}$.

The slope of the tangent line is $f'(3) = \frac{1}{9}$. Therefore, an equation is $y - \left(-\frac{1}{3}\right) = \frac{1}{9}(x-3)$, or $y = \frac{1}{9}x - \frac{2}{3}$.

22. $f(x) = \dfrac{3}{2x}$. First use the four-step process to find $f'(x) = -\dfrac{3}{2x^2}$. (This is similar to Exercise 21.) The slope of the tangent line is $f'(1) = -\frac{3}{2}$. Therefore, an equation is $y - \frac{3}{2} = -\frac{3}{2}(x-1)$ or $y = -\frac{3}{2}x + 3$.

23. a. $f(x) = 2x^2 + 1$.

Step 1 $f(x+h) = 2(x+h)^2 + 1 = 2x^2 + 4xh + 2h^2 + 1$.

Step 2 $f(x+h) - f(x) = \left(2x^2 + 4xh + 2h^2 + 1\right) - \left(2x^2 + 1\right)$

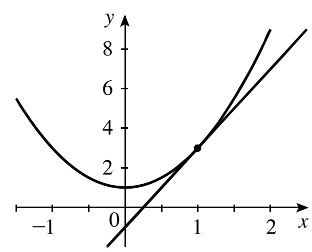
$= 4xh + 2h^2 = h(4x + 2h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x+2h)}{h} = 4x + 2h$.

Step 4 $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0}(4x + 2h) = 4x$.

b. The slope of the tangent line is $f'(1) = 4(1) = 4$. Therefore, an equation is $y - 3 = 4(x-1)$ or $y = 4x - 1$.

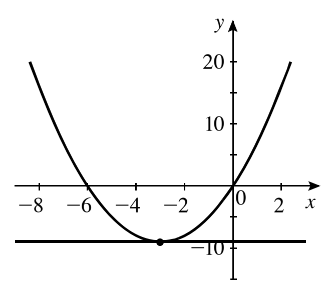
c.



24. a. $f(x) = x^2 + 6x$. Using the four-step process, we find that $f'(x) = 2x + 6$.

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x) = 0$. Then $2x + 6 = 0$, or $x = -3$. Therefore, $y = f(-3) = (-3)^2 + 6(-3) = -9$. The required point is $(-3, -9)$.

c.



25. a. $f(x) = x^2 - 2x + 1$. We use the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 2(x+h) + 1 = x^2 + 2xh + h^2 - 2x - 2h + 1$.

Step 2 $f(x+h) - f(x) = \left(x^2 + 2xh + h^2 - 2x - 2h + 1\right) - \left(x^2 - 2x + 1\right) = 2xh + h^2 - 2h$

$= h(2x + h - 2)$.

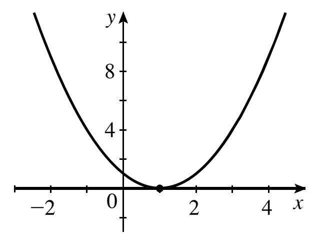
Step 3 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(2x+h-2)}{h} = 2x+h-2.$

Step 4 $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x+h-2)$

$= 2x-2.$

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x)=0$. Then $2x-2=0$, or $x=1$. Because $f(1)=1-2+1=0$, we see that the required point is $(1,0)$.

c.



d. It is changing at the rate of 0 units per unit change in x.

26. a. $f(x) = \dfrac{1}{x-1}.$

Step 1 $f(x+h) = \dfrac{1}{(x+h)-1} = \dfrac{1}{x+h-1}.$

Step 2 $f(x+h)-f(x) = \dfrac{1}{x+h-1} - \dfrac{1}{x-1} = \dfrac{x-1-(x+h-1)}{(x+h-1)(x-1)} = -\dfrac{h}{(x+h-1)(x-1)}.$

Step 3 $\dfrac{f(x+h)-f(x)}{h} = -\dfrac{1}{(x+h-1)(x-1)}.$

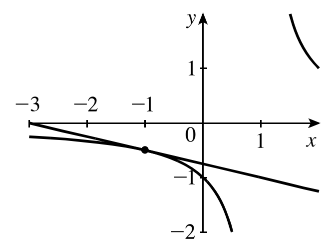
Step 4 $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} -\dfrac{1}{(x+h-1)(x-1)} = -\dfrac{1}{(x-1)^2}.$

b. The slope is $f'(-1) = -\frac{1}{4}$, so, an equation is

$y-\left(-\frac{1}{2}\right) = -\frac{1}{4}(x+1)$ or $y = -\frac{1}{4}x - \frac{3}{4}.$

c.



27. a. $f(x) = x^2+x$, so $\dfrac{f(3)-f(2)}{3-2} = \dfrac{(3^2+3)-(2^2+2)}{1} = 6,$

$\dfrac{f(2.5)-f(2)}{2.5-2} = \dfrac{(2.5^2+2.5)-(2^2+2)}{0.5} = 5.5$, and $\dfrac{f(2.1)-f(2)}{2.1-2} = \dfrac{(2.1^2+2.1)-(2^2+2)}{0.1} = 5.1.$

b. We first compute $f'(x)$ using the four-step process.

Step 1 $f(x+h) = (x+h)^2+(x+h) = x^2+2xh+h^2+x+h.$

Step 2 $f(x+h)-f(x) = \left(x^2+2xh+h^2+x+h\right)-\left(x^2+x\right) = 2xh+h^2+h = h(2x+h+1).$

Step 3 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(2x+h+1)}{h} = 2x+h+1.$

Step 4 $f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x+h+1) = 2x+1.$

The instantaneous rate of change of y at $x=2$ is $f'(2) = 2(2)+1$, or 5 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f at $x=2$ approach 5 as the interval $[2, 2+h]$ gets smaller and smaller ($h = 1$, 0.5, and 0.1). This number is the instantaneous rate of change of f at $x=2$ as computed in part (b).

28. a. $f(x) = x^2-4x$, so $\dfrac{f(4)-f(3)}{4-3} = \dfrac{(16-16)-(9-12)}{1} = 3,$

$\dfrac{f(3.5)-f(3)}{3.5-3} = \dfrac{(12.25-14)-(9-12)}{0.5} = 2.5$, and $\dfrac{f(3.1)-f(3)}{3.1-3} = \dfrac{(9.61-12.4)-(9-12)}{0.1} = 2.1.$

b. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 4(x+h) = x^2 + 2xh + h^2 - 4x - 4h$.

Step 2 $f(x+h) - f(x) = \left(x^2 + 2xh + h^2 - 4x - 4h\right) - \left(x^2 - 4x\right) = 2xh + h^2 - 4h = h(2x + h - 4)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x+h-4)}{h} = 2x + h - 4$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (2x + h - 4) = 2x - 4$.

The instantaneous rate of change of y at $x = 3$ is $f'(3) = 6 - 4 = 2$, or 2 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f over smaller and smaller intervals containing $x = 3$ approach the instantaneous rate of change of 2 units per unit change in x obtained in part (b).

29. a. $f(t) = 2t^2 + 48t$. The average velocity of the car over the time interval [20, 21] is $\dfrac{f(21) - f(20)}{21 - 20} = \dfrac{\left[2(21)^2 + 48(21)\right] - \left[2(20)^2 + 48(20)\right]}{1} = 130\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over [20, 20.1] is $\dfrac{f(20.1) - f(20)}{20.1 - 20} = \dfrac{\left[2(20.1)^2 + 48(20.1)\right] - \left[2(20)^2 + 48(20)\right]}{0.1} = 128.2\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over [20, 20.01] is $\dfrac{f(20.01) - f(20)}{20.01 - 20} = \dfrac{\left[2(20.01)^2 + 48(20.01)\right] - \left[2(20)^2 + 48(20)\right]}{0.01} = 128.02\ \dfrac{\text{ft}}{\text{s}}$.

b. We first compute $f'(t)$ using the four-step process.

Step 1 $f(t+h) = 2(t+h)^2 + 48(t+h) = 2t^2 + 4th + 2h^2 + 48t + 48h$.

Step 2 $f(t+h) - f(t) = \left(2t^2 + 4th + 2h^2 + 48t + 48h\right) - \left(2t^2 + 48t\right) = 4th + 2h^2 + 48h$

$= h(4t + 2h + 48)$.

Step 3 $\dfrac{f(t+h) - f(t)}{h} = \dfrac{h(4t + 2h + 48)}{h} = 4t + 2h + 48$.

Step 4 $f'(t) = \lim\limits_{t\to 0} \dfrac{f(t+h) - f(t)}{h} = \lim\limits_{t\to 0} (4t + 2h + 48) = 4t + 48$.

The instantaneous velocity of the car at $t = 20$ is $f'(20) = 4(20) + 48$, or 128 ft/s.

c. Our results show that the average velocities do approach the instantaneous velocity as the intervals over which they are computed decreases.

30. a. The average velocity of the ball over the time interval [2, 3] is $\dfrac{s(3) - s(2)}{3 - 2} = \dfrac{\left[128(3) - 16(3)^2\right] - \left[128(2) - 16(2)^2\right]}{1} = 48$, or 48 ft/s. Over the time interval [2, 2.5], it is $\dfrac{s(2.5) - s(2)}{2.5 - 2} = \dfrac{\left[128(2.5) - 16(2.5)^2\right] - \left[128(2) - 16(2)^2\right]}{0.5} = 56$, or 56 ft/s. Over the time interval [2, 2.1], it is $\dfrac{s(2.1) - s(2)}{2.1 - 2} = \dfrac{\left[128(2.1) - 16(2.1)^2\right] - \left[128(2) - 16(2)^2\right]}{0.1} = 62.4$, or 62.4 ft/s.

b. Using the four-step process, we find that the instantaneous velocity of the ball at any time t is given by $v(t) = 128 - 32t$. In particular, the velocity of the ball at $t = 2$ is $v(2) = 128 - 32(2) = 64$, or 64 ft/s.

c. At $t = 5$, $v(5) = 128 - 32(5) = -32$, so the speed of the ball at $t = 5$ is 32 ft/s and it is falling.

d. The ball hits the ground when $s(t) = 0$, that is, when $128t - 16t^2 = 0$, whence $t(128 - 16t) = 0$, so $t = 0$ or $t = 8$. Thus, it will hit the ground when $t = 8$.

31. a. We solve the equation $16t^2 = 400$ and find $t = 5$, which is the time it takes the screwdriver to reach the ground.

b. The average velocity over the time interval [0, 5] is $\dfrac{f(5)-f(0)}{5-0}=\dfrac{16(25)-0}{5}=80$, or 80 ft/s.

c. The velocity of the screwdriver at time t is

$$v(t)=\lim_{h\to0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to0}\frac{16(t+h)^2-16t^2}{h}=\lim_{h\to0}\frac{16t^2+32th+16h^2-16t^2}{h}$$
$$=\lim_{h\to0}\frac{(32t+16h)h}{h}=32t.$$

In particular, the velocity of the screwdriver when it hits the ground (at $t=5$) is $v(5)=32(5)=160$, or 160 ft/s.

32. **a.** We write $f(t)=\frac{1}{2}t^2+\frac{1}{2}t$. The height after 40 seconds is $f(40)=\frac{1}{2}(40)^2+\frac{1}{2}(40)=820$.

b. Its average velocity over the time interval [0, 40] is $\dfrac{f(40)-f(0)}{40-0}=\dfrac{820-0}{40}=20.5$, or 20.5 ft/s.

c. Its velocity at time t is

$$v(t)=\lim_{h\to0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to0}\frac{\frac{1}{2}(t+h)^2+\frac{1}{2}(t+h)-\left(\frac{1}{2}t^2+\frac{1}{2}t\right)}{h}$$
$$=\lim_{h\to0}\frac{\frac{1}{2}t^2+th+\frac{1}{2}h^2+\frac{1}{2}t+\frac{1}{2}h-\frac{1}{2}t^2-\frac{1}{2}t}{h}=\lim_{h\to0}\frac{th+\frac{1}{2}h^2+\frac{1}{2}h}{h}=\lim_{h\to0}\left(t+\frac{1}{2}h+\frac{1}{2}\right)=t+\frac{1}{2}.$$

In particular, the velocity at the end of 40 seconds is $v(40)=40+\frac{1}{2}$, or $40\frac{1}{2}$ ft/s.

33. **a.** We write $V=f(p)=\dfrac{1}{p}$. The average rate of change of V is $\dfrac{f(3)-f(2)}{3-2}=\dfrac{\frac{1}{3}-\frac{1}{2}}{1}=-\dfrac{1}{6}$, a decrease of $\frac{1}{6}$ liter/atmosphere.

b. $V'(t)=\lim_{h\to0}\dfrac{\dfrac{f(p+h)-f(p)}{h}}{h}=\lim_{h\to0}\dfrac{\dfrac{1}{p+h}-\dfrac{1}{p}}{h}=\lim_{h\to0}\dfrac{p-(p+h)}{hp(p+h)}=\lim_{h\to0}-\dfrac{1}{p(p+h)}=-\dfrac{1}{p^2}$. In particular, the rate of change of V when $p=2$ is $V'(2)=-\dfrac{1}{2^2}$, a decrease of $\frac{1}{4}$ liter/atmosphere.

34. $C(x)=-10x^2+300x+130$.

a. Using the four-step process, we find

$$C'(x)=\lim_{h\to0}\frac{C(x+h)-C(x)}{h}=\lim_{h\to0}\frac{h(-20x-10h+300)}{h}=-20x+300.$$

b. The rate of change is $C'(10)=-20(10)+300=100$, or \$100/surfboard.

35. **a.** $P(x)=-\frac{1}{3}x^2+7x+30$. Using the four-step process, we find that

$$P'(x)=\lim_{h\to0}\frac{P(x+h)-P(x)}{h}=\lim_{h\to0}\frac{-\frac{1}{3}\left(x^2+2xh+h^2\right)+7x+7h+30-\left(-\frac{1}{3}x^2+7x+30\right)}{h}$$
$$=\lim_{h\to0}\frac{-\frac{2}{3}xh-\frac{1}{3}h^2+7h}{h}=\lim_{h\to0}\left(-\frac{2}{3}x-\frac{1}{3}h+7\right)=-\frac{2}{3}x+7.$$

b. $P'(10)=-\frac{2}{3}(10)+7\approx0.333$, or approximately \$333 per \$1000 spent on advertising.
$P'(30)=-\frac{2}{3}(30)+7=-13$, a decrease of \$13,000 per \$1000 spent on advertising.

36. a. $f(x) = -0.1x^2 - x + 40$, so

$$\frac{f(5.05) - f(5)}{5.05 - 5} = \frac{\left[-0.1(5.05)^2 - 5.05 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.05} = -2.005,$$ or approximately $-\$2.01$ per 1000 tents. $$\frac{f(5.01) - f(5)}{5.01 - 5} = \frac{\left[-0.1(5.01)^2 - 5.01 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.01} = -2.001,$$ or approximately $-\$2.00$ per 1000 tents.

b. We compute $f'(x)$ using the four-step process, obtaining $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{h(-0.2x - 0.1h - 1)}{h} = \lim_{h \to 0} (-0.2x - 0.1h - 1) = -0.2x - 1$. The rate of change of the unit price if $x = 5000$ is $f'(5) = -0.2(5) - 1 = -2$, a decrease of \$2 per 1000 tents.

37. $N(t) = t^2 + 2t + 50$. We first compute $N'(t)$ using the four-step process.

Step 1 $N(t+h) = (t+h)^2 + 2(t+h) + 50 = t^2 + 2th + h^2 + 2t + 2h + 50$.

Step 2 $N(t+h) - N(t) = \left(t^2 + 2th + h^2 + 2t + 2h + 50\right) - \left(t^2 + 2t + 50\right) = 2th + h^2 + 2h = h(2t + h + 2)$.

Step 3 $\dfrac{N(t+h) - N(t)}{h} = 2t + h + 2$.

Step 4 $N'(t) = \lim_{h \to 0} (2t + h + 2) = 2t + 2$.

The rate of change of the country's GNP two years from now is $N'(2) = 2(2) + 2 = 6$, or \$6 billion/yr. The rate of change four years from now is $N'(4) = 2(4) + 2 = 10$, or \$10 billion/yr.

38. $f(t) = 3t^2 + 2t + 1$. Using the four-step process, we obtain $f'(t) = \lim_{t \to 0} \frac{f(t+h) - f(t)}{h} = \lim_{t \to 0} \frac{h(6t + 3h + 2)}{h} = \lim_{t \to 0} (6t + 3h + 2) = 6t + 2$. Next, $f'(10) = 6(10) + 2 = 62$, and we conclude that the rate of bacterial growth at $t = 10$ is 62 bacteria per minute.

39. a. $f'(h)$ gives the instantaneous rate of change of the temperature with respect to height at a given height h, in $^\circ$F per foot.

b. Because the temperature decreases as the altitude increases, the sign of $f'(h)$ is negative.

c. Because $f'(1000) = -0.05$, the change in the air temperature as the altitude changes from 1000 ft to 1001 ft is approximately -0.05° F.

40. a. $\dfrac{f(b) - f(a)}{b - a}$ measures the average rate of change in revenue as the advertising expenditure changes from a thousand dollars to b thousand dollars. The units of measurement are thousands of dollars per thousands of dollars.

b. $f'(x)$ gives the instantaneous rate of change in the revenue when x thousand dollars is spent on advertising. It is measured in thousands of dollars per thousands of dollars.

c. Because $f'(20) \cdot (21 - 20) = 3 \cdot 1 = 3$, the approximate change in revenue is \$3000.

41. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the seal population over the time interval $[a, a+h]$. $\displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the seal population at $x = a$.

42. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the prime interest rate over the time interval $[a, a+h]$. $\displaystyle\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the prime interest rate at $x = a$.

43. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the country's industrial production over the time interval $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the country's industrial production at $x=a$.

44. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the cost incurred in producing the commodity over the production level $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the cost of producing the commodity at $x=a$.

45. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the atmospheric pressure over the altitudes $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the atmospheric pressure with respect to altitude at $x=a$.

46. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the fuel economy of a car over the speeds $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the fuel economy at $x=a$.

47. **a.** f has a limit at $x=a$.
b. f is not continuous at $x=a$ because $f(a)$ is not defined.
c. f is not differentiable at $x=a$ because it is not continuous there.

48. **a.** f has a limit at $x=a$.
b. f is continuous at $x=a$.
c. f is differentiable at $x=a$.

49. **a.** f has a limit at $x=a$.
b. f is continuous at $x=a$.
c. f is not differentiable at $x=a$ because f has a kink at the point $x=a$.

50. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.
b. f is not continuous at $x=a$ because the limit does not exist there.
c. f is not differentiable at $x=a$ because it is not continuous there.

51. **a.** f does not have a limit at $x=a$ because it is unbounded in the neighborhood of a.
b. f is not continuous at $x=a$.
c. f is not differentiable at $x=a$ because it is not continuous there.

52. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.
b. f is not continuous at $x=a$ because the limit does not exist there.
c. f is not differentiable at $x=a$ because it is not continuous there.

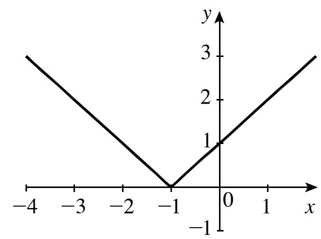
53. $s(t)=-0.1t^3+2t^2+24t$. Our computations yield the following results: 32.1, 30.939, 30.814, 30.8014, 30.8001, and 30.8000. The motorcycle's instantaneous velocity at $t=2$ is approximately 30.8 ft/s.

54. $C(x) = 0.000002x^3 + 5x + 400$. Our computations yield the following results: 5.060602, 5.06006002, 5.060006, 5.0600006, and 5.0600001. The rate of change of the total cost function when the level of production is 100 cases a day is approximately \$5.06.

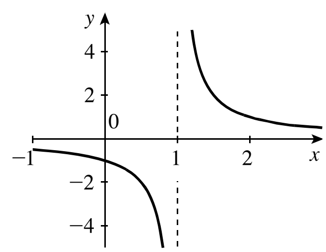
55. False. Let $f(x) = |x|$. Then f is continuous at $x = 0$, but is not differentiable there.

56. True. If g is differentiable at $x = a$, then it is continuous there. Therefore, the product fg is continuous, and so $\lim_{x\to a} f(x)g(x) = \left[\lim_{x\to a} f(x)\right]\left[\lim_{x\to a} g(x)\right] = f(a)g(a)$.

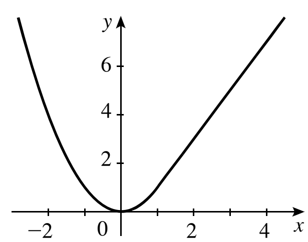
57. Observe that the graph of f has a kink at $x = -1$. We have $\dfrac{f(-1+h) - f(-1)}{h} = 1$ if $h > 0$, and -1 if $h < 0$, so that $\lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h}$ does not exist.



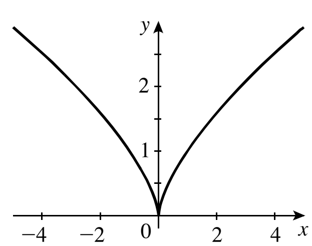
58. f does not have a derivative at $x = 1$ because it is not continuous there.



59. For continuity, we require that $f(1) = 1 = \lim_{x\to 1^+}(ax + b) = a + b$, or $a + b = 1$. Next, using the four-step process, we have $f'(x) = \begin{cases} 2x & \text{if } x < 1 \\ a & \text{if } x > 1 \end{cases}$ In order that the derivative exist at $x = 1$, we require that $\lim_{x\to 1^-} 2x = \lim_{x\to 1^+} a$, or $2 = a$. Therefore, $b = -1$ and so $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$



60. f is continuous at $x = 0$, but $f'(0)$ does not exist because the graph of f has a vertical tangent line at $x = 0$.



61. We have $f(x) = x$ if $x > 0$ and $f(x) = -x$ if $x < 0$. Therefore, when $x > 0$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$, and when $x < 0$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-x-h-(-x)}{h} = \lim_{h \to 0} \frac{-h}{h} = -1$. Because the right-hand limit does not equal the left-hand limit, we conclude that $\lim_{h \to 0} f(x)$ does not exist.

62. From $f(x) - f(a) = \left[\frac{f(x) - f(a)}{x-a}\right](x-a)$, we see that $\lim_{x \to a}[f(x) - f(a)] = \lim_{x \to a}\left[\frac{f(x) - f(a)}{x-a}\right]\lim_{x \to a}(x-a) = f'(a) \cdot 0 = 0$, and so $\lim_{x \to a} f(x) = f(a)$. This shows that f is continuous at $x = a$.

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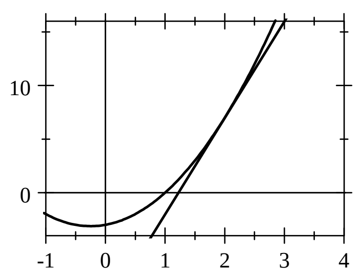
1. a. 9

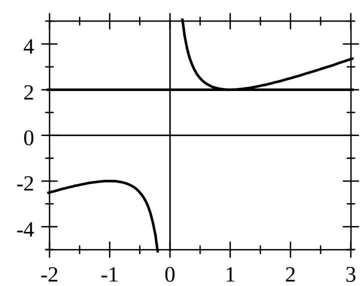
b.



c. $y = 9x - 11$

2. a. 0

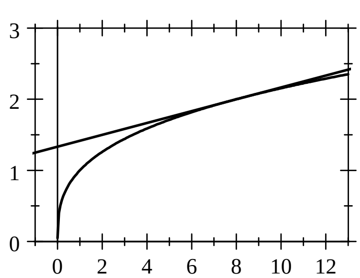
b.



c. $y = 2$

3. a. 0.08

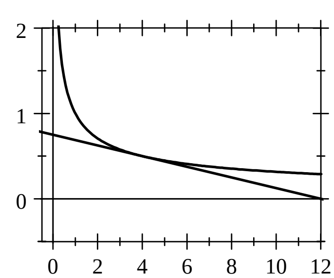
b.



c. $y = \frac{1}{12}x + \frac{4}{3}$

4. a. -0.06

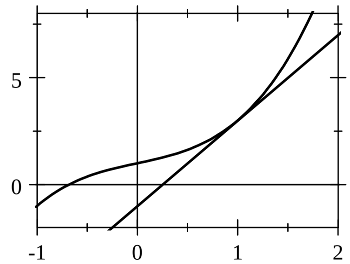
b.



c. $y = -\frac{1}{16}x + \frac{3}{4}$

5. a. 4

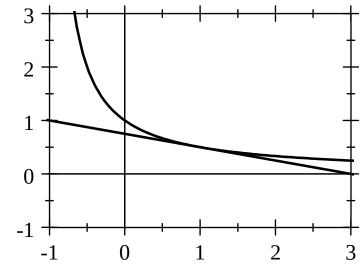
b.



c. $y = 4x - 1$

6. a. -0.25

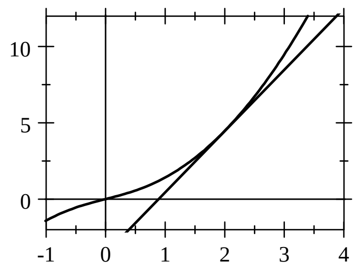
b.



c. $y = -\frac{1}{4}x + \frac{3}{4}$

7. a. 4.02

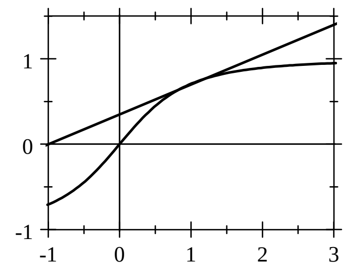
b.



c. $y = 4.02x - 3.57$

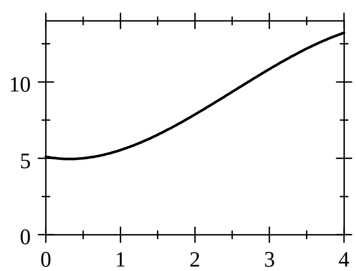
8. a. 0.35

b.



c. $y = 0.35x + 0.36$

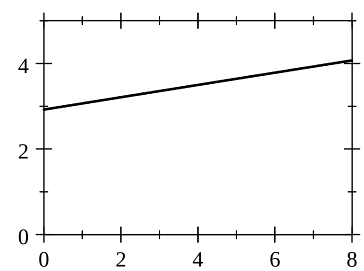
9. a.



b. $f'(3) = 2.8826$ (million per decade)

10. a. $S(t) = -0.000114719t^2 + 0.144618t + 2.92202$

b.



c. \$3.786 billion

d. \$143 million/yr

CHAPTER 2 Concept Review Questions page 156

1. domain, range, B

2. domain, $f(x)$, vertical, point

3. $f(x) \pm g(x)$, $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$, $A \cap B$, $A \cap B$, 0

4. $g(f(x))$, f, $f(x)$, g

5. a. $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$ and n is a positive integer

b. linear, quadratic, cubic **c.** quotient, polynomials **d.** x^r, where r is a real number

6. $f(x)$, L, a

7. a. L^r **b.** $L \pm M$ **c.** LM **d.** $\dfrac{L}{M}$, $M \neq 0$

8. a. L, x **b.** M, negative, absolute

9. a. right **b.** left **c.** L, L

10. a. continuous **b.** discontinuous **c.** every

11. a. a, a, $g(a)$ **b.** everywhere **c.** Q

12. a. $[a, b]$, $f(c) = M$ **b.** $f(x) = 0$, (a, b)

13. a. $f'(a)$ **b.** $y - f(a) = m(x - a)$

14. a. $\dfrac{f(a+h) - f(a)}{h}$ **b.** $\displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

CHAPTER 2 Review Exercises page 157

1. a. $9 - x \geq 0$ gives $x \leq 9$, and the domain is $(-\infty, 9]$.

b. $2x^2 - x - 3 = (2x - 3)(x + 1)$, and $x = \frac{3}{2}$ or -1. Because the denominator of the given expression is zero at these points, we see that the domain of f cannot include these points and so the domain of f is $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

2. a. We must have $2 - x \geq 0$ and $x + 3 \neq 0$. This implies $x \leq 2$ and $x \neq -3$, so the domain of f is $(-\infty, -3) \cup (-3, 2]$.

b. The domain is $(-\infty, \infty)$.

3. a. $f(-2) = 3(-2)^2 + 5(-2) - 2 = 0$.

b. $f(a+2) = 3(a+2)^2 + 5(a+2) - 2 = 3a^2 + 12a + 12 + 5a + 10 - 2 = 3a^2 + 17a + 20$.

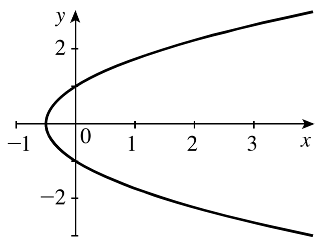
c. $f(2a) = 3(2a)^2 + 5(2a) - 2 = 12a^2 + 10a - 2$.

d. $f(a+h) = 3(a+h)^2 + 5(a+h) - 2 = 3a^2 + 6ah + 3h^2 + 5a + 5h - 2$.

4. a.
$$\begin{aligned} f(x-1) + f(x+1) &= \left[2(x-1)^2 - (x-1) + 1\right] + \left[2(x+1)^2 - (x+1) + 1\right] \\ &= \left(2x^2 - 4x + 2 - x + 1 + 1\right) + \left(2x^2 + 4x + 2 - x - 1 + 1\right) = 4x^2 - 2x + 6. \end{aligned}$$

b. $f(x + 2h) = 2(x + 2h)^2 - (x + 2h) + 1 = 2x^2 + 8xh + 8h^2 - x - 2h + 1$.

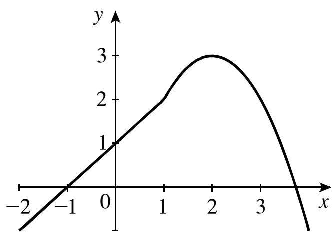
5. a.



b. For each value of $x > 0$, there are two values of y. We conclude that y is not a function of x. (We could also note that the function fails the vertical line test.)

c. Yes. For each value of y, there is only one value of x.

6.



7. a. $f(x)\,g(x) = \dfrac{2x+3}{x}$.

b. $\dfrac{f(x)}{g(x)} = \dfrac{1}{x(2x+3)}$.

c. $f(g(x)) = \dfrac{1}{2x+3}$.

d. $g(f(x)) = 2\left(\dfrac{1}{x}\right) + 3 = \dfrac{2}{x} + 3$.

8. a. $(f \circ g)(x) = f(g(x)) = 2g(x) - 1 = 2\left(x^2+4\right) - 1 = 2x^2 + 7$ and
$(g \circ f)(x) = g(f(x)) = \left[f(x)\right]^2 + 4 = (2x-1)^2 + 4 = 4x^2 - 4x + 5$.

b. $(f \circ g)(x) = f(g(x)) = 1 - g(x) = 1 - \dfrac{1}{3x+4} = \dfrac{3x+3}{3x+4} = \dfrac{3(x+1)}{3x+4}$ and
$(g \circ f)(x) = g(f(x)) = \dfrac{1}{3f(x)+4} = \dfrac{1}{3(1-x)+4} = \dfrac{1}{7-3x}$.

c. $(f \circ g)(x) = f(g(x)) = g(x) - 3 = \dfrac{1}{\sqrt{x+1}} - 3$ and
$(g \circ f)(x) = g(f(x)) = \dfrac{1}{\sqrt{f(x)+1}} = \dfrac{1}{\sqrt{(x-3)+1}} = \dfrac{1}{\sqrt{x-2}}$.

9. a. Take $f(x) = 2x^2 + x + 1$ and $g(x) = \dfrac{1}{x^3}$.

b. Take $f(x) = x^2 + x + 4$ and $g(x) = \sqrt{x}$.

10. We have $c(4)^2 + 3(4) - 4 = 2$, so $16c + 12 - 4 = 2$, or $c = -\dfrac{6}{16} = -\dfrac{3}{8}$.

11. $\lim\limits_{x\to 0}(5x-3) = 5(0) - 3 = -3$.

12. $\lim\limits_{x\to 1}\left(x^2+1\right) = (1)^2 + 1 = 1 + 1 = 2$.

13. $\lim\limits_{x\to -1}\left(3x^2+4\right)(2x-1) = \left[3(-1)^2+4\right]\left[2(-1)-1\right] = -21$.

14. $\lim\limits_{x\to 3}\dfrac{x-3}{x+4} = \dfrac{3-3}{3+4} = 0$

15. $\lim\limits_{x\to 2}\dfrac{x+3}{x^2-9} = \dfrac{2+3}{4-9} = -1$.

16. $\lim\limits_{x\to -2}\dfrac{x^2-2x-3}{x^2+5x+6}$ does not exist. (The denominator is 0 at $x = -2$.)

17. $\lim_{x\to 3}\sqrt{2x^3-5}=\sqrt{2(27)-5}=7.$

18. $\lim_{x\to 3}\dfrac{4x-3}{\sqrt{x+1}}=\dfrac{12-3}{\sqrt{4}}=\dfrac{9}{2}.$

19. $\lim_{x\to 1^+}\dfrac{x-1}{x(x-1)}=\lim_{x\to 1^+}\dfrac{1}{x}=1.$

20. $\lim_{x\to 1^-}\dfrac{\sqrt{x}-1}{x-1}=\lim_{x\to 1^-}\dfrac{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1^-}\dfrac{x-1}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1^-}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{2}.$

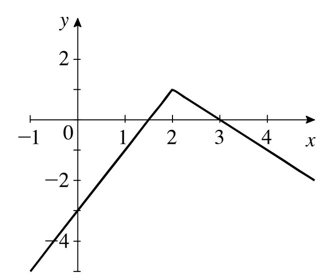
21. $\lim_{x\to\infty}\dfrac{x^2}{x^2-1}=\lim_{x\to\infty}\dfrac{1}{1-\dfrac{1}{x^2}}=1.$

22. $\lim_{x\to-\infty}\dfrac{x+1}{x}=\lim_{x\to-\infty}\left(1+\dfrac{1}{x}\right)=1.$

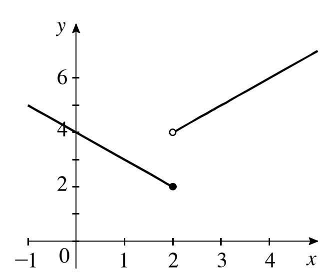
23. $\lim_{x\to\infty}\dfrac{3x^2+2x+4}{2x^2-3x+1}=\lim_{x\to\infty}\dfrac{3+\dfrac{2}{x}+\dfrac{4}{x^2}}{2-\dfrac{3}{x}+\dfrac{1}{x^2}}=\dfrac{3}{2}.$

24. $\lim_{x\to-\infty}\dfrac{x^2}{x+1}=\lim_{x\to-\infty}\left(x\cdot\dfrac{1}{1+\dfrac{1}{x}}\right)=-\infty$, so the limit does not exist.

25. $\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(-x+3)=-2+3=1$ and
$\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}(2x-3)=2(2)-3=4-3=1.$
Therefore, $\lim_{x\to 2}f(x)=1.$



26. $\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(x+2)=4$ and
$\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}(4-x)=2$. Therefore, $\lim_{x\to 2}f(x)$ does not exist.



27. The function is discontinuous at $x=2$.

28. Because the denominator $4x^2-2x-2=2\left(2x^2-x-1\right)=2(2x+1)(x-1)=0$ if $x=-\frac{1}{2}$ or 1, we see that f is discontinuous at these points.

29. Because $\lim\limits_{x \to -1} f(x) = \lim\limits_{x \to -1} \dfrac{1}{(x+1)^2} = \infty$ (does not exist), we see that f is discontinuous at $x = -1$.

30. The function is discontinuous at $x = 0$.

31. a. Let $f(x) = x^2 + 2$. Then the average rate of change of y over $[1, 2]$ is $\dfrac{f(2) - f(1)}{2-1} = \dfrac{(4+2) - (1+2)}{1} = 3$.
Over $[1, 1.5]$, it is $\dfrac{f(1.5) - f(1)}{1.5 - 1} = \dfrac{(2.25+2) - (1+2)}{0.5} = 2.5$. Over $[1, 1.1]$, it is
$\dfrac{f(1.1) - f(1)}{1.1 - 1} = \dfrac{(1.21+2) - (1+2)}{0.1} = 2.1$.

b. Computing $f'(x)$ using the four-step process., we obtain
$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{h(2x+h)}{h} = \lim\limits_{h \to 0} (2x+h) = 2$. Therefore, the instantaneous rate of change of f at $x = 1$ is $f'(1) = 2$, or 2 units per unit change in x.

32. $f(x) = 4x + 5$. We use the four-step process:
Step 1 $f(x+h) = 4(x+h) + 5 = 4x + 4h + 5$.
Step 2 $f(x+h) - f(x) = 4x + 4h + 5 - 4x - 5 = 4h$.
Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4h}{h} = 4$.
Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (4) = 4$.

33. $f(x) = \frac{3}{2}x + 5$. We use the four-step process:
Step 1 $f(x+h) = \frac{3}{2}(x+h) + 5 = \frac{3}{2}x + \frac{3}{2}h + 5$.
Step 2 $f(x+h) - f(x) = \frac{3}{2}x + \frac{3}{2}h + 5 - \frac{3}{2}x - 5 = \frac{3}{2}h$.
Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3}{2}$.
Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{3}{2} = \dfrac{3}{2}$.
Therefore, the slope of the tangent line to the graph of the function f at the point $(-2, 2)$ is $\frac{3}{2}$. To find the equation of the tangent line to the curve at the point $(-2, 2)$, we use the point-slope form of the equation of a line, obtaining $y - 2 = \frac{3}{2}[x - (-2)]$ or $y = \frac{3}{2}x + 5$.

34. $f(x) = -x^2$. We use the four-step process:
Step 1 $f(x+h) = -(x+h)^2 = -x^2 - 2xh - h^2$.
Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2\right) - \left(-x^2\right) = -2xh - h^2 = h(-2x - h)$.
Step 3 $\dfrac{f(x+h) - f(x)}{h} = -2x - h$.
Step 4 $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} (-2x - h) = -2x$.
The slope of the tangent line is $f'(2) = -2(2) = -4$. An equation of the tangent line is $y - (-4) = -4(x - 2)$, or $y = -4x + 4$.

35. $f(x) = -\frac{1}{x}$. We use the four-step process:

Step 1 $f(x+h) = -\frac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\frac{1}{x+h} - \left(-\frac{1}{x}\right) = -\frac{1}{x+h} + \frac{1}{x} = \frac{h}{x(x+h)}$.

Step 3 $\frac{f(x+h) - f(x)}{h} = \frac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}$.

36. a. f is continuous at $x = a$ because the three conditions for continuity are satisfied at $x = a$; that is, 1. $f(x)$ is defined. 2. $\lim_{x \to a} f(x)$ exists. 3. $\lim_{x \to a} f(x) = f(a)$.

b. f is not differentiable at $x = a$ because the graph of f has a kink at $x = a$.

37. $S(4) = 6000(4) + 30{,}000 = 54{,}000$.

38. a. The line passes through $(0, 2.4)$ and $(5, 7.4)$ and has slope $m = \frac{7.4 - 2.4}{5 - 0} = 1$. Letting y denote the sales, we see that an equation of the line is $y - 2.4 = 1(t - 0)$, or $y = t + 2.4$. We can also write this in the form $S(t) = t + 2.4$.

b. The sales in 2011 are $S(3) = 3 + 2.4 = 5.4$, or \$5.4 million.

39. a. $C(x) = 6x + 30{,}000$.

b. $R(x) = 10x$.

c. $P(x) = R(x) - C(x) = 10x - (6x + 30{,}000) = 4x - 30{,}000$.

d. $P(6000) = 4(6000) - 30{,}000 = -6000$, or a loss of \$6000. $P(8000) = 4(8000) - 30{,}000 = 2000$, or a profit of \$2000. $P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, or a profit of \$18,000.

40. Substituting the first equation into the second yields $3x - 2\left(\frac{3}{4}x + 6\right) + 3 = 0$, so $\frac{3}{2}x - 12 + 3 = 0$ and $x = 6$. Substituting this value of x into the first equation then gives $y = \frac{21}{2}$, so the point of intersection is $\left(6, \frac{21}{2}\right)$.

41. The profit function is given by $P(x) = R(x) - C(x) = 20x - (12x + 20{,}000) = 8x - 20{,}000$.

42. We solve the system $\begin{cases} 3x + p - 40 = 0 \\ 2x - p + 10 = 0 \end{cases}$ Adding these two equations, we obtain $5x - 30 = 0$, or $x = 6$. Thus, $p = 2x + 10 = 12 + 10 = 22$. Therefore, the equilibrium quantity is 6000 and the equilibrium price is \$22.

43. The child should receive $D(35) = \frac{500(35)}{150} \approx 117$, or approximately 117 mg.

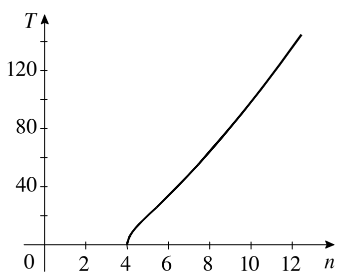
44. When 1000 units are produced, $R(1000) = -0.1(1000)^2 + 500(1000) = 400{,}000$, or \$400,000.

45. $R(30) = -\frac{1}{2}(30)^2 + 30(30) = 450$, or \$45,000.

46. $N(0) = 200(4 + 0)^{1/2} = 400$, and so there are 400 members initially. $N(12) = 200(4 + 12)^{1/2} = 800$, and so there are 800 members after one year.

47. The population will increase by $P(9) - P(0) = \left[50{,}000 + 30(9)^{3/2} + 20(9)\right] - 50{,}000$, or 990, during the next 9 months. The population will increase by $P(16) - P(0) = \left[50{,}000 + 30(16)^{3/2} + 20(16)\right] - 50{,}000$, or 2240 during the next 16 months.

48. $T = f(n) = 4n\sqrt{n-4}$. $f(4) = 0$, $f(5) = 20\sqrt{1} = 20$, $f(6) = 24\sqrt{2} \approx 33.9$, $f(7) = 28\sqrt{3} \approx 48.5$, $f(8) = 32\sqrt{4} = 64$, $f(9) = 36\sqrt{5} \approx 80.5$, $f(10) = 40\sqrt{6} \approx 98$, $f(11) = 44\sqrt{7} \approx 116$, and $f(12) = 48\sqrt{8} \approx 135.8$.



49. We need to find the point of intersection of the two straight lines representing the given linear functions. We solve the equation $2.3 + 0.4t = 1.2 + 0.6t$, obtaining $1.1 = 0.2t$ and thus $t = 5.5$. This tells us that the annual sales of the Cambridge Drug Store first surpasses that of the Crimson Drug store $5\frac{1}{2}$ years from now.

50. We solve $-1.1x^2 + 1.5x + 40 = 0.1x^2 + 0.5x + 15$, obtaining $1.2x^2 - x - 25 = 0$, $12x^2 - 10x - 250 = 0$, $6x^2 - 5x - 125 = 0$, and $(x-5)(6x+25) = 0$. Therefore, $x = 5$. Substituting this value of x into the second supply equation, we have $p = 0.1(5)^2 + 0.5(5) + 15 = 20$. So the equilibrium quantity is 5000 and the equilibrium price is \$20.

51. The life expectancy of a female whose current age is 65 is $C(65) = 0.0053694(65)^2 - 1.4663(65) + 92.74 \approx 20.12$ (years). The life expectancy of a female whose current age is 75 is $C(75) = 0.0053694(75)^2 - 1.4663(75) + 92.74 \approx 12.97$ (years).

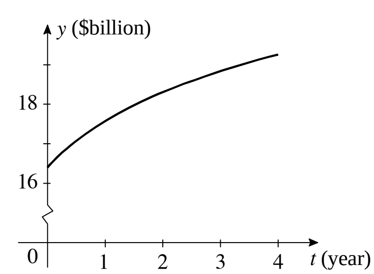
52. **a.** The amount of Medicare benefits paid out in 2010 is $B(0) = 0.25$, or \$250 billion.

b. The amount of Medicare benefits projected to be paid out in 2040 is $B(3) = 0.09(3)^2 + (0.102)(3) + 0.25 = 1.366$, or \$1.366 trillion.

53. $N(0) = 648$, or 648,000, $N(1) = -35.8 + 202 + 87.7 + 648 \approx 902$ or 902,000, $N(2) = -35.8(2)^3 + 202(2)^2 + 87.8(2) + 648 = 1345.2$ or 1,345,200, and $N(3) = -35.8(3)^3 + 202(3)^2 + 87.8(3) + 648 = 1762.8$ or 1,762,800.

54. **a.** $A(0) = 16.4$, or \$16.4 billion; $A(1) = 16.4(1+1)^{0.1} \approx 17.58$, or \$17.58 billion; $A(2) = 16.4(2+1)^{0.1} \approx 18.30$, or \$18.3 billion; $A(3) = 16.4(3+1)^{0.1} \approx 18.84$, or \$18.84 billion; and $A(4) = 16.4(4+1)^{0.1} \approx 19.26$, or \$19.26 billion. The nutritional market grew over the years 1999 to 2003.

b.



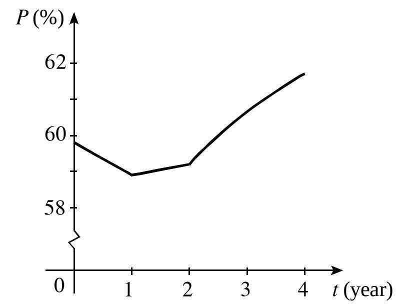
55. **a.** $f(t) = 267$; $g(t) = 2t^2 + 46t + 733$.

b. $h(t) = (f+g)(t) = f(t) + g(t) = 267 + \left(2t^2 + 46t + 733\right) = 2t^2 + 46t + 1000$.

c. $h(13) = 2(13)^2 + 46(13) + 1000 = 1936$, or 1936 tons.

56. **a.** $P(0) = 59.8$, $P(1) = 0.3(1) + 58.6 = 58.9$,
$P(2) = 56.79(2)^{0.06} \approx 59.2$, $P(3) = 56.79(3)^{0.06} \approx 60.7$, and
$P(4) = 56.79(4)^{0.06} \approx 61.7$.

b.



c. $P(3) \approx 60.7$, or 60.7%.

57. **a.** $f(r) = \pi r^2$.

b. $g(t) = 2t$.

c. $h(t) = (f \circ g)(t) = f(g(t)) = \pi[g(t)]^2 = 4\pi t^2$.

d. $h(30) = 4\pi(30^2) = 3600\pi$, or 3600π ft^2.

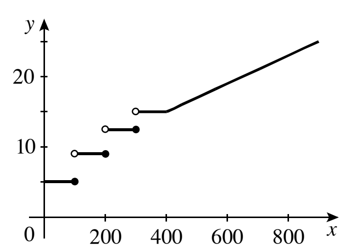
58. Measured in inches, the sides of the resulting box have length $20 - 2x$ and the height is x, so its volume is $V = x(20 - 2x)^2$ in^3.

59. Let h denote the height of the box. Then its volume is $V = (x)(2x)h = 30$, so that $h = \dfrac{15}{x^2}$. Thus, the cost is

$$\begin{aligned} C(x) &= 30(x)(2x) + 15[2xh + 2(2x)h] + 20(x)(2x) \\ &= 60x^2 + 15(6xh) + 40x^2 = 100x^2 + (15)(6)x\left(\frac{15}{x^2}\right) \\ &= 100x^2 + \frac{1350}{x}. \end{aligned}$$

60.

$$C(x) = \begin{cases} 5 & \text{if } 1 \le x \le 100 \\ 9 & \text{if } 100 < x \le 200 \\ 12.50 & \text{if } 200 < x \le 300 \\ 15.00 & \text{if } 300 < x \le 400 \\ 7 + 0.02x & \text{if } x > 400 \end{cases}$$



61. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty}\left(20 + \dfrac{400}{x}\right) = 20$. As the level of production increases without bound, the average cost of producing the commodity steadily decreases and approaches \$20 per unit.

62. **a.** $C'(x)$ gives the instantaneous rate of change of the total manufacturing cost c in dollars when x units of a certain product are produced.

b. Positive

c. Approximately \$20.

63. True. If $x < 0$, then $\sqrt{x}$ is not defined, and if $x > 0$, then $\sqrt{-x}$ is not defined. Therefore $f(x)$ is defined nowhere, and is not a function.

64. False. Let $f(x) = x^{1/3} + 1$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, so $f'(1) = \frac{1}{3}$ and an equation of the tangent line to the graph of f at the point $(1, 2)$ is $y - 2 = \frac{1}{3}(x - 1)$ or $y = \frac{1}{3}x + \frac{5}{3}$. This tangent line intersects the graph of f at the point $(-8, -1)$, as can be easily verified.

CHAPTER 2 Before Moving On... page 160

1. a. $f(-1) = -2(-1) + 1 = 3$.

b. $f(0) = 2$.

c. $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 2 = \frac{17}{4}$.

2. a. $(f + g)(x) = f(x) + g(x) = \dfrac{1}{x+1} + x^2 + 1$.

b. $(fg)(x) = f(x)\,g(x) = \dfrac{x^2+1}{x+1}$.

c. $(f \circ g)(x) = f(g(x)) = \dfrac{1}{g(x)+1} = \dfrac{1}{x^2+2}$.

d. $(g \circ f)(x) = g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{(x+1)^2} + 1$.

3. $4x + h = 108$, so $h = 108 - 4x$. The volume is $V = x^2h = x^2(108 - 4x) = 108x^2 - 4x^3$.

4. $\displaystyle\lim_{x\to -1} \frac{x^2+4x+3}{x^2+3x+2} = \lim_{x\to -1} \frac{(x+3)(x+1)}{(x+2)(x+1)} = 2$.

5. a. $\displaystyle\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(x^2 - 1\right) = 0$.

b. $\displaystyle\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^3 = 1$.

Because $\displaystyle\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$, f is not continuous at 1.

6. The slope of the tangent line at any point is

$$\begin{aligned}
\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) + 1 - \left(x^2 - 3x + 1\right)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x - 1}{h} \\
&= \lim_{h\to 0} \frac{h(2x + h - 3)}{h} = \lim_{h\to 0} (2x + h - 3) = 2x - 3.
\end{aligned}$$

Therefore, the slope at 1 is $2(1) - 3 = -1$. An equation of the tangent line is $y - (-1) = -1(x - 1)$, or $y + 1 = -x + 1$, or $y = -x$.

CHAPTER 2 Explore & Discuss

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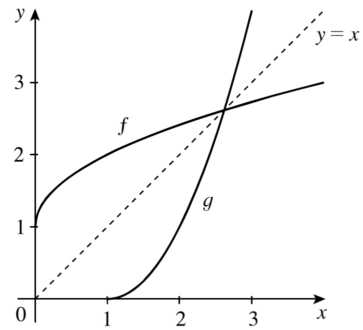
1. $(g \circ f)(x) = g(f(x)) = [f(x) - 1]^2 = [(\sqrt{x} + 1) - 1]^2 = (\sqrt{x})^2 = x$ and
$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} + 1 = \sqrt{(x-1)^2} + 1 = (x-1) + 1 = x$.

2. Refer to the figure at right. If a mirror is placed along the line $y = x$, then the graphs are reflections of each other.



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1. As x approaches 0 from either direction, $h(x)$ oscillates more and more rapidly between -1 and 1 and therefore cannot approach a specific number. But this says $\lim_{x \to 0} h(x)$ does not exist.

2. The function f fails to have a limit at $x = 0$ because $f(x)$ approaches 1 from the right but -1 from the left. The function g fails to have a limit at $x = 0$ because $g(x)$ is unbounded on either side of $x = 0$. The function h here does not approach any number from either the right or the left and has no limit at 0, as explained earlier.

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1. $\lim_{x \to \infty} f(x)$ does not exist because no matter how large x is, $f(x)$ takes on values between -1 and 1. In other words, $f(x)$ does not approach a definite number as x approaches infinity. Similarly, $\lim_{x \to -\infty} f(x)$ fails to exist.

2. The function of Example 10 fails to have a limit at infinity (negative infinity) because $f(x)$ increases (decreases) without bound as x approaches infinity (negative infinity). On the other hand, the function whose graph is depicted here, though bounded (its values lie between -1 and 1), does not approach any specific number as x increases (decreases) without bound and this is the reason it fails to have a limit at infinity or negative infinity.

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The average rate of change of a function f is measured over an interval. Thus, the average rate of change of f over the interval $[a, b]$ is the number $\dfrac{f(b) - f(a)}{b - a}$. On the other hand, the instantaneous rate of change of a function measures the rate of change of the function at a point. As we have seen, this quantity can be found by taking the limit of an appropriate difference quotient. Specifically, the instantaneous rate of change of f at $x = a$ is $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

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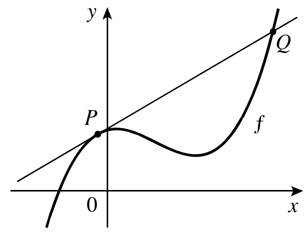
Yes. Here the line tangent to the graph of f at P also intersects the graph at the point Q lying on the graph of f.



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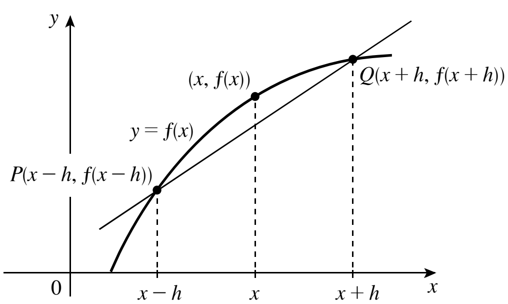
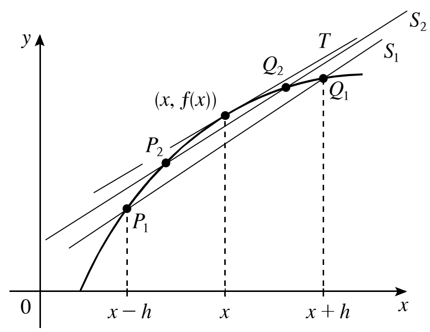
1. The quotient gives the slope of the secant line passing through $P\,(x-h,\ f\,(x-h))$ and $Q\,(x+h,\ f\,(x+h))$. It also gives the average rate of change of f over the interval $[x-h, x+h]$.



2. The limit gives the slope of the tangent line to the graph of f at the point $(x,\ f\,(x))$. It also gives the instantaneous rate of change of f at the point $(x,\ f\,(x))$. As h gets smaller and smaller, the secant lines approach the tangent line T.



3. The observation in part (b) suggests that this definition makes sense. We can also justify this observation as follows: From the definition of $f'\,(x)$, we have $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x)}{h}$. Replacing h by $-h$ gives $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x-h) - f\,(x)}{-h} = \lim_{h\to 0} \dfrac{f\,(x) - f\,(x-h)}{h}$. Thus, $2f'\,(x) = \lim_{h\to 0} \left[\dfrac{f\,(x+h) - f\,(x)}{h} + \dfrac{f\,(x) - f\,(x-h)}{h}\right]$, and so $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x-h)}{2h}$, in agreement with the result of Example 3.

4. **Step 1** Compute $f\,(x+h)$ and $f\,(x-h)$.
   Step 2 Form the difference $f\,(x+h) - f\,(x-h)$.
   Step 3 Form the quotient $\dfrac{f\,(x+h) - f\,(x-h)}{2h}$.
   Step 4 Compute $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x-h)}{2h}$.
   For the function $f\,(x) = x^2$, we have the following:
   Step 1 $f\,(x+h) = (x+h)^2 = x^2 + 2xh + h^2$ and $f\,(x-h) = (x-h)^2 = x^2 - 2xh + h^2$.
   Step 2 $f\,(x+h) - f\,(x-h) = \left(x^2 + 2xh + h^2\right) - \left(x^2 - 2xh + h^2\right) = 4xh$.
   Step 3 $\dfrac{f\,(x+h) - f\,(x)}{2h} = \dfrac{4xh}{2h} = 2x$.
   Step 4 $f'\,(x) = \lim_{h\to 0} \dfrac{f\,(x+h) - f\,(x)}{2h} = \lim_{h\to 0} 2x = 2x$, in agreement with the result of Example 3.

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No. The slope of the tangent line to the graph of f at $(a, f(a))$ is defined by $f'(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$, and because the limit must be unique (see the definition of a limit), there is only one number $f'(a)$ giving the slope of the tangent line. Furthermore, since there can only be one straight line with a given slope, $f'(a)$, passing through a given point, $(a, f(a))$, our conclusion follows.

CHAPTER 2 Exploring with Technology

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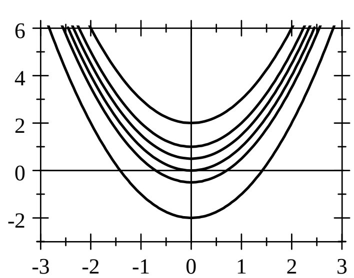
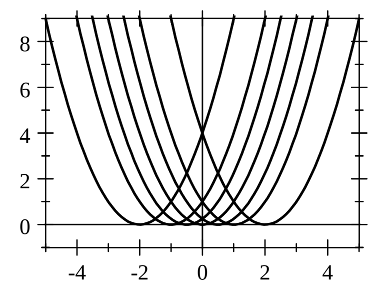
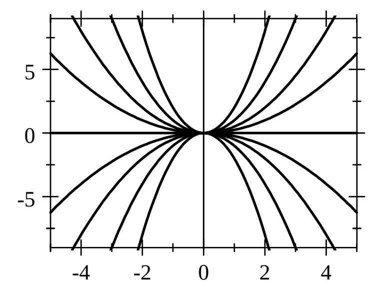
1.



2.



3.



4. The graph of $f(x) + c$ is obtained by translating the graph of f along the y-axis by c units. The graph of $f(x + c)$ is obtained by translating the graph of f along the x-axis by c units. Finally, the graph of cf is obtained from that of f by "expanding"(if $c > 1$) or "contracting"(if $0 < c < 1$) that of f. If $c < 0$, the graph of cf is obtained from that of f by reflecting it with respect to the x-axis as well as expanding or contracting it.

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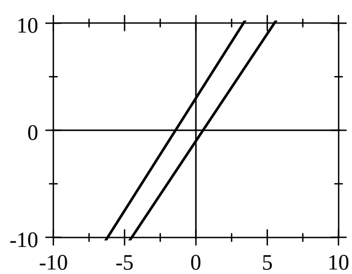
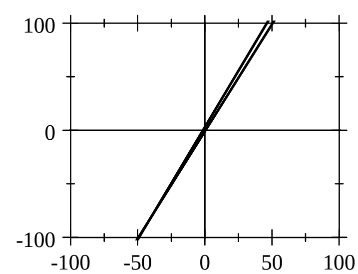
1. a.



The lines seem to be parallel to each other and do not appear to intersect.

b.

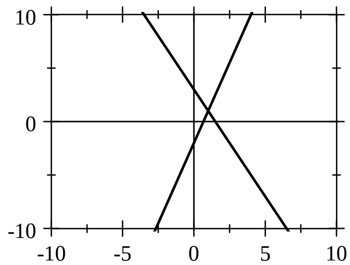


They appear to intersect. But finding the point of intersection using TRACE and ZOOM with any degree of accuracy seems to be an impossible task. Using the intersection feature of the graphing utility yields the point of intersection $(-40, -81)$ immediately.

c. Substituting the first equation into the second gives $2x - 1 = 2.1x + 3$, $-4 = 0.1x$, and thus $x = -40$. The corresponding y-value is -81.

d. Using TRACE and ZOOM is not effective. The intersection feature gives the desired result immediately. The algebraic method also yields the answer with little effort and without the use of a graphing utility.

2. a.



Plotting the straight lines L_1 and L_2 and using TRACE and ZOOM repeatedly, you will see that the iterations approach the answer $(1, 1)$. Using the intersection feature of the graphing utility gives the result $x = 1$ and $y = 1$, immediately.

b. Substituting the first equation into the second yields $3x - 2 = -2x + 3$, so $5x = 5$ and $x = 1$. Substituting this value of x into either equation gives $y = 1$.

c. The iterations obtained using TRACE and ZOOM converge to the solution $(1, 1)$. The use of the intersection feature is clearly superior to the first method. The algebraic method also yields the desired result easily.

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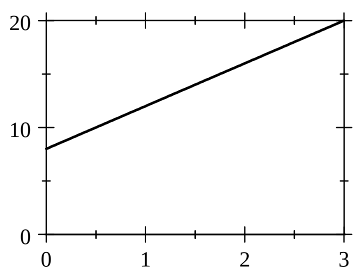
1.



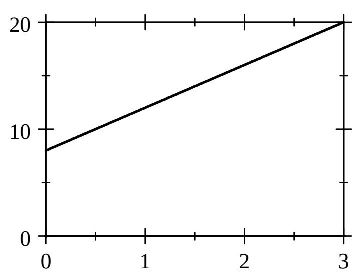
2. Using TRACE and ZOOM repeatedly, we find that $g(x)$ approaches 16 as x approaches 2.

3. If we try to use the evaluation function of the graphing utility to find $g(2)$ it will fail. This is because $x = 2$ is not in the domain of g.

4. The results obtained here confirm those obtained in the preceding example.

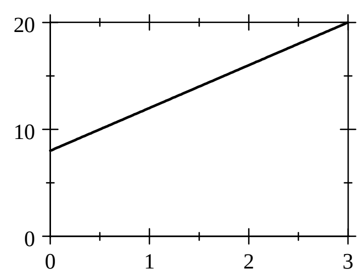
Page 109 (First Box)

1.



Using TRACE, we find $\lim_{x \to 2} \dfrac{4(x^2 - 4)}{x - 2} = 16$.

2.



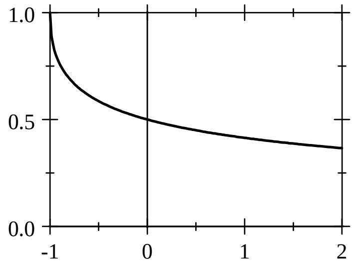
Using TRACE, we find $\lim_{x \to 2} 4(x + 2) = 16$. When $x = 2$, $y = 16$. The function $f(x) = 4(x + 2)$ is defined at $x = 2$ and so $f(2) = 16$ is defined.

3. No.

4. As we saw in Example 5, the function f is not defined at $x = 2$, but g is defined there.

Page 109 (Second Box)

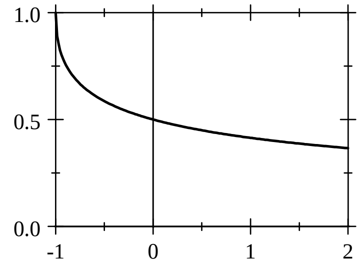
1.



Using TRACE and ZOOM, we see that

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=0.5.$$

2.



The graph of f is the same as that of g except that the domain of f includes $x=0$. (This is not evident from simply looking at the graphs!) Using the evaluation function to find the value of y, we obtain $y=0.5$ when $x=0$. This is to be expected since $x=0$ lies in the domain of g.

3. As mentioned in part 2, the graphs are indistinguishable even though $x=0$ is in the domain of g but not in the domain of f.

4. The functions f and g are the same everywhere except at $x=0$ and so $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1}{x}=\lim\limits_{x\to 0}\dfrac{1}{\sqrt{1+x}+1}=\dfrac{1}{2}$, as seen in Example 6.

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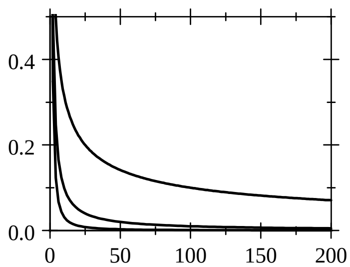
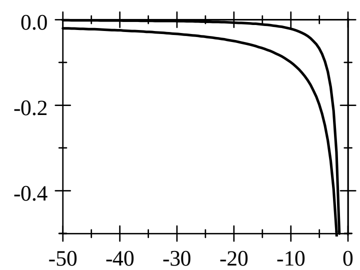
1.



The results suggest that $\dfrac{1}{x^n}$ goes to zero (as x increases) with increasing rapidity as n gets larger, as predicted by Theorem 2.

2.



The results suggest that $\dfrac{1}{x^n}$ goes to zero (as negative x increases in absolute value) with increasing rapidity as n gets larger, as predicted by Theorem 2.

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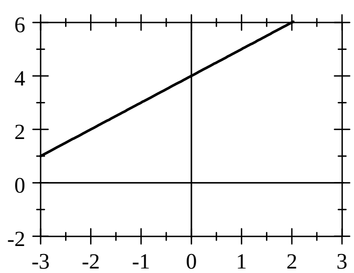
1.



2. Using ZOOM repeatedly, we find $\lim\limits_{x\to 0} g(x)=4$.

3. The fact that the limit found in part 2 is $f'(2)$ is an illustration of the definition of a derivative.

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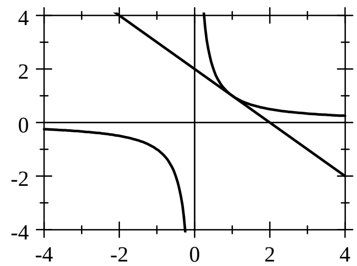

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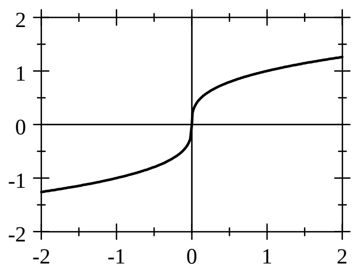
1.



2. The graphing utility will indicate an error when you try to draw the tangent line to the graph of f at $(0, 0)$. This happens because the slope of the tangent line to the graph of $f(x)$ is not defined at $x = 0$.